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2.007 Design and Manufacturing I  
Spring 2009

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# 2.007 –Design and Manufacturing I

## Mechanisms:

### Kinematic Analysis and Synthesis

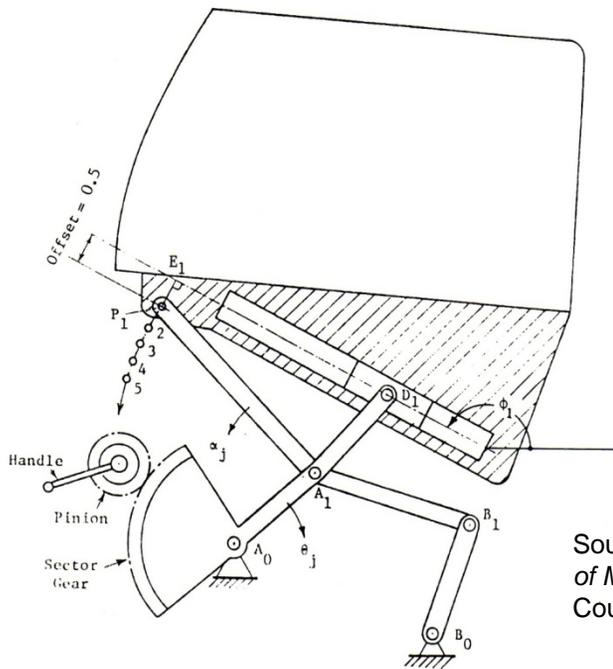


Image removed due to copyright restrictions. Please see  
<http://www.automotive-illustrations.com/img/suspensions/accord-rear-sus.jpg>

Source: Fig. 4-31 in Suh, C. H. *Computer Aided Design of Mechanisms Part A*. Self-published book, 1989.  
Courtesy of Dr. C. H. Suh. Used with permission.

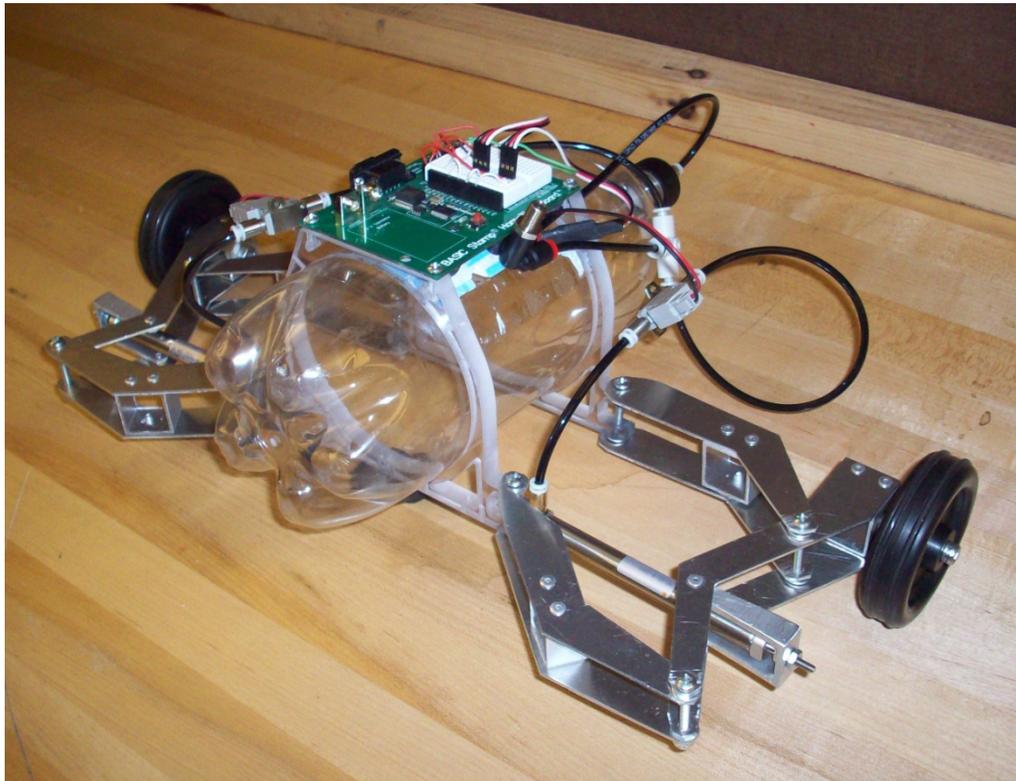
Presented by Dan Frey on 24 FEB 2009

# Today's Agenda

- Collect homework #1
- Distribute homework #2
- Begin mechanisms
  - Mobility / degrees of freedom
  - Some basic mechanisms
  - Kinematic analysis
  - Mechanism synthesis
  - Solving systems of equations

# 2D Mechanisms

- Where are they found?
- What do they do?



# 2D Mechanisms

- Where are they found?
- What do they do?

