MASSACHUSETTS INSTITUTE OF TECHNOLOGY
DEPARTMENT OF MECHANICAL ENGINEERING

2.004 *Dynamics and Control II*
Spring Term 2008

Lecture 5[1]

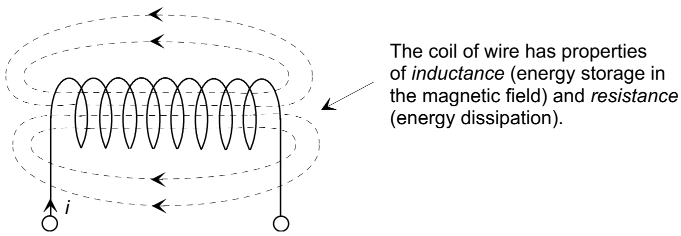
Reading:

- Class Handout: *Modeling Part 1: Energy and Power Flow in Linear Systems*
  Sec. 1 (Introduction)
  Sec. 4 (Electrical System Elements)

1 Modeling of Physical Systems

We will develop a unified modeling method that will allow the generation of a transfer function in several energy domains. The method is based on an extension of *impedance* concepts used in electrical systems. For that reason we will start with electrical systems.

Lumped parameter modeling involves approximating physical elements (where the parameters may be distributed in space) with discrete elements that are assumed to be concentrated at poins (or "lumped") in space. For example, a real electrical inductor is made from a coil of wire.



The coil has the primary property of electrical *inductance*, BUT it also has property of *resistance* associated with the wire - this resistance is distributed throughout the coil. In lumped parameter modeling we approximate the coil as having two elements - an inductance L and a lumped resistance R.

2 Modeling Electrical Systems

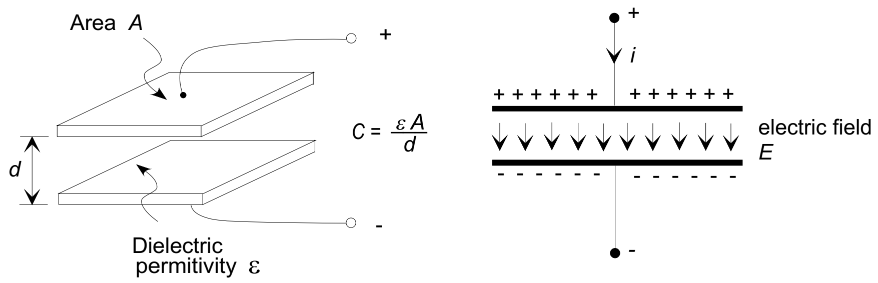
Step 1: Identify the variables to be used. We will concentrate on the use of *power variables*, that is a pair of variables whose product is power. In electrical systems:

$$P = vi$$

where P is power, v is the voltage drop (volts) across an element, and i is the current (amps) through an element. We therefore use v and i as our modeling variables.

Step 2: Identify the "primitive" lumped modeling elements. In each energy domain we will identify 3 elements, two elements that store energy, and an element that dissipates energy. In the electrical domain the energy storage elements are the *capacitor* and the *inductor*, and the dissipative element is the *resistor*.

The Capacitor: The capacitor stores energy in the electrostatic field between its "plates".



For a capacitor the current and voltage drop are related by

$$i = C\frac{dv}{dt}$$

where C is defined as the capacitance (Farad), or alternatively

$$v = \frac{1}{C}\int_0^t i dt + v(0)$$

Also, if we define charge as $q = \int_0^t i dt$ then

$$v = \frac{1}{C}q$$

For a parallel plate capacitor,where the plates have area A, and are separated by a distance d

$$C \propto \frac{A}{d} \Longrightarrow C = \epsilon\frac{A}{d}$$

where ϵ is defined to be the permittivity of the (non-conducting) dielectric material between the plates.

Note that power can flow in and out of a capacitor, since

$$P = vi = Cv\frac{dv}{dt}$$

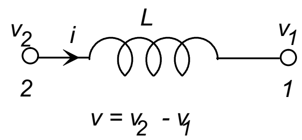
and v and dv/dt may have same or opposite signs. The capacitor is therefore an energy storage element. The energy stored in a capacitor is

$$E_C(t) = \int_0^t P dt + E(0) = \frac{1}{2}Cv^2.$$

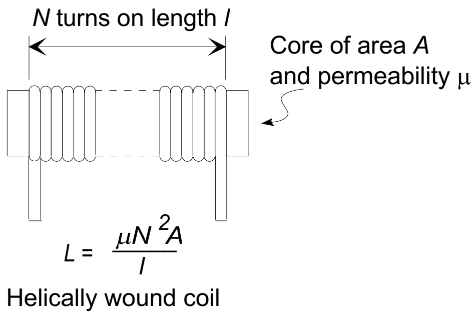
Notes:
1) The Farad is a very large unit. Most practical capacitors are measured in units of
- microfarads (μ F) $= 10^{-6}$F,
- nanofarads (nF) $= 10^{-9}$F, or
- picofarads (pF) $= 10^{-12}$F

2) A capacitor will not pass a dc current. Current only flows when the applied voltage is changing.

