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2.007 Design and Manufacturing I  
Spring 2009

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NAME: \_\_\_\_\_ "Solution" \_\_\_\_\_

**2.007 – Design and Manufacturing I**  
**Exam on Gears, Springs, Sensors, Programming, Drawing, etc.**

You have 1.5 hours to answer the following 11 questions. Please show your work and explain your reasoning to the extent possible given the allotted time. Point allocations (out of 100 total) are listed for each question.

1. (5 points) Define the term "conjugate action." Briefly (using one or two sentences) explain its significance for design of power transmission systems.

**"Conjugate action" is the property of a transmission that delivers constant rate of rotation of the output shaft if rate of rotation of the input shaft is also constant. It is important to note that this not not simply valuable in and of itself, but is also a requirement for smooth transmission of power. It reduces vibration, noise, and premature failure due to fatigue of the machine elements.**

2. (10 points) You fill a one liter container with air at 60 psi gauge pressure and plan to use it as a source of power for a machine. The air in the bottle is at thermal equilibrium with the air in the room at 20 degrees Centigrade. Estimate (within  $\pm 10\%$  is fine) the force applied if a valve is opened connecting the reservoir of air to a piston with a one inch internal diameter and a two inch stroke assuming the piston extends its full stroke in about 2 seconds.

**Displacement of the piston**

$$\frac{\pi}{4} \cdot (1\text{-in})^2 \cdot 2\text{-in} = 0.026\text{L}$$

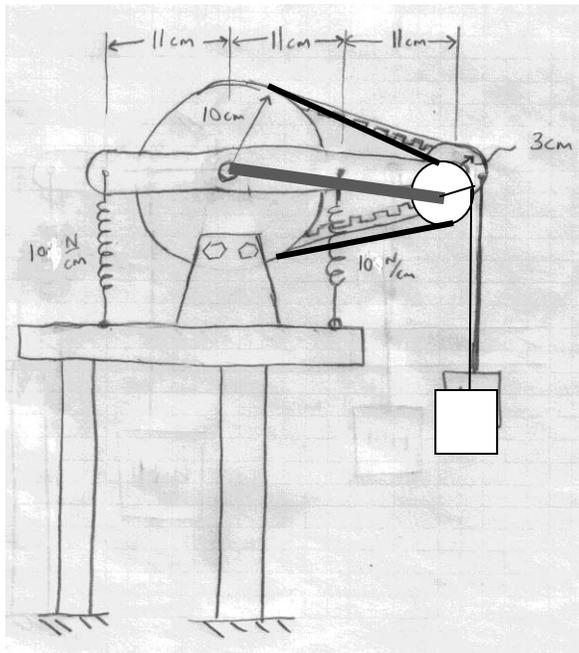
**Since the piston is only 2% of the volume of the reservoir and we only need an estimate with 10%, it's good enough to just multiply the original gauge pressure times the area of the face.**

$$\frac{\pi}{4} \cdot (1\text{-in})^2 \cdot 60\text{psi} = 47.124\text{bf} \qquad \frac{\pi}{4} \cdot (1\text{-in})^2 \cdot 60\text{psi} = 209.618\text{N}$$

3. (15 points) Sketch the mechanism in a position that places it in static equilibrium. Base your drawing on an estimate of the displacements to within  $\pm 20\%$ . Assume the joints have negligible friction and that the pulleys and links have negligible weight. The 10 cm radius pulley is fixed with respect to the table and cannot rotate. You can assume the toothed belt will not slip. The extension springs are at their natural length. You may assume that the extension springs apply tension with the spring constant indicated but cannot apply any appreciable force in compression (they buckle). Briefly justify your solution with a couple equations, schematic diagrams, and/or a few sentences of explanation. Be sure to indicate clearly:
- Whether you think the 3cm pulley rotates clockwise or counterclockwise
  - Whether you think the arm connecting the pulleys rotates clockwise or counterclockwise
  - Whether you think the arm 1 kg mass is higher or lower at equilibrium than at the starting position

Consider a modest clockwise rotation of the arm, say 0.1 radian (6 deg). To maintain the proper position of the belt, tangent to both pulleys, the belt will have to wrap around the top of the 10 cm rad pulley by  $10\text{cm} \cdot 0.1 \text{ rad}$  or 1cm. The kinematics of a belt drive demand that the same 1cm of belt feeds off of the 3cm pulley which requires a  $1\text{cm}/3\text{cm} = 0.33 \text{ rad}$  (18 deg) rotation of the 3cm dia pulley relative to the arm or 0.23 with respect to the fixed frame of reference.