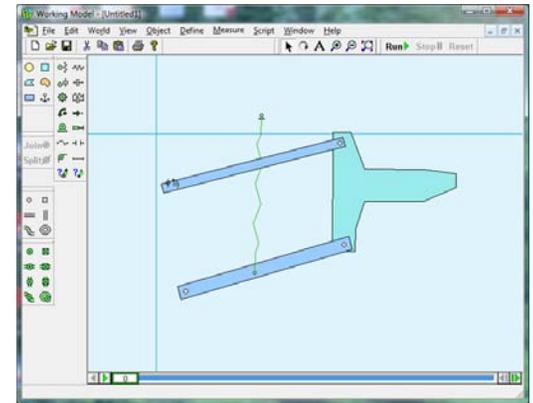


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<http://mossmotors.com/Graphics/Products/Schematics/SPM-032.gif>

# 2D Mechanisms

- Where are they found?
- What do they do?

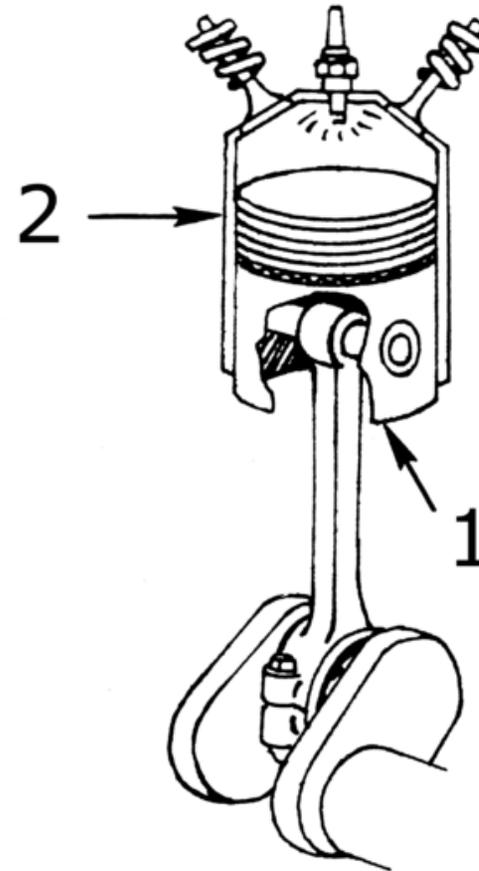
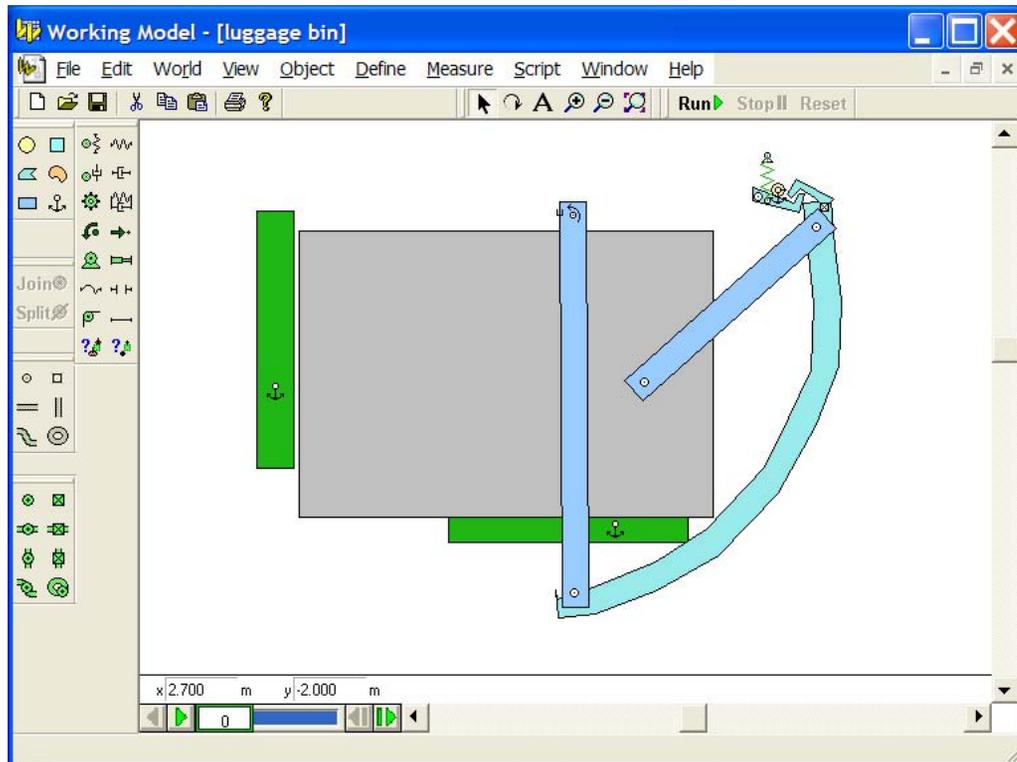


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# 2D Mechanisms

- Where are they found?
- What do they do?

