

31 Number squares (games)

Start with a square, with a natural number at each corner. Now write, at the midpoint of each edge, the absolute value $|a - b|$ of the difference of numbers (a, b) at the endpoints of that edge. Draw a smaller square connecting those midpoints, and repeat. The game stops if you get to $(0, 0, 0, 0)$. For instance,

$$(1, 3, 9, 7) \rightarrow (2, 6, 2, 6) \rightarrow (4, 4, 4, 4) \rightarrow (0, 0, 0, 0).$$

For a longer game, try $(44, 24, 13, 7)$.

Are there games of infinite length, or of arbitrarily long finite length? Given a bound on the size of the initial numbers, what can one say about the maximum length of the game?

One can also play this game with real numbers, for instance $(1, 2, 3, \pi)$. In that case, in addition to the questions raised above, the probabilistic aspect (taking random starting numbers) becomes more interesting.

Of course, there are many ways to vary the rules of the game...

MIT OpenCourseWare
<http://ocw.mit.edu>

18.821 Project Laboratory in Mathematics
Spring 2013

For information about citing these materials or our Terms of Use, visit: <http://ocw.mit.edu/terms>.