This 0.1 deg rotation of the arm would also extend the left spring by 1.1cm leading to a 11N downward force. The extension spring on the right side is placed in compression and we assume it buckles out of the way or otherwise goes slack and applies negligible force.



Due to the small angles and generous  $\pm 20\%$  allowance for our estimate, we can linearize. We have a motion of the weight due to two factors:  
 Rotation of the arm  $\Rightarrow 0.1 \text{ rad} \cdot 24 \text{ cm} = 2.4 \text{ cm}$  downward  
 Rotation of the 3cm pulley  $\Rightarrow 0.33 \text{ rad} \cdot 3 \text{ cm} = 1 \text{ cm}$  upward  
 So a net motion of 1.4 cm downward.

Kinematically, the pulley arrangement is similar to attaching the weight directly to the arm at 14 cm. I infer the guess of 0.1 rad of rotation was a bit low. I update my guess by 30% throughout to:

0.13 rad (7.5deg) CW rotation of the arm  
0.23 rad (17deg) CCW rotation of the 3cm pulley  
 net motion of 1.8 cm downward of the 1 kg mass  
 as a check I see if the energy stored in the spring equals the energy lost by the mass  $mgh = 0.18\text{J}$

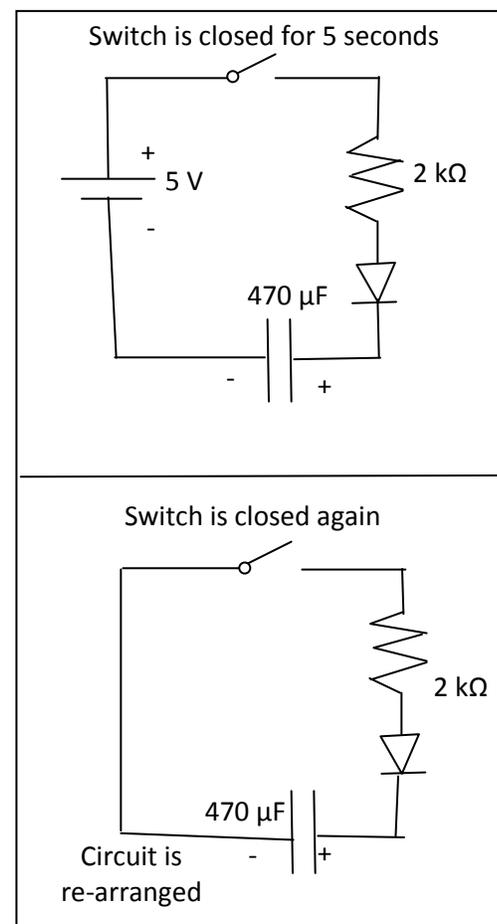
$$0.5 \cdot K \cdot x^2 = 0.5 \cdot 10 \cdot (\text{N/cm}) \cdot (0.13 \cdot 11 \text{ cm})^2 = 0.19\text{J} \text{ close enough.}$$

4. (5 points) The components shown here (resistor, LED, capacitor, battery, and switch) are connected in series. The switch is closed for 5 seconds. During that time, the LED lights and then gradually dims. The circuit composed of the very same components is then reconfigured and reconnected as shown. The switch is then closed. Which statement best describes what happens after the switch is closed again:
- The LED lights up and then slowly dims over the course of 5 seconds
  - The LED starts dim then slowly brightens over the course of 5 seconds
  - The LED lights up steadily
  - LED does not light

**Many students did not notice that the scenario differed in an important way from the problem on the last exam. The capacitor was left in the same orientation this time whereas it was reversed previously between charging and discharging. In this case, in the new problem, the capacitor cannot discharge at all due to the orientation of the diode. The + and – on the lower figure should have helped to cue you in to that.**

**I note that I didn't cover diodes very effectively in 2.007.**

**I covered them in 2.670 but a good number of students didn't take that course this year. I put a diode on the first exam, but that didn't raise the question and prompt us to discuss diodes in 2.007. In any case, my bad. The wikipedia entry for diodes states "Diodes have two active electrodes between which the signal of interest may flow, and most are used for their unidirectional electric current property." This is the main thing you need to know about a diode and what it does. The symbol for the element is a good reminder.**



5. (10 points) Consider a rear wheel drive vehicle with a four speed manual transmission connected to a conventional differential (such as the one depicted to the right). The vehicle is placed in first gear, the engine is turned off.

