MASSACHUSETTS INSTITUTE OF TECHNOLOGY
DEPARTMENT OF MECHANICAL ENGINEERING

2.004 *Dynamics and Control II*

Spring Term 2008

Lecture 1[1]

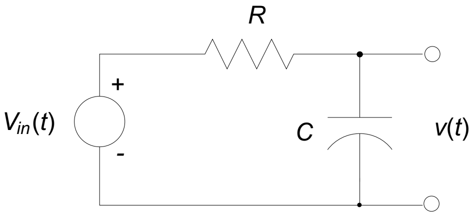
Reading:

- Nise: Chapter 1

1 Elements of the Course

(a) System Dynamics: *System dynamics* provides a unified approach to the modeling and dynamic behavior of linear systems in many energy domains. For example:

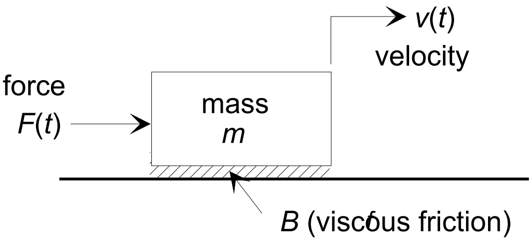
a) An electrical network:



Using KVL & KCL:

$$RC\frac{dv}{dt} + v = V_{in}(t)$$

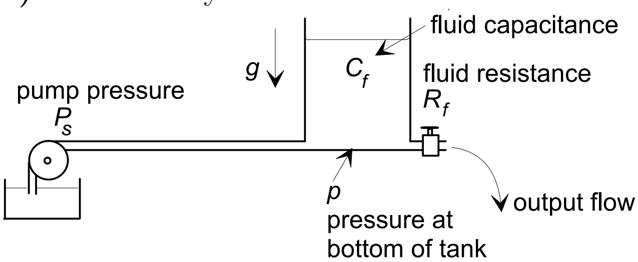
b) A simple mechanical system:



From a force balance:

$$\frac{m}{B}\frac{dv}{dt} + v = \frac{1}{B}F(t)$$

c) A fluidic system:



Using fluid junction equations

$$R_f C_f \frac{dp}{dt} + p = P(t)$$

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We recognize a common form to the ODE describing each system and create **analogs** in the various energy domains, for example

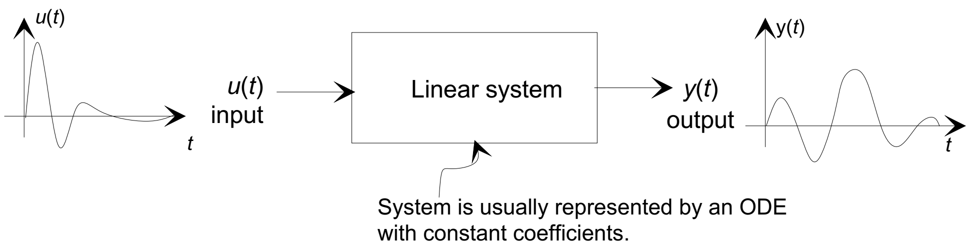
$$\left\{ \begin{array}{l} \text{voltage} \\ \text{velocity} \\ \text{pressure} \end{array} \right\} \text{ and } \left\{ \begin{array}{l} \text{current} \\ \text{force} \\ \text{volume flow rate} \end{array} \right\} \text{ in the } \left\{ \begin{array}{l} \text{electrical} \\ \text{mechanical} \\ \text{fluidic} \end{array} \right\} \text{ domains.}$$

Then for each system, we can write a differential equation:

$$\tau \frac{dy}{dt} + y = u(t)$$

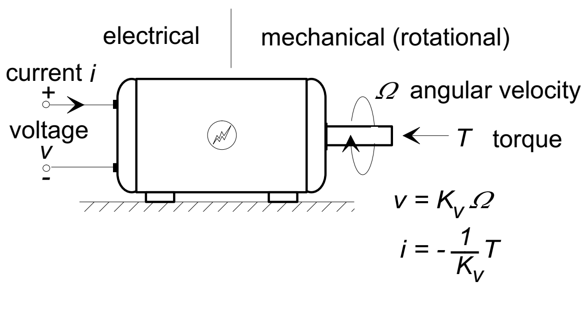
where $u(t)$ is the system **input**
$y(t)$ is the system **output**, and
τ is a system **parameter** (time-constant).

We will frequently use **block diagrams** to represent the input/output relationships of systems



usually represented by an ODE with constant coefficients.

In addition we will investigate **energy transducers** that convert power/energy from one domain to another, for example a motor converts power P from the electrical domain to the rotational domain.



