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2.004 Dynamics and Control II
Spring 2008

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Lecture 12¹

Reading:

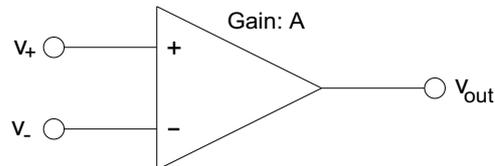
- Class Handout - *Introduction to the Operational Amplifier*
- Nise: pages 55–58 (Operational Amplifiers)

This lecture is inserted in the course at this point to coincide with Laboratory Session 3, which requires the construction of a proportional controller using a differential amplifier.

1 Introduction to the Operational Amplifier

The integrated-circuit operational-amplifier (op-amp) is a fundamental building block for many electronic circuits, including analog control systems. In this hand-out we examine some of the basic circuits that can be used to implement control systems. We take simplified approaches to the analysis, and this discussion is by no means complete or exhaustive.

What is an operational amplifier? It is simply a very high gain electronic amplifier, with a pair of differential inputs. Its functionality comes about through the use of *feedback* around the amplifier, as we show below.



The op-amp has the following characteristics:

- It is basically a “three terminal” amplifier, with two inputs and an output. It is a *differential* amplifier, that is the output is proportional to the *difference* in the voltages applied to the two inputs, with very high gain A ,

$$v_{out} = A(v_+ - v_-)$$

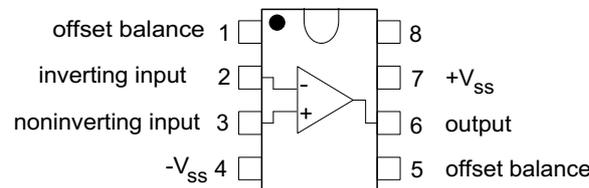
where A is typically $10^4 - 10^5$, and the two inputs are known as the *non-inverting* (v_+) and *inverting* (v_-) inputs respectively. In the ideal op-amp we assume that the gain A is infinite.

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- In an ideal op-amp no current flows into either input, that is they are voltage-controlled and have infinite input resistance. In a practical op-amp the input current is in the order of pico-amps (10^{-12}) amp, or less.
- The output acts as a voltage source, that is it can be modeled as a Thevenin source with a very low source resistance.

Op-amps come in many forms and with a bewildering array of specifications. They range in cost from a few cents to many dollars, depending on the specs. These specifications include input impedance, input bias current, output offset voltage, external power requirements, etc. Higher grade amplifiers are known as precision, or instrumentation amplifiers.

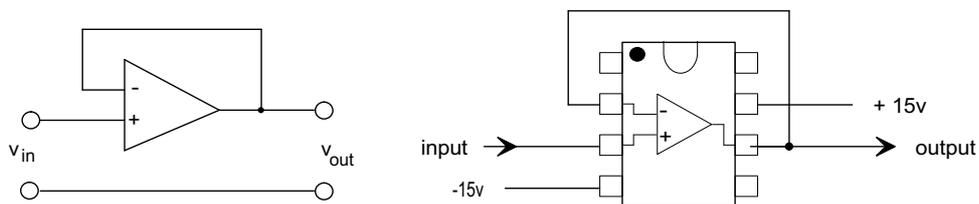
Op-amps come in a variety of packages. A common inexpensive op-amp, the 741, has an 8 pin DIP (dual in-line package) form. Many amps use a common basic pin-out for this package as shown below:



The pins are numbered counter-clockwise from the top left as shown above. (Note that pin 1 is identified by a notch at the top or a dot beside pin 1.) The basic amplifier is connected between pins 2, 3 and 6. The amplifier requires a pair of external supply voltages to operate, these are typically ± 15 volts and are connected to pins 7 (positive) and 4 (negative). Pins 1 and 5 are usually used for optional external offset nulling circuitry - the actual connection is dependent on the type. We will not use this feature if we can get away without it.

