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2.002 MECHANICS AND MATERIALS II
EXAMPLE PROBLEM IN FRACTURE

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A spherical thin-walled pressure vessel is to be fabricated from formed plates. The capacity of the vessel is to be $14\,m^3$, and the fluid it contains will be under a peak internal pressure of $50\,MPa$. You may use the standard stress analysis of thin-walled spherical pressure vessels to relate stress to pressure and vessel geometry.

Three grades of mar-ageing steel are under consideration for constructing the vessel. The table gives the tensile yield strength (σ_y) and fracture toughness (K_{Ic}) of each grade. Each of the steels has mass density $\rho = 7.8 Mg/m^3$.

Grade	σ_y (MPa)	K_{Ic} (MPa $\sqrt{m}$)
200	1482	111
250	1690	94
300	2000	66

You are to specify the material and corresponding wall thickness so that the structure will

1. be **of least weight**, while

2. **retaining a factor of safety of at least** 2 **against both**

   (a) yielding and
   (b) brittle fracture.

In carrying out your calculations, you may assume that the critical defect is a small-scale circular surface crack of radius "a". The calibration for maximum stress intensity factor along a shallow semi-circular surface crack front in a large plate of thickness $t \gg a$, subject to tensile stress "σ" perpendicular to the crack plane is:

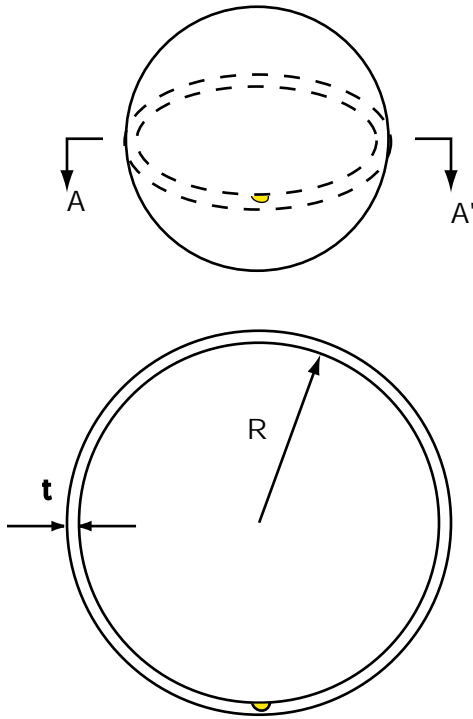$$K_{I(max)} = 0.617\sigma\sqrt{\pi a}$$

Figure 1: Schematic of spherical pressure vessel with small, semi-circular surface crack on inner surface.

Furthermore, assume that the non-destructive examination (NDE) technology available to inspect the fabricated vessel has a "reliable" resolution limit of $10\,mm$. (In this context, the resolution limit can be taken as the *surface length* of the assumed semi-circular surface crack.)

Suppose instead that enhanced NDE methods are available, with a reliable detection resolution of $5\,mm$. <u>Now</u> which material would you choose to optimize the vessel, and what wall thickness would you specify? Discuss.