2.004 Dynamics and Control II
Spring 2008

Massachusetts Institute of Technology
Department of Mechanical Engineering

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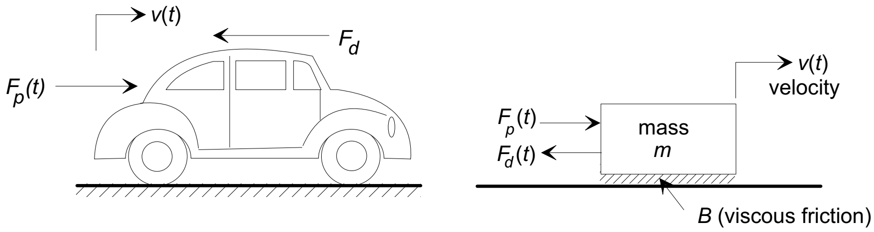
Lecture 2[1]

Reading:

- Nise: Chapter 1

1 Cruise Control Example (continued from Lecture 1)

From Lecture 1 the model for the car is:

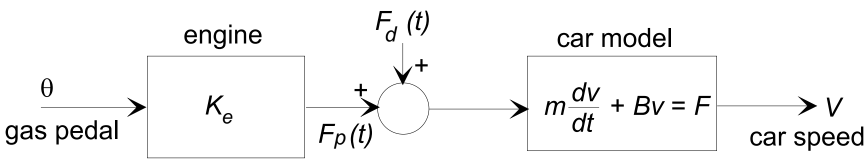


where the propulsion force $F_p(t)$ is proportional to the gas pedal depression θ:

$$F_p(t) = K\theta(t)$$

so that

$$\boxed{m\frac{dv}{dt} + Bv = K_e\theta(t) + F_d(t)}$$



The "Open-Loop"Dynamic Response: Let's examine how the car will respond to commands at the gas-pedal. Assume $F_d(t) = 0$, so that we can write the differential equation as

$$\boxed{\frac{m}{B}\dot{v} + v = \frac{K_e}{B}\theta}$$

and compare this to the standard form for a first-order ODE:

$$\tau\dot{y} + y = f(t)$$

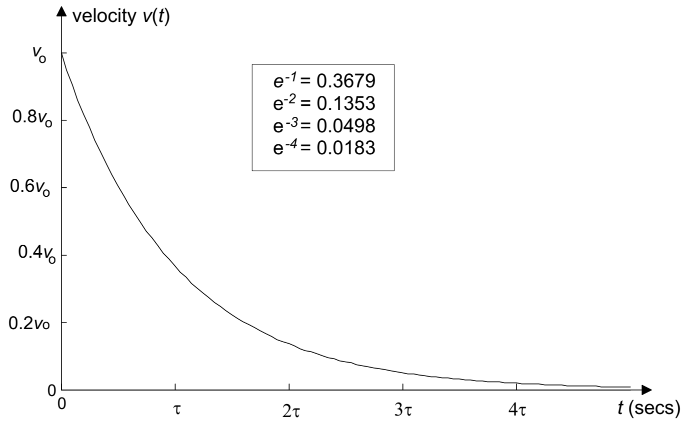
where the time-constant $\tau = m/B$, and the forcing function $f(t) = \frac{K_e}{B}\theta(t)$.

Consider

1) The "coast-down" response from an initial speed $v(0) = v_0$ with $\theta = 0$. The homogeneous response is

$$\boxed{v(t) = v_0 e^{-\frac{t}{\tau}} = v_0 e^{-\frac{B}{m}t}}$$

which has the form



and after a period 4τ we have $v < 0.02v_0$.

2) The response to a "step" in the command $\theta(t)$. Assume that the car is at rest, that is $v(0) = 0$, and we "step on the gas" so that $\theta(t) = \theta$. The differential equation is then

$$\frac{m}{B}\dot{v} + v = \frac{K_e}{B}\theta$$

with $v(0) = 0$.