Non-Inverting Configurations

The Unity Gain Non-Inverting amplifier: The simplest configuration of the op-amp is as a unity gain “buffer” amplifier:



The output is connected directly to the inverting input. Then

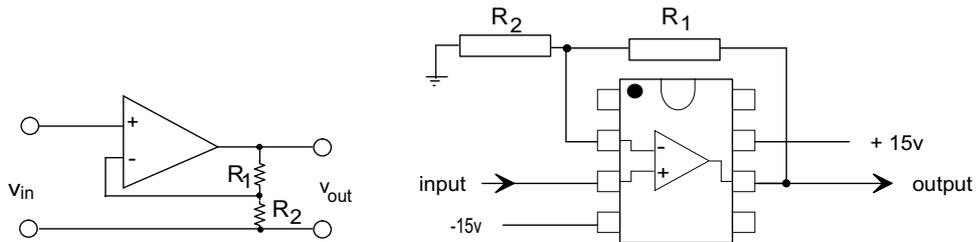
$$\begin{aligned}
 v_{out} &= A(v_+ - v_-) \\
 &= A(v_+ - v_{out}) \\
 &= \frac{A}{1 + A} v_+
 \end{aligned}$$

and if, as noted above, A is typically 10^4 or much higher we may simply say

$$v_{out} = v_+$$

The unity gain amplifier is used as a *buffer* to minimize loading on a circuit, because it has a very high input impedance (draws no current) yet provides a low output resistance output.

The Non-Inverting amplifier with gain: The above circuit may be modified to feed back only a fraction of the output voltage by including a two resistor voltage divider.



In this case R_1 and R_2 act as a voltage divider, and

$$v_- = \frac{R_2}{R_1 + R_2} v_{out}$$

Then

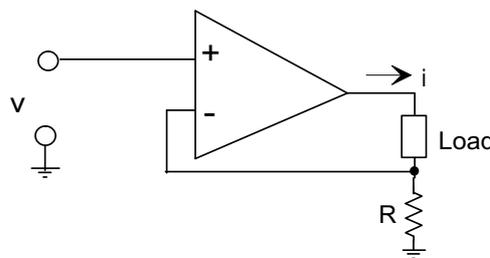
$$\begin{aligned} v_{out} &= A(v_+ - v_-) \\ &= A \left(v_{in} - \frac{R_2}{R_1 + R_2} v_{out} \right) \\ &= \frac{A(R_1 + R_2)}{R_1 + R_2 + AR_2} v_{in} \end{aligned}$$

and when $A \gg 1$

$$v_{out} = \frac{R_1 + R_2}{R_2} v_{in}$$

For example if $R_1 = 47 \text{ k}\Omega$, and $R_2 = 10 \text{ k}\Omega$, the amplifier voltage gain will be 5.7.

The Voltage-Controlled Current-Source: The above circuit can be modified to produce a current source.



Assume that the op-amp drives a load of unknown resistance. A small resistor R is placed in series with the load. Then the inverting input is $v_- = iR$. At the output of the amplifier

$$\begin{aligned} v_{out} &= A(v_+ - v_-) \\ &= A(v_{in} - iR) \end{aligned}$$

Then

$$i = \frac{1}{R} \left(v_{in} - \frac{1}{A} v_{out} \right)$$

and if $A \gg 1$

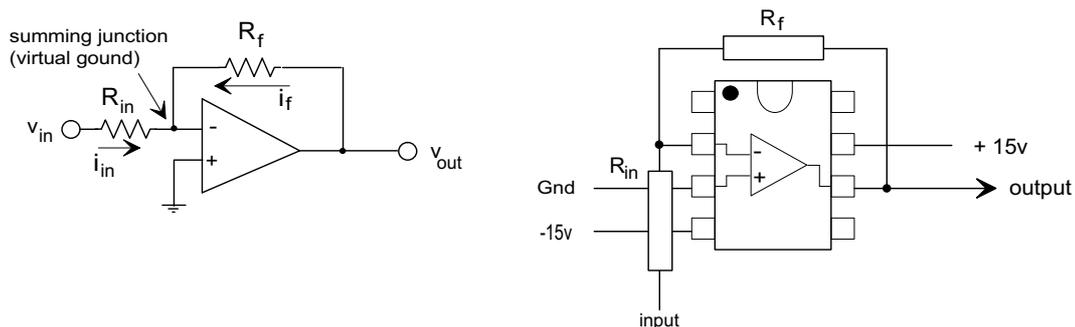
$$i = \frac{1}{R} v_{in}$$

which is independent of the load. For example, if $R = 0.1\Omega$, then the current in the load will be $i = 10v_{in}$ amps.