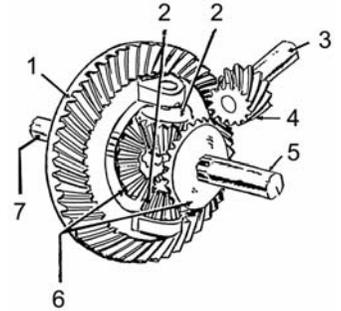
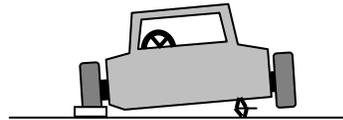


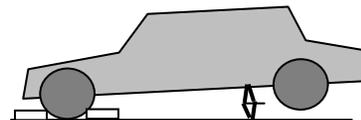
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Please also see  
[http://commons.wikimedia.org/wiki/File:Transmission\\_diagram.JPG](http://commons.wikimedia.org/wiki/File:Transmission_diagram.JPG)

- A) (5 points) The front and rear driver's side rear tires are "chocked" by placing wedges in front and behind them. A jack is used to raise the passenger's side tires slightly off the ground. Using your hands, you apply a clock-wise torque to the passenger's side rear tire. Briefly describe what you think will happen.



**With the engine off and the manual transmission in first gear, the drive shaft into the differential will be fixed. With the driver's side wheel chocked, it is also fixed. The differential has 2 kinematic DOF so it has been frozen and the passenger's side wheel cannot be turned by hand.**

- B) 5 points) Again, the engine is off and the manual transmission is in first gear. The front tires are chocked and a jack is used to raise BOTH the rear tires slightly off the ground. Using your hands, you apply a clock-wise torque to the driver's side rear tire. Briefly describe what you think will happen.



**Again, with the engine off and the manual transmission in first gear, the drive shaft into the differential will be fixed. With the driver's side wheel up in the air now, it is free to turn. If you apply a torque and turn by hand either rear wheel, the opposite side will also turn BUT IN THE OPPOSITE DIRECTION. So you'll observe CCW rotation of the passenger's side wheel.**

6. (15 points) The subproblems below refer to the page from a bearing catalog provided here.

A) (5 points) If gear PX24B-8 and PX24B-22 are mated together in a gear train, how far apart should the centers of their shafts be placed?

$$(0.333 + 0.916) / 2 =$$

**0.625 inches apart**

B) (5 points) If gear PX24B-8 and PX24B-22 are mated together in a gear train and a torque of 2 ft lbs is applied to PX24B-8, what is the separation force?

