MIT 18.996: Topic in TCS: Internet Research Problems **Spring 2002**

Lecture 11 — April 24, 2002

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11.1 Open Loop Congestion Control

From Powerpoint slides.

11.1.1 Erlang Loss Model

Assume that we have k channels and that calls arrive at an arrival rate λ and each call departs at a rate $\mu = 1$. We can then construct a Markov model for the process as shown in Figure 11.1.

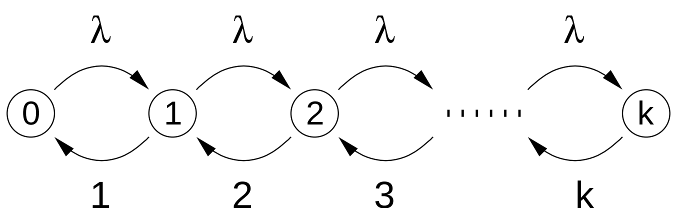

Figure 11.1. Erlang Loss Model

The balance equations for the model are as follows:

$$\begin{aligned}
\lambda \Pi_0 &= \Pi_1 \\
\lambda \Pi_1 &= \Pi_2 \\
&\vdots \\
\lambda \Pi_{k-1} &= \Pi_k \\
\sum_{i=1}^{k} \Pi_i &= 1
\end{aligned}$$

The solution to these balance equations yields:

$$\Pi_k = \frac{\frac{\lambda^k}{k!}}{\sum_{i=1}^{k} \frac{\lambda^i}{i!}}$$

Hence, we will accept up to n connections where

$$\frac{\frac{(\lambda n)^k}{k!}}{\sum_{i=1}^{k} \frac{(\lambda n)^i}{i!}} < \rho_\epsilon$$

Suppose $\frac{(\lambda n)}{k} \to \rho^* > 1$, then

$$\begin{aligned} P[X=k] &= \left(\frac{1}{\rho^*}\right)\left(1-\frac{1}{\rho^*}\right)^k \\ \Pi_k &\approx \left(\frac{1}{\rho^*}\right)\left(1-\frac{1}{\rho^*}\right)^k < \rho_\epsilon \end{aligned}$$

We can obtain n by solving

$$\left(1-\frac{k}{\lambda n}\right)^k \left(\frac{k}{\lambda n}\right) < \rho_\epsilon$$

The main difference between circuit switching and packet switching is that the former has fixed capacity channels and a finite number of channels.

11.1.2 Small Buffer Model

In the small buffer model, we have n sources $X_i, 1 \le i \le n$. Loss occurs when $\sum_{i=1}^n X_i(t) > C$. We assume that $X_i(t)$ are independent identically distributed random variables and we want to find n such that

$$P[\sum_{i=1}^n X_i(t) > C] < \rho_\epsilon$$

Although we often apply the Central Limit Theorem: $\sum_{i=1}^n X_i \sim N(n\mu, n\sigma^2)$ when we wish to approximate $P[\sum_{i=1}^n X_i > \mu\bar{X} + c_n]$, this approximation does not work well when we are working in the tail region. Hence we use the Chernoff Bound.

Assume that $M_x(s) = \log E(e^{sX})$ exists. Then for any random variable Y,

$$\begin{aligned} P[Y \ge 0] &= P[e^{sX} \ge 1] \\ &\le E[e^{sX}] \quad (\forall\ s \ge 0) \\ P[\sum_{i=1}^n X_i \ge 0] &\le E[e^{s\sum X_i}] \\ &= E[e^{sX_i}]^n \\ \frac{1}{n}\log P[\sum_{i=1}^n X_i \ge 0] &\le M_x(s) \\ \log P[\sum_{i=1}^n X_i \ge 0] &= \log E[e^{s(\sum X_i - C)}] \\ &\le nM_x(s) - sC \end{aligned}$$

Therefore, to obtain n, we let $nM_x(s) - sC \le \rho_\epsilon$, and find s which minimizes the RHS of:

$$n \le \frac{\log \rho_\epsilon + sC}{M_x(s)}$$

11.1.3 Large Buffer Model

In this model, the input is given by $A = A(t) : t \geq 0$. Suppose $\psi(s)$ exists, where

$$\psi(s) = \lim_{t\to\infty} \frac{1}{t} \log E[e^{sA(t)}]$$

and the system serves work at rate M.

Theorem 11.1 (Glynn & Whitt - 1994). *Where s^* is the root of $\psi(s) = sM$,*

$$\frac{1}{\lambda} \log P[W > x] \to -s^*$$

A good approximation is given by:

$$P[W > x] \approx Ce^{-s^* x}$$

Where there are k sources, each with the cumulative moment generating functions $\psi_i(s)$,

$$\psi(s) = \sum_{k=1}^{n} n_k \psi_k(s)$$

To keep the loss probability less than $e^{-\delta x}$, we let

$$\begin{aligned} \psi(s) - \delta M &\leq 0 \\ \Rightarrow \sum_{k=1}^{n} n_k \frac{\psi_k(s)}{\delta} &\leq M \end{aligned}$$