2.004 Dynamics and Control II
Spring 2008

Massachusetts Institute of Technology
Department of Mechanical Engineering

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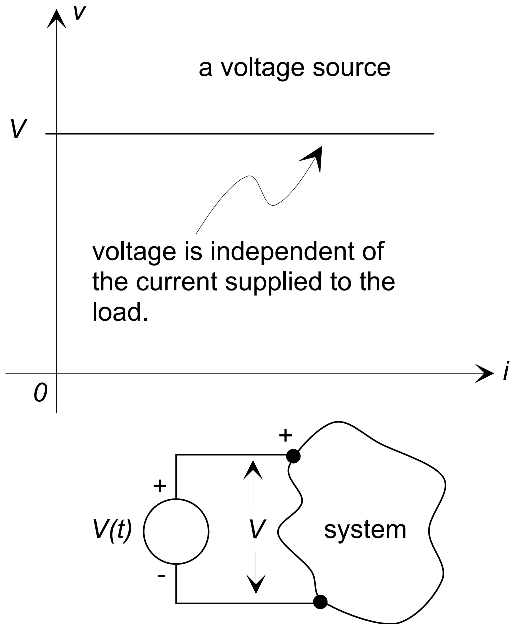
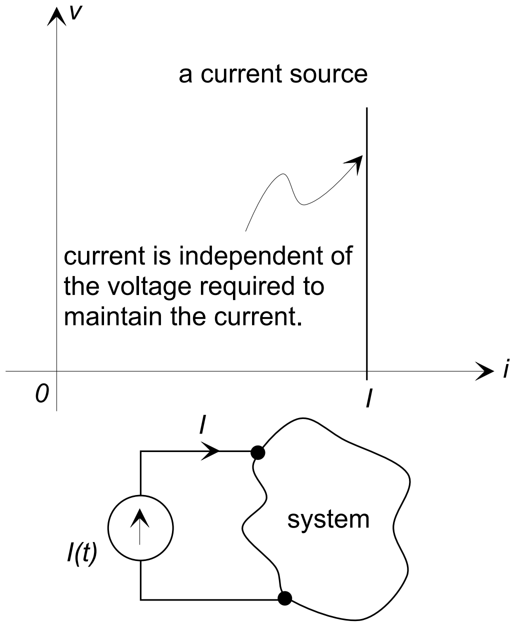
Spring Term 2008

Lecture 8[1]

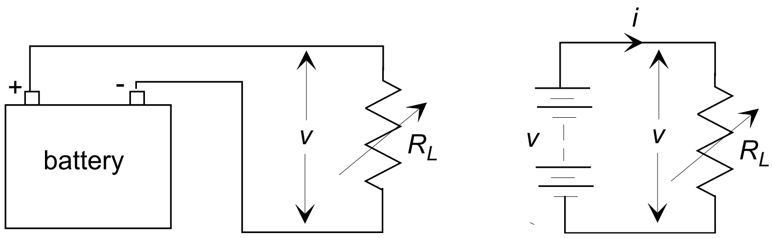
1 Electrical System Modeling (continued)

Modeling Real Sources: Up to this point we have considered only *ideal* voltage and current sources:

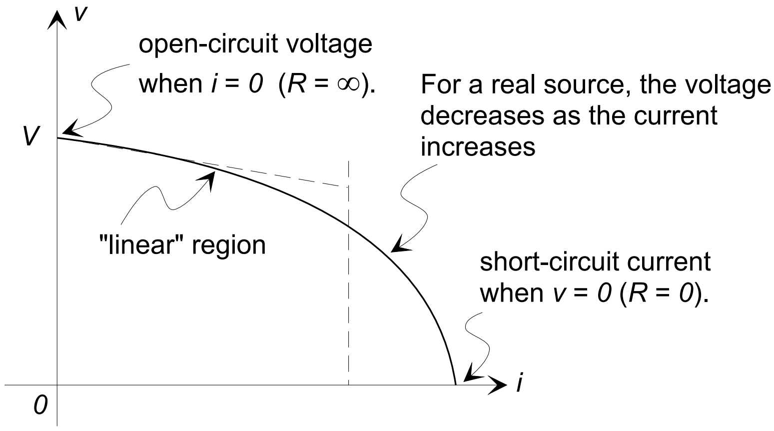




However, practical (real) sources do not have an ideal characteristic – they are power limited. For example we may think of a battery as an ideal voltage source - say with a 9 volt output, but if we were to measure the terminal voltage under increasing load currents



we would find that at high currents the voltage decreases in a nonlinear manner:

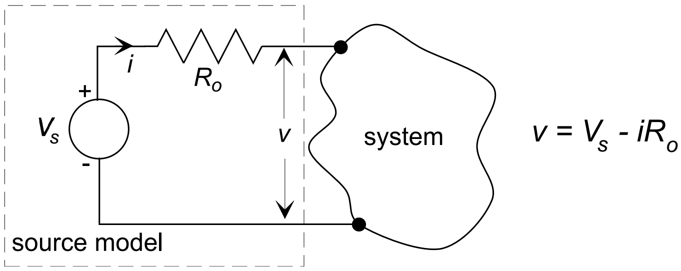


The voltage–current relationship is determined by the electrochemical reaction within the battery. It may be possible to define a region of operation at low currents in which there is an approximately linear relationship between voltage and current

$$\boxed{v(i) \approx v_{oc} - R_s i}$$

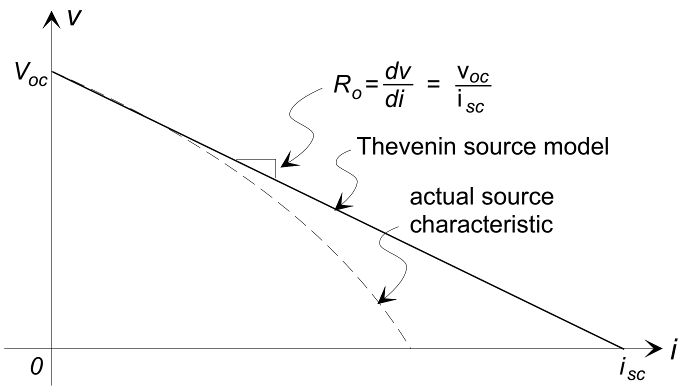
where v_{oc} is the open-circuit voltage (when $i = 0$), and R_o is a resistance.

How can we model this linear part of the characteristic?



One way to do this is to model the real source(the battery) as an ideal voltage source V_s in series with a resistor R_o (the value of which is found experimentally). The voltage drop $v_{R_o} = iR_o$ across the resistor accounts for the "droop" in the source characteristic.

This is known as the *Thévenin equivalent source* model.



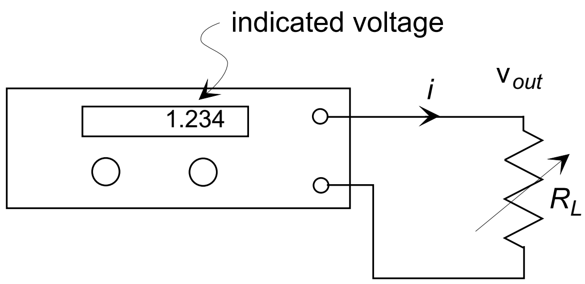
Note that

(1) The open-circuit ($i = 0$) voltage is V_s.

(2) The short-circuit ($v = 0$) current is $i_{sc} = V_s/R$.

■ Example 1

Electronic instruments are often designed to have a specific output impedance (resistance) R_o. For example when the Tektronix function generator in the 2.004 laboratory is connected to an arbitrary load resistance R_L the measured output voltage v_{out} will not necessarily be equal to the value set on the front panel:



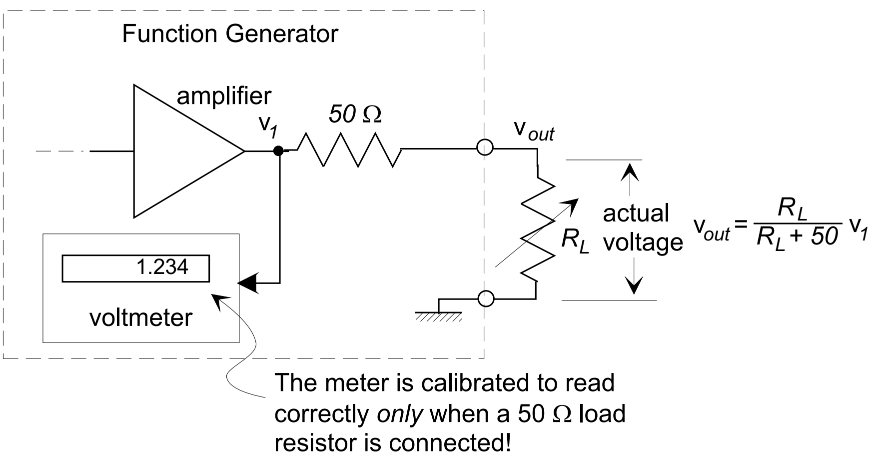
(a) If $R_L = \infty$ (no load connected) the true voltage at the terminals is twice the indicated value.

(b) If $R_L = 0$ (short circuit) the actual voltage is zero but the indicated voltage is unchanged.

(c) If $R_L = 50\ \Omega$ the indicated and measured output voltages are the same.