It should be realized, however, that typical op-amps are not capable of supplying more than a few milli-amps of current. A power op-amp or an additional power amplifier will generally be necessary to supply significant current.

Inverting Configurations

The Inverting Amplifier: The basic *inverting* amplifier has a configuration as shown below:



The non-inverting input is connected to ground and a pair of resistors, R_{in} and R_f (the feedback resistor) is used to define the gain. To simplify the analysis we make the following assumptions:

- (a) The gain A is very high and we let it tend to infinity. Because $v_{out} = A(v_+ - v_-)$ and $v_+ = 0$, this means that $v_- = \lim_{A \rightarrow \infty} v_{out}/A = 0$. In other words, the inverting input voltage is so small (μ volts) that we declare it to be a “virtual ground”, that is in our simplified analyses we assume $v_- = 0$.
- (b) No current flows into either of the inputs. This allows us to apply Kirchoff’s current law at the junction of the resistors.

With the above assumptions, when we apply Kirchoff’s current law at the junction of the resistors (the inverting input terminal):

$$i_{in} + i_f = 0$$

Since the junction is at the virtual ground ($v_- \approx 0$) we can simply use Ohm's law:

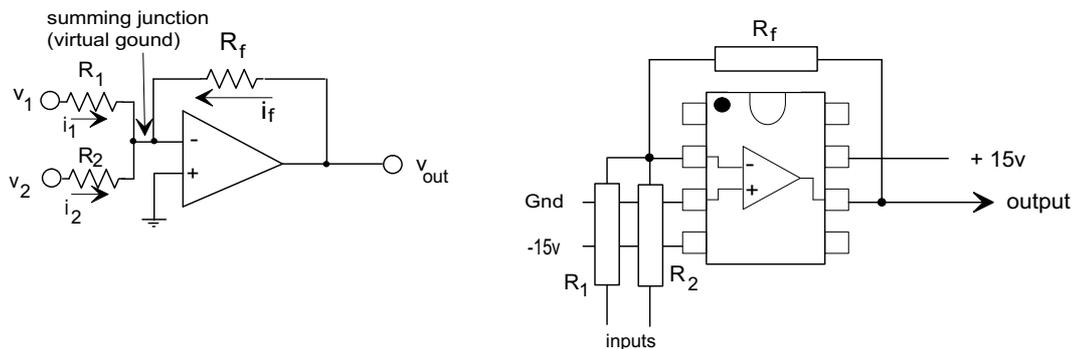
$$\frac{v_{in} - v_-}{R_{in}} + \frac{V_{out} - v_-}{R_f} \approx \frac{v_{in}}{R_{in}} + \frac{V_{out}}{R_f} = 0$$

or

$$v_{out} = -\frac{R_f}{R_{in}}v_{in}$$

In other words the voltage gain is defined by the ratio of the two resistors. The term *inverting* amplifier comes about because of the sign change.