**24 inch lbs / radius of 0.333/2 leads to a tangential force of**

**144 lbs.**

**To get separation force, multiply by tan(20deg)**

**= 52.5lbs**

## Spur Gears

**24, 32, 48, and 64 Pitch 1/8" Bore AGMA Quality 4  
Cold Rolled Steel and Brass 20° Pressure Angle**

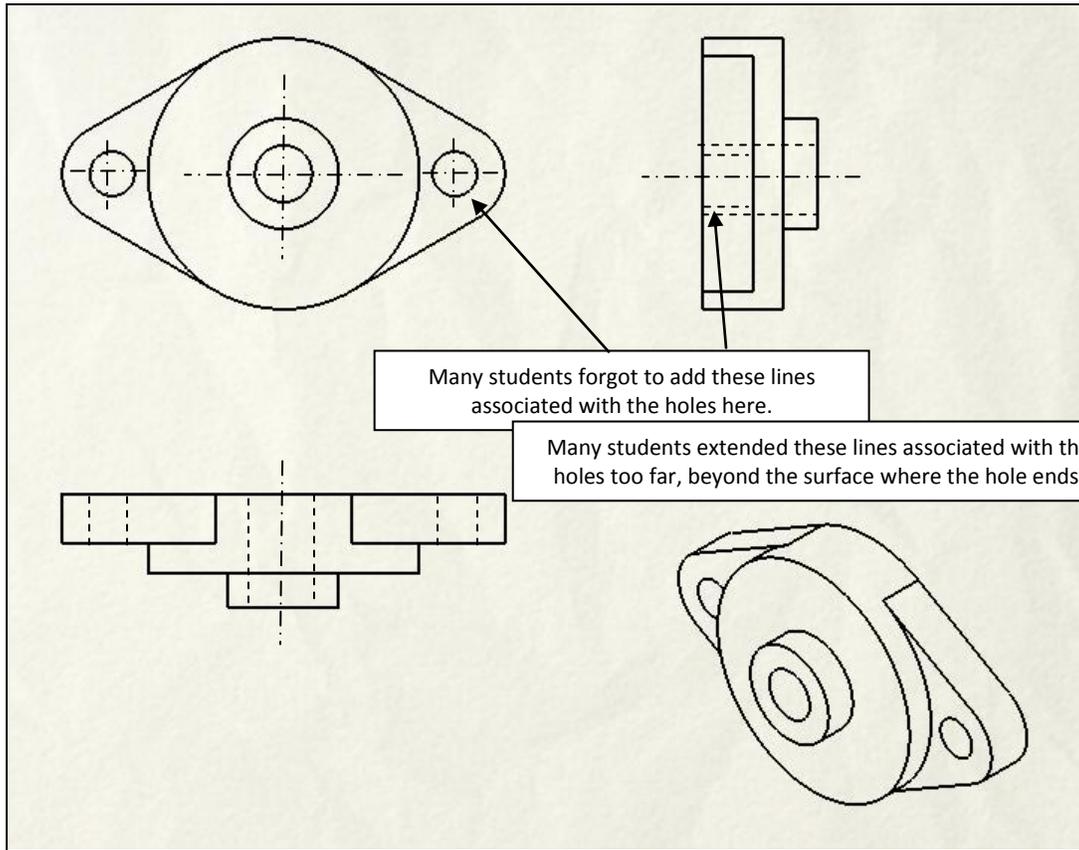
COLD ROLLED STEEL C12L14 OR C12L15 WITH SELENIUM		BRASS ALLOY 360		NO OF TEETH	PITCH DIA.	OUTSIDE DIA.	H	F
STOCK NUMBER	STOCK NUMBER							
24 PITCH (.1309)								
PX24S-8	PX24B-8	8	.333	.416	.208	1/4		
PX24S-9	PX24B-9	9	.375	.458	.250			
PX24S-10	PX24B-10	10	.417	.500	.291			
PX24S-12	PX24B-12	12	.500	.583	.375			
PX24S-16	PX24B-16	16	.666	.750	.542			
PX24S-18	PX24B-18	18	.750	.833	.625			
—	PX24B-22	22	.916	1.000	.792			
32 PITCH (.0981)								
PX32S-10	PX32B-10	10	.312	.375	.218	1/4		
PX32S-11	PX32B-11	11	.344	.406	.250			
PX32S-12	PX32B-12	12	.375	.437	.281			
PX32S-14	PX32B-14	14	.438	.500	.343			
PX32S-15	PX32B-15	15	.489	.531	.375			
PX32S-16	PX32B-16	16	.500	.562	.406			
PX32S-18	PX32B-18	18	.563	.625	.468			
PX32S-20	PX32B-20	20	.625	.688	.532			
—	PX32B-24	24	.750	.813	.656			
48 PITCH (.0654)								
PX48S-14	PX48B-14	14	.292	.333	.229	1/8		
PX48S-15	PX48B-15	15	.312	.353	.250			
PX48S-16	PX48B-16	16	.333	.375	.271			
PX48S-18	PX48B-18	18	.375	.417	.312			
PX48S-24	PX48B-24	24	.500	.542	.437			
—	PX48B-32	32	.666	.708	.604			
—	PX48B-36	36	.750	.792	.687			
—	PX48B-40	40	.833	.875	.770			
64 PITCH (.0490)								
PX64S-15	PX64B-15	15	.234	.265	.187	1/8		
PX64S-16	PX64B-16	16	.250	.281	.203			
PX64S-18	PX64B-18	18	.281	.312	.234			
—	PX64B-24	24	.375	.406	.328			
—	PX64B-40	40	.625	.656	.578			
—	PX64B-48	48	.750	.781	.703			

