6.4 Partial Fourier matrices satisfying the Restricted Isometry Property

While the results above are encouraging, rarely one has the capability of designing random gaussian measurements. A more realistic measurement design is to use rows of the Discrete Fourier Transform: Consider the random $M \times N$ matrix obtained by drawing rows uniformly with replacement from the $N \times N$ discrete Fourier transform matrix. It is known [CT06] that if $M = \Omega_\delta(K \operatorname{polylog} N)$, then the resulting partial Fourier matrix satisfies the restricted isometry property with high probability.

A fundamental problem in compressed sensing is determining the order of the smallest number M of random rows necessary. To summarize the progress to date, Candès and Tao [CT06] first found that $M = \Omega_\delta(K \log^6 N)$ rows suffice, then Rudelson and Vershynin [RV08] proved $M = \Omega_\delta(K \log^4 N)$, and more recently, Bourgain [Bou14] achieved $M = \Omega_\delta(K \log^3 N)$; Nelson, Price and Wootters [NPW14] also achieved $M = \Omega_\delta(K \log^3 N)$, but using a slightly different measurement matrix. The latest result is due to Haviv and Regev [HR] giving an upper bound of $M = \Omega_\delta(K \log^2 k \log N)$. As far as lower bounds, in [BLM15] it was shown that $M = \Omega_\delta(K \log N)$ is necessary. This draws a contrast with random Gaussian matrices, where $M = \Omega_\delta(K \log(N/K))$ is known to suffice.

Open Problem 6.1 *Consider the random $M \times N$ matrix obtained by drawing rows uniformly with replacement from the $N \times N$ discrete Fourier transform matrix. How large does M need to be so that, with high probability, the result matrix satisfies the Restricted Isometry Property (for constant δ)?*

References

[BLM15] A. S. Bandeira, M. E. Lewis, and D. G. Mixon. Discrete uncertainty principles and sparse signal processing. *Available online at arXiv:1504.01014 [cs.IT]*, 2015.

[Bou14] J. Bourgain. An improved estimate in the restricted isometry problem. *Lect. Notes Math.*, 2116:65–70, 2014.

[CT06] E. J. Candès and T. Tao. Near optimal signal recovery from random projections: universal encoding strategies? *IEEE Trans. Inform. Theory*, 52:5406–5425, 2006.

[HR] I. Haviv and O. Regev. The restricted isometry property of subsampled fourier matrices. *SODA 2016*.

[NPW14] J. Nelson, E. Price, and M. Wootters. New constructions of RIP matrices with fast multiplication and fewer rows. *SODA*, pages 1515–1528, 2014.

[RV08] M. Rudelson and R. Vershynin. On sparse reconstruction from Fourier and Gaussian measurements. *Comm. Pure Appl. Math.*, 61:1025–1045, 2008.

18.S096 Topics in Mathematics of Data Science
Fall 2015