(a) The steady-state (final) speed is

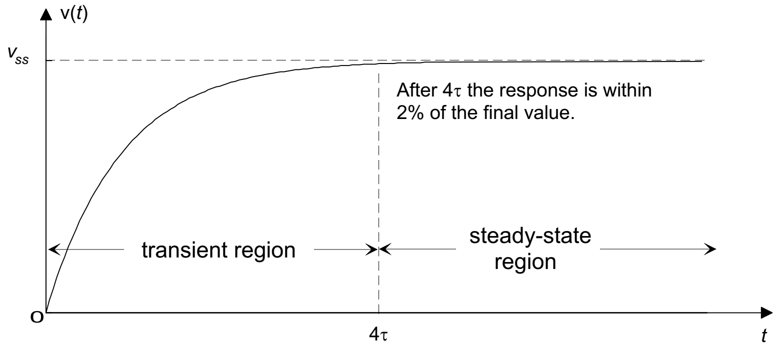
$$\boxed{v_{ss} = \frac{K_e}{B}\theta}$$

(found by letting all derivatives to be zero), and

(b) the differential equation may be solved to give the dynamic response:

$$\boxed{v(t) = v_{ss}(1 - e^{-\frac{t}{\tau}}) = \frac{K_e\theta}{B}(1 - e^{-\frac{B}{m}t})}$$

which is shown below:



2 Closed-Loop Control

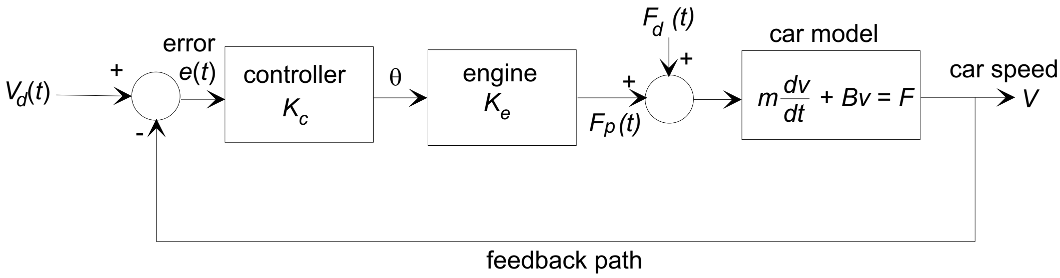
Now design the controller. Assume we will use **error-based** control. In other words, given a **desired speed for the car** v_d, and the **measured car speed** $v(t)$, we define the **error** $e(t)$ as the difference

$$e(t) = v_d(t) - v(t),$$

and choose a **control law** that is based on $e(t)$

$$\boxed{\theta(t) = K_c e(t) = K_c(v_d(t) - v(t))}$$

where K_c is the **controller gain**. Thus the control **effort** is proportional to the error, and the block diagram becomes:



Note that the control law states:

- if $v < v_d$ then $e > 0$ - depress gas pedal.
- if $v = v_d$ then $e = 0$ - set $\theta = 0$ (do nothing).
- if $v > v_d$ then $e < 0$ - set $\theta < 0$ (apply brakes).

The new differential equation is

$$m\frac{dv}{dt} + Bv = K_c K_e(v_d(t) - v) + F_d(t)$$

and rearranging

$$\boxed{m\frac{dv}{dt} + (B + K_c K_e)v = K_c K_e v_d(t) + F_d(t)}$$

which is the **closed-loop differential equation**. We can write this as

$$\boxed{\frac{m}{B + K_cK_e}\frac{dv}{dt} + v = \frac{K_cK_e}{B + K_cK_e}v_d(t) + \frac{1}{B + K_cK_e}F_d(t)}$$

and compare with the standard first-order form

$$\tau\dot{y} + y = u(t)$$

where $\tau = m/(B + K_cK_e)$ is the **closed-loop time-constant**.

The important thing to note is that **feedback has modified the ODE**.

3 Questions:

Assume there are no external disturbance forces, that is $F_d(t) \equiv 0$ for now,

(a) If we command the car to travel at a steady speed v_d, what speed will it actually reach?
To find the steady-state speed v_{ss}, set $\frac{dv}{dt} = 0$ and solve for v_{ss}, giving