The Inverting Summer: The inverting amplifier may be extended to include multiple inputs:



As before we assume that the inverting input is at a virtual ground ($v_- \approx 0$) and apply Kirchoff's current law at the summing junction

$$i_1 + i_2 + i_f = 0$$

$$\frac{v_1}{R_1} + \frac{v_2}{R_2} + \frac{V_{out}}{R_f} = 0$$

or

$$v_{out} = -\frac{R_f}{R_1}v_1 - \frac{R_f}{R_2}v_2$$

which is a weighted sum of the inputs. If $R_1 = R_2 = R_{in}$ the output is proportional to the sum of the input voltages.

$$v_{out} = -\frac{R_f}{R_{in}}(v_1 + v_2)$$

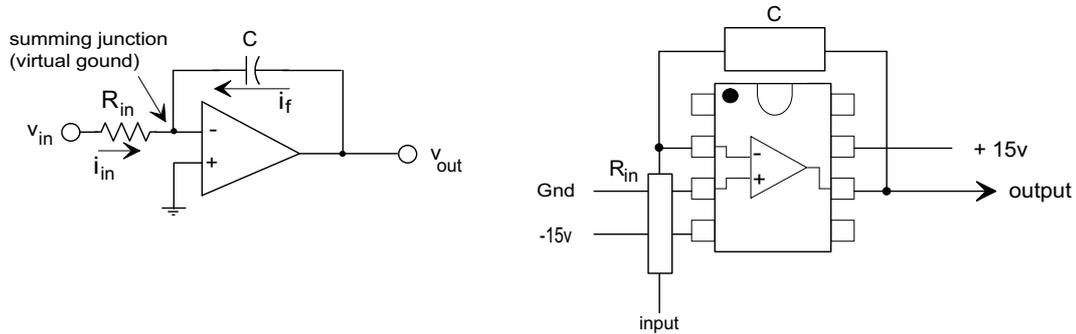
The summer may be extended to many inputs by simply adding input resistors R_i

$$v_{out} = -\sum_{i=1}^n \frac{R_f}{R_i}v_i$$

If all of the input resistors are equal ($R_i = R_{in}$)

$$v_{out} = -\frac{R_f}{R_{in}}\sum_{i=1}^n v_i$$

The Integrator: If the feedback resistor in the inverting amplifier is replaced by a capacitor C the amplifier becomes an integrator.



At the summing junction we apply Kirchoff's current law as before but the feedback current is now defined by the elemental relationship for the capacitor:

$$i_{in} + i_f = 0$$

$$\frac{v_{in}}{R_{in}} + C \frac{dv_{out}}{dt} = 0$$

Then

$$\frac{dv_{out}}{dt} = -\frac{1}{R_{in}C}v_{in}$$

or

$$v_{out} = -\frac{1}{R_{in}C} \int_0^t v_{in} dt + v_{out}(0)$$

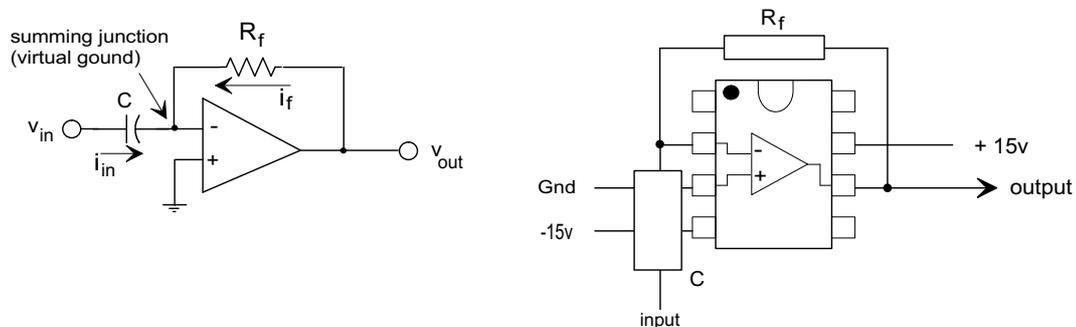
As above, the integrator may be extended to a summing configuration by simply adding extra input resistors:

$$v_{out} = -\frac{1}{C} \int_0^t \left(\sum_{i=1}^n \frac{v_i}{R_i} \right) dt + v_{out}(0)$$

and if all input resistors have the same value R

$$v_{out} = -\frac{1}{RC} \int_0^t \left(\sum_{i=1}^n v_i \right) dt + v_{out}(0)$$

The Differentiator: If the input resistor R_{in} in the inverting amplifier is replaced by a capacitor, the circuit becomes a differentiator:



At the summing junction we apply Kirchoff's current law as before but the input current is now defined by the elemental relationship for the capacitor:

$$i_{in} + i_f = 0$$

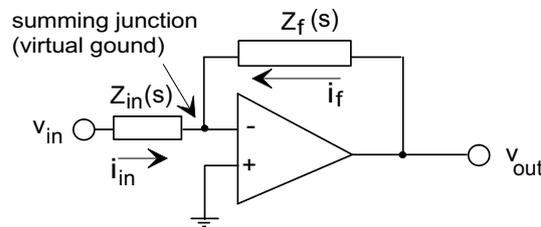
$$C \frac{dv_{in}}{dt} + \frac{v_{out}}{R_f} + = 0$$

Then

$$v_{out} = -\frac{1}{R_{in}C} \frac{dv_{in}}{dt}$$

Pure differentiators are rarely used because of their susceptibility to noise. In practice it is common to use a pseudo-differentiator by including a resistor in series with the capacitor.

A General Linear Transfer Function: The inverting configuration may be extended to synthesize many general linear transfer functions by including appropriate networks in the feedback and input paths. Consider the general circuit shown below with elements described by impedances $Z_{in}(s)$ and $Z_f(s)$ in the input and output paths:



At the summing junction

$$i_{in} + i_f = 0$$

$$\frac{V_{in}(s)}{Z_{in}(s)} + \frac{V_{out}(s)}{Z_f(s)} + = 0$$

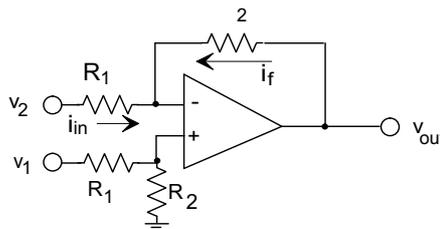
so that the amplifier becomes a linear system with a transfer function

$$H(s) = \frac{V_{out}(s)}{V_{in}(s)} = -\frac{Z_f(s)}{Z_{in}(s)}$$

This configuration may be used to synthesize lead-lag compensators for control, active filters for signal processing, and many other functions.

The Differential Amplifier

A differential amplifier produces an output that is proportional to the difference between two inputs.



At the non-inverting input

$$v_+ = \frac{R_2}{R_1 + R_2} v_1$$

If the op-amp gain is assumed to be infinite, $v_- = v_+$. The input current at the inverting amplifier is:

$$\begin{aligned} i_{in} &= \frac{v_2 - v_-}{R_1} = \frac{v_2 - v_+}{R_1} \\ &= \frac{1}{R_1} v_2 - \frac{R_2}{R_1(R_1 + R_2)} v_1 \end{aligned}$$

Similarly the feedback current is

$$\begin{aligned} i_f &= \frac{v_{out} - v_-}{R_2} = \frac{v_{out} - v_+}{R_2} \\ &= \frac{1}{R_2} v_{out} - \frac{1}{(R_1 + R_2)} v_1 \end{aligned}$$

Applying Kirchoff's current law at the summing junction $i_{in} + i_f = 0$, then yields

$$v_{out} = \frac{R_2}{R_1} (v_1 - v_2)$$

so that the output is proportional to the voltage difference between the two inputs.

Practical Considerations:

The above simplified description does not include many practical considerations that must be taken into account when designing high performance op-amp circuits, including:

- Most op-amps are low current devices. Most circuits are designed to operate with signal currents in the mA or μA range. This means that resistor values should be in the range 10 k Ω and above.
- An op-amp cannot supply voltage levels greater than the supply voltage levels. Care must be taken to ensure that the output voltage remains within the limits defined by the supply voltage.
- Our simplified analysis has not taken bandwidth limitations into account. Practical op-amps exhibit a high-frequency roll-off of gain because of internal capacitances within the chip.
- A similar phenomenon is known as "slew-rate" limiting. In practice, op-amps have a limit on the slope (v/s) of the output voltage. This nonlinear effect has a limiting effect on the high frequency performance.