Berg Manufacturing "The Mark of Quality" **1-800-232-BERG**  
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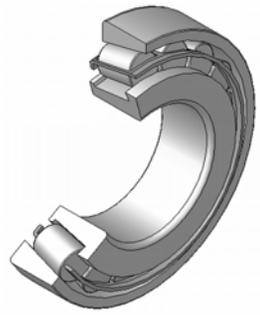
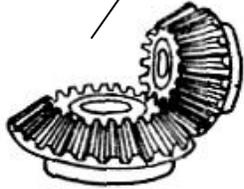
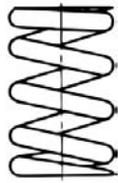
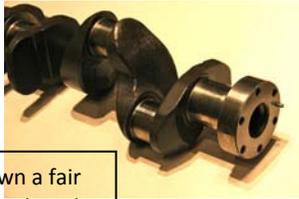
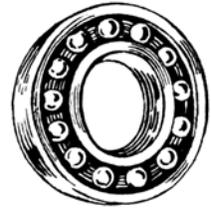
C) (5 points) If gear PX24B-8 and PX24B-22 are mated together 0.02 inches farther apart than you determined in part A, what are the primary consequences? Would your answer to part B change and if so would it rise or fall?

**Such as small displacement with leave the gears in mesh still. Not much of a problem. Mainly the backlash would increase. Conjugate action would be preserved. The separation force would rise a small amount due to the larger pressure angle as the gears mesh at point farther from the gear's centers.**

7. (5pts) Add hidden lines and center lines to the views below.



8. (15 points) Match the items below to the terms that describe them.



- Torsion spring
- Extension spring
- Tapered roller bearing
- Bevel gear
- Cam shaft
- Worm gear
- V belt pulley
- Constant force spring
- Crank shaft
- Compression spring
- Timing belt pulley
- Rack
- Ball bearing

The crank shaft seems to have thrown a fair number of people off. Many students thought it might be a cam shaft. The lobes on this item aren't cams, but rather counterweights.

here are cams.

These features on a timing belt pulley engage with the teeth on a toothed belt.

9. (5 points) A vehicle is composed of a box shaped structure with permanent magnet DC motors placed at all four corners and driving all four wheels (through a gear train). All these motors are attached to a 5V NiCad battery pack. The vehicle is climbing a 20 degree incline. The vehicle is proceeding very slowly since the motors are nearly stalled. The coefficient of static friction between the wheels and the inclined surface is 50% greater than that needed to maintain a steady climb up that grade. At a particular moment, the mass of the vehicle is reduced by 25% because a robot arm on the vehicle takes an object off the vehicle and lays it to the side. Circle the item below that best describes the events that would likely transpire.
- The vehicle will continue up the ramp at the same speed as before.
  - The normal force of the wheels on the surface will decrease causing the wheels will to begin to slip with respect to the surface of the incline reducing the vehicle's speed.
  - The vehicle's velocity will increase instantaneously when the block is released so that the vehicle's momentum will remain constant even as the mass is reduced.
  - The vehicle will accelerate smoothly up the ramp until it reaches a higher steady-state velocity.
  - The wheels will not slip but will turn backwards causing the vehicle to go back down the ramp.

Answer C seems to have caught quite a few people. The phrase “velocity will increase instantaneously” should be a cue that something is wrong. A body cannot increase in speed “instantaneously” unless a huge force is applied for a very short period, such as a bullet in the chamber of a pistol. The phrase “the vehicle's momentum remains constant even as the mass is reduced” also must have sounded attractive. Indeed, a system of bodies will have a constant total momentum when there is no net external force applied. In this case, the block that is dropped would have its share of the system's momentum and would initially keep moving at the same velocity as the robot. Subsequently, it could have significant forces applied to it by the ramp to stop it, but this would not cause the robot to accelerate to make up for its lost momentum since there are no forces between the two bodies once released.

