8.6 An interesting conjecture regarding cuts and bisections

Given d and n let $G^{reg}(n,d)$ be a random d-regular graph on n nodes, drawn from the uniform distribution on all such graphs. An interesting question is to understand the typical value of the Max-Cut such a graph. The next open problem is going to involve a similar quantity, the Maximum Bisection. Let n be even, the Maximum Bisection of a graph G on n nodes is

$$\mathrm{MaxBis}(G) = \max_{S\colon |S|=\frac{n}{2}} \mathrm{cut}(S),$$

and the related Minimum Bisection (which will play an important role in next lectures), is given by

$$\mathrm{MinBis}(G) = \min_{S\colon |S|=\frac{n}{2}} \mathrm{cut}(S),$$

A typical bisection will cut half the edges, meaning $\frac{d}{4}n$. It is not surprising that, for large n, $\mathrm{MaxBis}(G)$ and $\mathrm{MinBis}(G)$ will both fluctuate around this value, the amazing conjecture [ZB09] is that their fluctuations are the same.

Conjecture 8.5 ([ZB09]) *Let $G \sim G^{reg}(n,d)$, then for all d, as n grows*

$$\frac{1}{n}\left(\mathrm{MaxBis}(G) + \mathrm{MinBis}(G)\right) = \frac{d}{2} + o(1),$$

where $o(1)$ is a term that goes to zero with n.

Open Problem 8.5 *Prove or disprove Conjecture 8.5.*

Recently, it was shown that the conjecture holds up to $o(\sqrt{d})$ terms [DMS15]. We also point the reader to this paper [Lyo14], that contains bounds that are meaningful already for $d = 3$.

References

[DMS15] A. Dembo, A. Montanari, and S. Sen. Extremal cuts of sparse random graphs. *Available online at arXiv:1503.03923 [math.PR]*, 2015.

[ZB09] L. Zdeborova and S. Boettcher. Conjecture on the maximum cut and bisection width in random regular graphs. *Available online at arXiv:0912.4861 [cond-mat.dis-nn]*, 2009.