Open Problem 5.2 *Theorem 5.14 guarantees that if we take A to have i.i.d. Gaussian entries then it should be RIP for $s \approx \frac{M}{\log(N)}$. If we were able to, given A, certify that it indeed is RIP for some s then one could have a randomized algorithm to build RIP matrices (but that is guaranteed to eventually find one). This motives the following question*

1. *Let $N = 2M$, for which s is there a polynomial time algorithm that is guaranteed to, with high probability, certify that a gaussian matrix A is $\left(s, \frac{1}{3}\right)$-RIP?*
2. *In particular, a $\left(s, \frac{1}{3}\right)$-RIP matrix has to not have s sparse vectors in its nullspace. This motivates a second question: Let $N = 2M$, for which s is there a polynomial time algorithm that is guaranteed to, with high probability, certify that a gaussian matrix A does not have s-sparse vectors in its nullspace?*

The second question is tightly connected to the question of sparsest vector on a subspace (for which $s \approx \sqrt{M}$ is the best known answer), we refer the reader to [SWW12, QSW14, BKS13b] for more on this problem and recent advances. Note that checking whether a matrix has RIP or not is, in general, NP-hard [BDMS13, TP13].

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