10. (5 points) Explain in a few sentences why a lead acid battery is a reasonable choice for use in typical commercially available automobiles today, but may not be a reasonable choice for a plug-in electric vehicle.

The high power density of lead acid batteries is useful for starting a car since it takes a lot of power to turn over a cold engine. The low energy density is not a big deal for most cars today because you don't take much time to start the car (1 sec of cranking?), so the battery doesn't have to be so large and is a small fraction of the weight of the car anyway. It helps too that lead acid batteries perform well at cold temperatures. The low number of cycles to failure is not usually a problem because starting is really not a cycle. Letting the battery go fully dead is bad practice. My advice is –do not leave your head lights on all night or you'll have to replace your lead acid battery frequently.

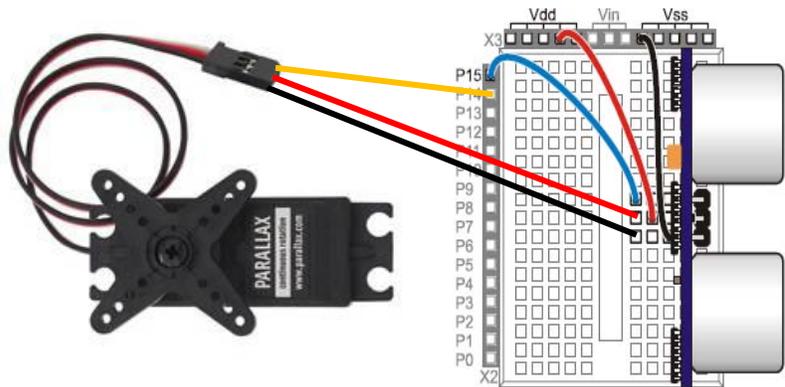
For a plug-in electric car, the demands on the battery are very different. You need much greater energy density and far more cycles of charging and re-charging. The speed of charging might be an issue also, although some people propose that plug-in vehicles should have batteries swapped in and out rather than charging on the car and tying up that valuable asset waiting for the charge to complete.

11. (10 points) A Homework board is wired with an ultrasonic sensor and a continuous rotation servo as shown below and the code below has been uploaded onto the EEPROM. The sensor points forward on the vehicle and the servo is configured to drive the wheels. Describe in a few sentences how the system behaves.

```

time VAR Word
speed VAR Word
LABEL1:
PAUSE 14
PULSOUT 15, 5
PULSIN 15, 1, time
IF (time<150) THEN
' time of 150 is about two inches
  PULSOUT 14, 750
' this sends a 1.5 millisec pulse which will
command the servo to remain still
  GOTO LABEL1
ELSE
  GOTO LABEL2
ENDIF
LABEL2:
PULSOUT 15, 5
PULSIN 15, 1, time
IF (time>450) THEN
  PULSOUT 14, 1000
' command the servo to go forward
ELSEIF (time<300) THEN
  PULSOUT 14, 500
' command the servo to go backward
ELSE
  PULSOUT 14, 750
' command the servo to remain still
ENDIF
PAUSE 14
GOTO LABEL2

```



**This code would have the vehicle wait and idle as long as a barrier (the door of the robot's house?) is no more than 2 inches from the front of the vehicle. Once that barrier is removed, the vehicle would drive forward until it meets an object. It should stop when the object (the rail of the contest field?) is 6 inches from the nose and wait. If subsequently, something new (a botherbot?) is placed between the vehicle and the object, the vehicle backs up to seek a 4 inch spacing. So the polite little bot will step aside for the bother bot. Then it wil close up the gap again to place the rail 6 inches away again.**

**The most common concern I had with student's answers is that they mixed up concerns of the first "waiting" phase with the subsequent object seeking phase. It was common to see descriptions like "when the object is beteen two and four inches from an object it will...". But the two inch distance is only in the waiting phase and the four inch distance is only relevant in the "seeking" phase. This is a little like describing a frog jumping off a lilypad and then breathing in the water using its gills. It used to have gills when it was a tadpole, but, when it was a tadpole, it couldn't jump off a lily pad.**