The Inductor:



Inductance is a property that represents energy stored in the electromagnetic field in a current carrying conductor. A practical inductor consists of a coil of wire



Lenz' Law relates the voltage across an inductor to yje current flowing through it:

$$v = L\frac{di}{dt}$$

or

$$i = \frac{1}{L}\int_0^t v dt + i(0)$$

where L is the inductance, with units of the Henry. Power can flow in and out of an inductor since

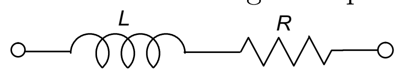
$$P = vi = Li\frac{di}{dt},$$

and i and di/dt may have the same or opposite signs. The stored energy is

$$E = \int_0^t P dt + E(0) = \frac{1}{2}Li^2$$

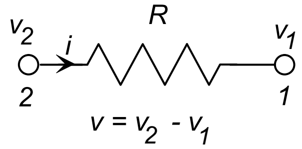
Notes:
1) The Henry is a very large unit. Practical inductors have values in
- millihenrys (mH) - 10^{-3}H
- microhenrys (μH) - 10^{-6}H

2) Notice that a pure inductance will have $v = 0$ if $\frac{di}{dt} = 0$ (a dc current)
3) Practical inductors are wound with (copper) wire and will have a finite resistance R. The lumped model including both phenomena is



The relative importance of the resistance is dependent on the operating conditions.

The Resistor:



The current and voltage in a resistor are governed by Ohm's law:

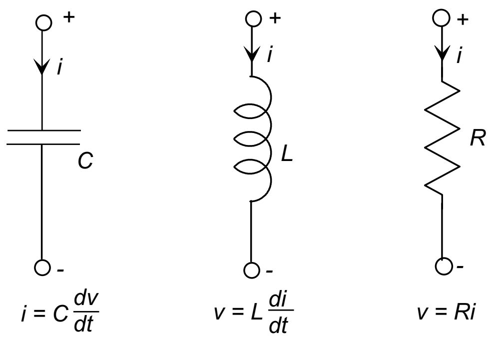
$$V = iR \qquad \text{or} \qquad i = \frac{1}{R}v$$

Resistance is a dissipative property since v and i always have the same sign, and the power is always positive

$$P = i^2R = v^2/R > 0$$

Energy always flows into a resistor, and cannot be recovered.

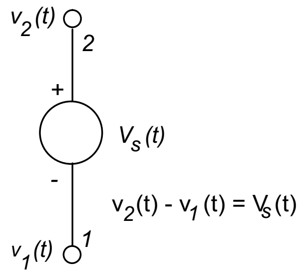
Summary: In modeling electrical systems we use three passive modeling elements:



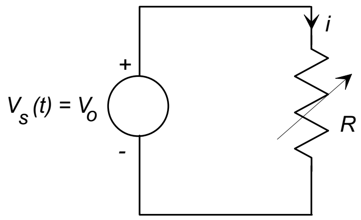
The capacitor and inductor are energy storage elements in that power can flow into the element, and recovered. The resistor is a dissipative element because the voltage an current are algebraically related and power always flows *into* the element.

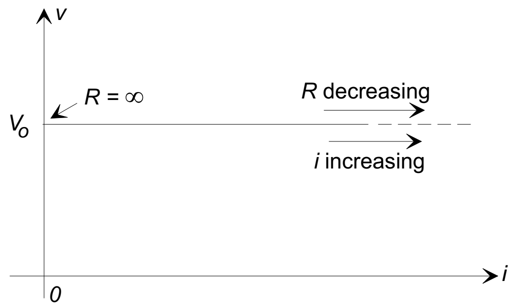
Sources: In addition to the three passive elements we will use two ideal electrical power sources:

The Voltage Source:

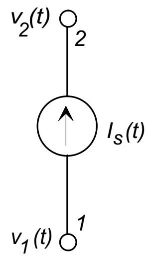


A voltage source will *maintain the voltage at its terminals* regardless of the current it must supply to the connected load.





The Current Source:



A current source will *maintain the current supplied to the attached circuit* regardless of the voltage at it must generate to do so.