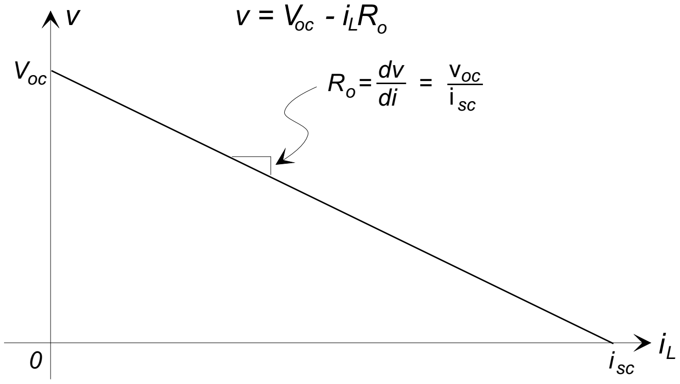
Why?

Answer: The function generator is specified as having an output impedance of 50Ω. In fact, if you look at the circuit schematic, you will find that there is a 50Ω resistor in series with the output terminal:

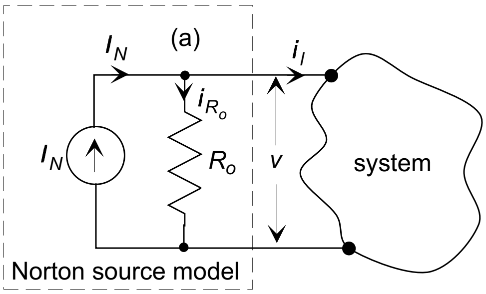


In the lab we have 50Ω terminator resistors connected across the output terminals so that the voltage reads correctly.

A second approach to modeling the real source characteristic



is to use a current source I_N in parallel with the resistor R_o



This is known as the *Norton equivalent source* model.

If no load resistor is connected (open-circuit)

$$v_{os} = I_N R_o.$$

The short-circuit current is

$$i_{sc} = I_N.$$

If the load is a resistor R_L

$$v = \frac{R_L R_o}{R_L + R_o} = \frac{R_L}{R_L + R_o} v_{oc} = \frac{v/i}{v1 + R_o}$$

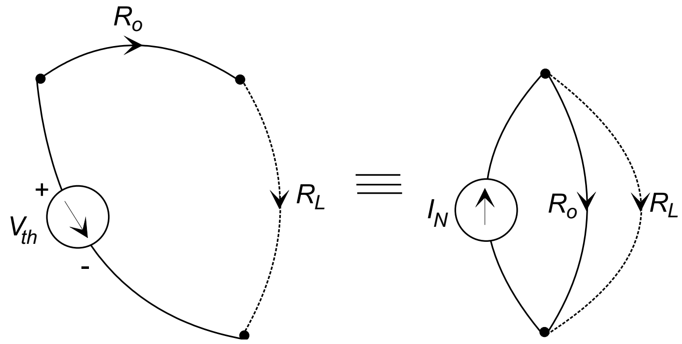
and rearranging

$$v = v_{oc} - iR_L.$$

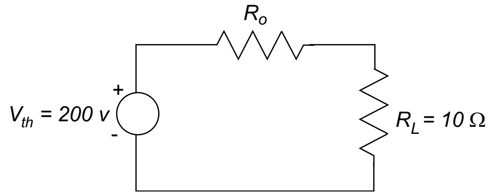
The Norton and Thévenin source models are equivalent and cannot be distinguished by measurements at the terminals.

■ Example 2

Measurements on an industrial dc power supply showed it to have an open-circuit voltage of 200 v, but when 10 Ω resistor is connected, the voltage drops to 133v. Find Thevénin and Norton equivalent source models.



Thevénin Model:



$$V_{th} = v_{oc} = 200 \text{ v.}$$

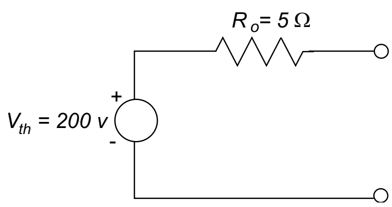
Using the voltage divider relationship

$$v_o = v_{R_L} = \frac{10}{10 + R_o} = 133\text{volts}$$

which gives

$$R_o = 5\Omega.$$

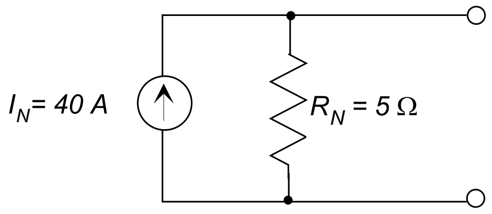
The Thevénin model is:



Norton Model: From above, the short-circuit current is

$$i_{sc} = \frac{V_{th}}{R_o} = \frac{200}{5} = 40 \text{ amps}$$

and the Norton model is:


I_N= 40 A
R_N = 5 Ω