A transducer is bi-directional, that is it can transmit power P in both directional, so that the motor can also act as a generator.

$$\begin{array}{c} \\ P = vi \\ \text{electrical} \end{array} \left\{ \begin{array}{c} \textbf{transduction} \\ \Rightarrow \text{motor} \Rightarrow \\ \Leftarrow \text{generator} \Leftarrow \end{array} \right\} \begin{array}{c} \\ P = T\Omega \\ \text{rotational} \end{array}$$

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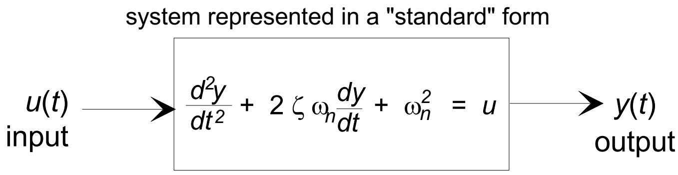
(b) Linear System Theory A generalized method for the description of the dynamic response of systems described by linear ordinary differential equations (ODEs) with constant coefficients without regard to the particular energy domain.



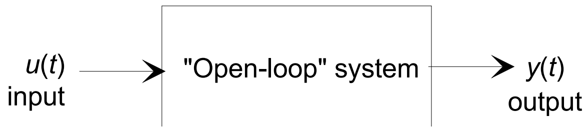
Typical questions we might ask about the system might be what is the response to:

(a) a sinusoidal input?

(b) a step input ?

(c) a short pulse ?

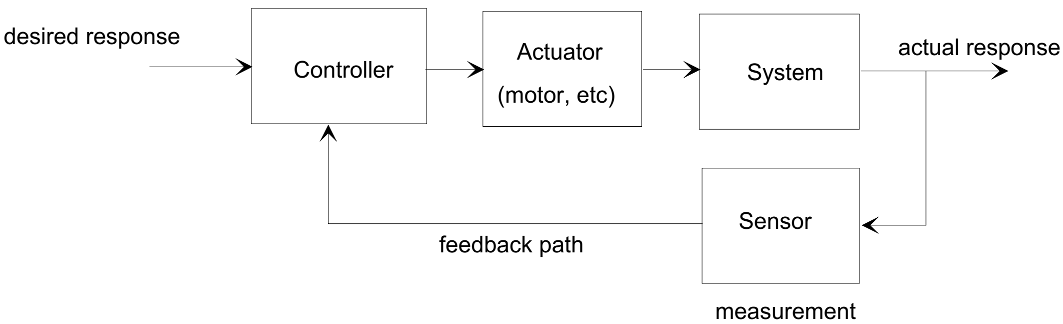
(c) Feedback Control Theory The use of **feedback** to modify the dynamic behavior of a system



Often the inherent system behavior is unsatisfactory, for example:

- The response might be **too slow**.
- The response might be **unstable**.
- The system might be **susceptible to external influences**.
- Components inside the system might **change their values as they age**, causing the system response to change.

Often control involves monitoring the system response - comparing the response to the desired behavior - and generating a system input so as to *drive the system toward the desired response.*



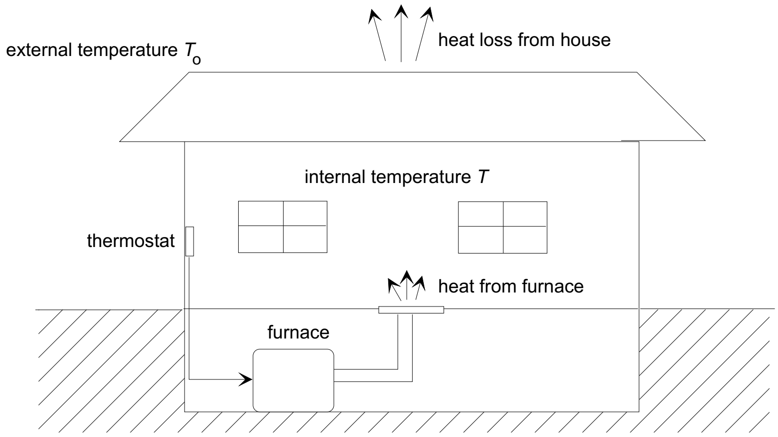
(1) monitor the response in real-time.

(2) compare the actual response with the desired response.

(3) decide how to modify the response by changing the input.

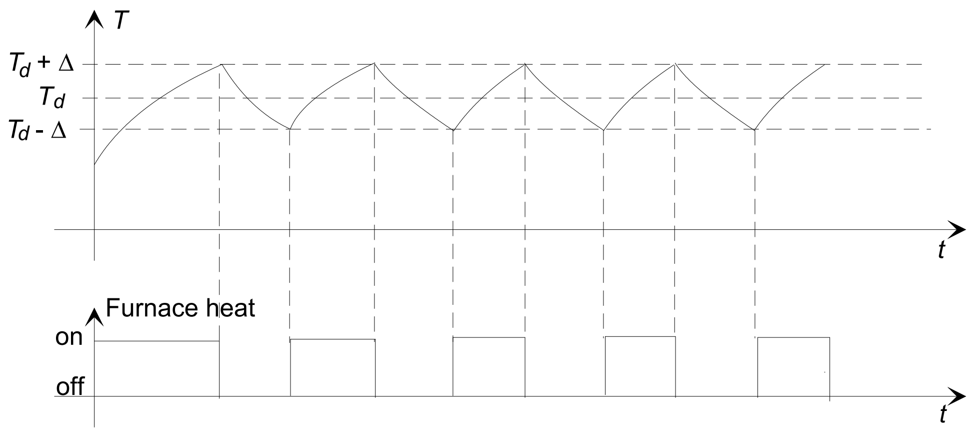
■ Example 1

Temperature control in a home:



- Desired behavior - maintain temperature at T
- Control law (algorithm) using a thermostat:
  (a) if $T > T_0 + \Delta$ - turn off furnace.
  (b) if $T < T_0 - \Delta$ - turn on furnace.
  (c) if $T_0 - \Delta \leq T \leq T_0 + \Delta$ - do nothing.

A typical response as the thermostat turns on and off might be:

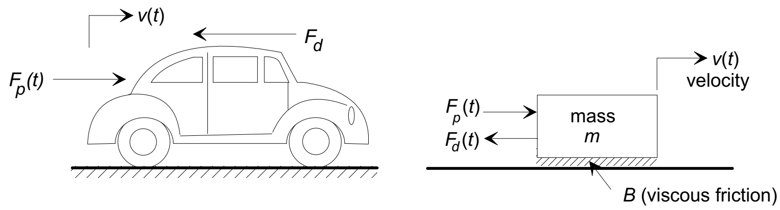


Notice that the temperature rises while the furnace is on, and falls while it is off. The thermostat acts to keep the temperature centered about the desired value T_d (known as the *set-point*).

■ Example 2

Cruise Control for a car:
Goals - maintain the speed of a car at a prescribed value in the presence of external disturbances **(external forces such as wind gusts, gravitational forces on a incline, etc)**.
Also - improve the dynamic response of the car as the driver "steps on the gas".

(a) Form a dynamic model of the car. Assume a simplified model

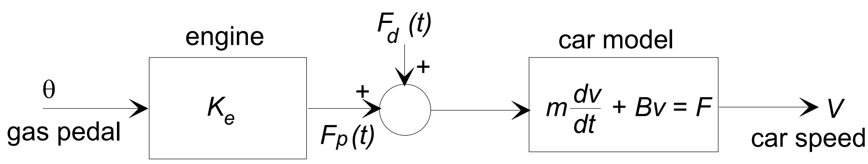


- Model the car as a simple lumped mass element m, sliding on a viscous friction element, $F_B = BV_B$ - simplification
- Assume two external forces
  $F_p(t)$ - the propulsive force $F_p(t)$ from the engine.
  $F_d(t)$ - a "disturbance" force $F_d(t)$ from the environment.
  Also assume $F_p(t) = K_e\theta(t)$ where $\theta(t)$ is the gas-pedal depression and K_e is a constant. Then from a simple force balance:

$$m\frac{dv}{dt} + Bv = F_p(t) + F_d(t)$$

$$\boxed{m\frac{dv}{dt} + Bv = K_e\theta(t) + F_d(t)}$$

and draw a block diagram



(b) Closed-loop Control:
Now design the controller. Assume we will use error-based control. In other words, given a desired speed v_d, and the measured car speed $v(t)$, we define the error $e(t)$ as

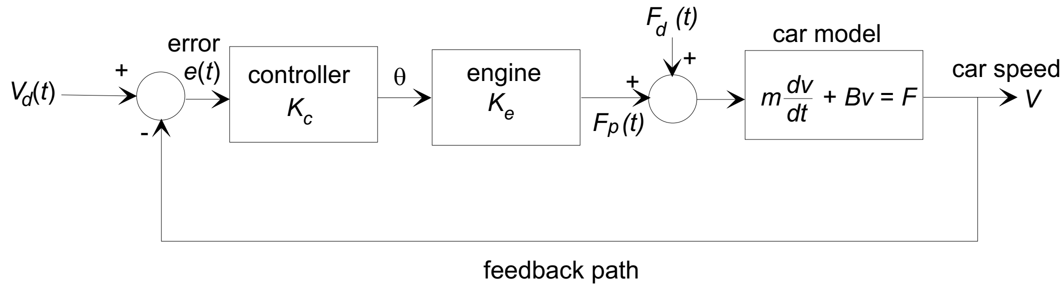
$$e(t) = v_d(t) - v(t)$$

and choose a control law that tells us to depress the gas pedal by an amount proportional to the error:

$$\theta(t) = K_c e(t) = K_c(v_d(t) - v(t)),$$

so that the propulsive force acting on the car is

$$F_p(t) = K_e K_p e(t) = K_e K_c(v_d(t) - v(t)),$$



This is known as *proportional control.*