$$v_{ss} = \frac{K_cK_e}{B + K_cK_e}v_d$$

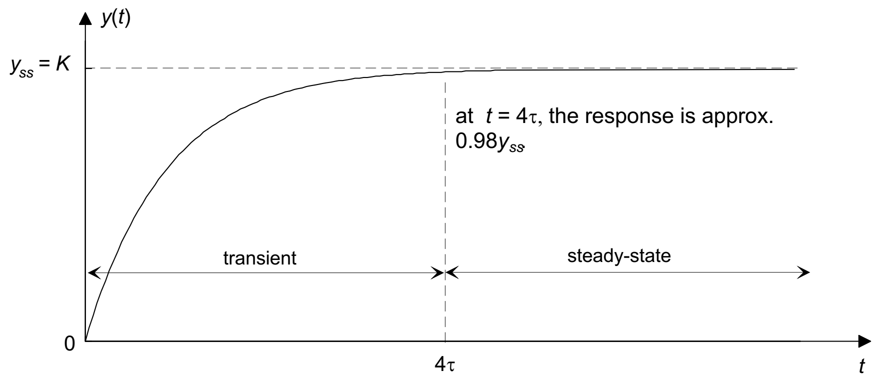
or $v_{ss} < v_d$, for $B > 0$.
Note that as we increase the controller gain so that $K_cK_e \gg B$ then $v_{ss} \to v_d$.

(b) How has the dynamic response affected by the feedback?
Consider the first-order ODE

$$\tau\frac{dy}{dt} + y = Ku(t).$$

The response to a steady input $u(t) = 1$ with initial condition $y(0) = 0$ is

$$y(t) = K(1 - e^{-\frac{t}{\tau}})$$



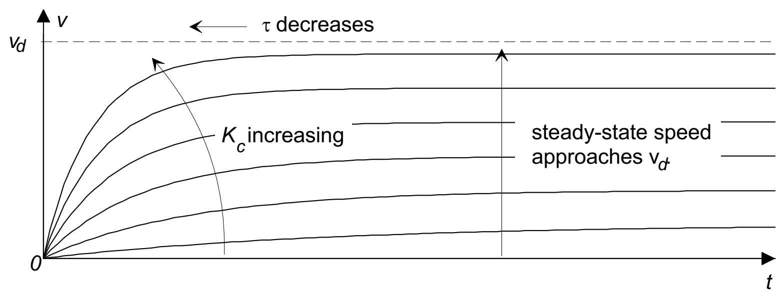
For the car under feedback control, this translates to a steady-state speed

$$v_{ss} = \frac{K_c K_e}{B + K_c K_e} v_d$$

which is a function of the controller gain K_c, and a closed-loop time constant τ that is

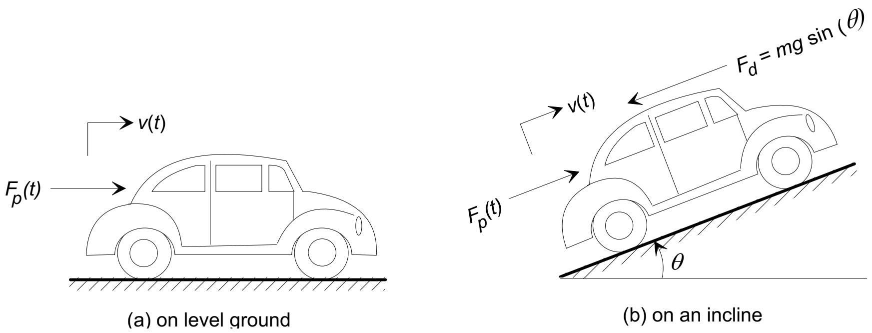
$$\tau = \frac{m}{B + K_c K_e}$$

which is also a function of K_c, and as K_c increases, τ decreases, meaning that the car responds more quickly to changes in the gas pedal.



4 The Effect of an External Disturbance Force

Now consider the car on an incline under closed-loop control:



(a) On horizontal ground $F_d = 0$, and we showed:

$$v_{ss} = \frac{K_c K_e}{B + K_c K_e} v_d$$

(b) On the incline there is a constant disturbance force $F_d = -mg \sin \phi$ acting down the incline, and from the closed-loop differential equation:

$$\frac{m}{B + K_c K_e} \frac{dv}{dt} + v = \frac{K_c K_e}{B + K_c K_e} v_d(t) + \frac{1}{B + K_c K_e} F_d(t)$$

the steady-state speed will be

$$\boxed{v_{ss} = \frac{K_c K_e}{B + K_c K_e} v_d - \frac{mg \sin \phi}{B + K_p K_e},}$$

but we note that as the controller gain K_p is increased the impact of the disturbance is reduced.

Conclusion: **Feedback control reduces the effect of external disturbances on the system behavior.**