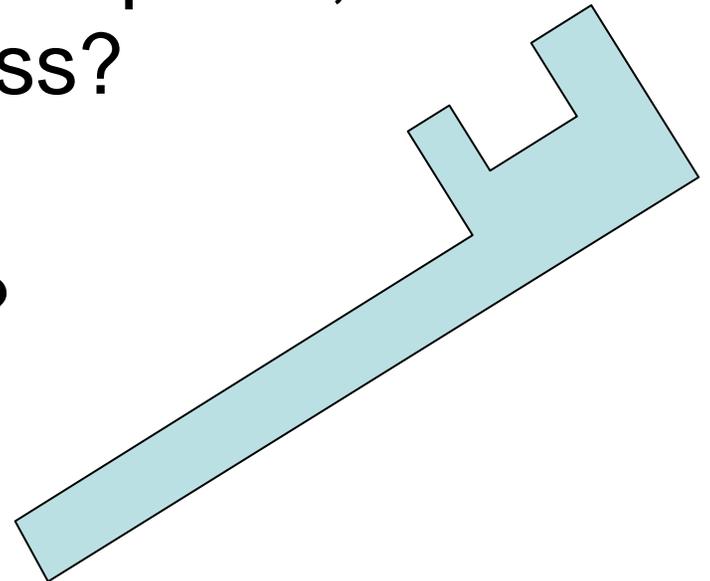


# Two Goals in our Study of Mechanisms

- Analysis
  - Given a mechanism, predict its behavior
  - Degrees of freedom
  - Path of motion
  - Dynamics -- velocity, acceleration
  - Force transmission
- Synthesis
  - Given a desired behavior, develop a mechanism

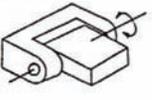
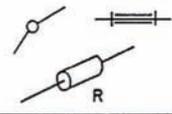
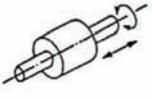
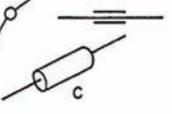
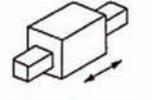
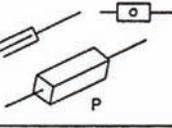
# Degrees of Freedom

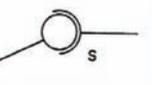
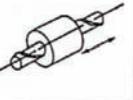
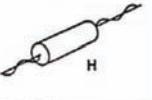
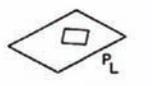
- DOF = the number of independent parameters required to completely define the entity's location and orientation
- If a body is constrained to a plane, how many DOF does it possess?
- If a pin joint is added?
- If a sliding joint is added?
- If it rolls without slipping?



# Types of “Joints”

- How do these attachments affect the degrees of freedom?
- What about welding / clamping?

Name of pair	Mechanical joint forms of pairs	Schematic Representations
1. Revolute (R)		
2. Cylinder (C)		
3. Prism (P)		

4. Sphere (S)		
5. Helix (H)		
6. Plane (P <sub>L</sub> )		

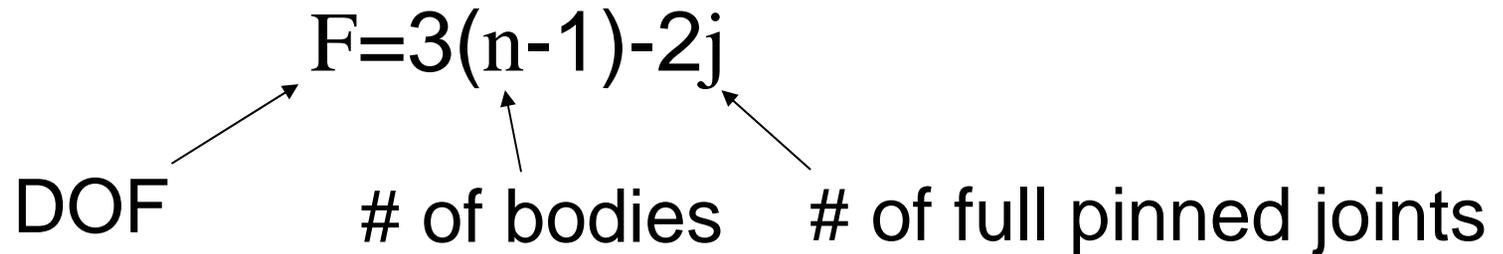
Source: Figure 4 in Suh, C. H. *Computer Aided Design of Mechanisms Part B*. Self-published book, 1992.  
 Courtesy of Dr. C. H. Suh. Used with permission.

# Mobility Criteria in 2D

- Kutzbach criterion (to find the DOF)

$$F = 3(n-1) - 2j$$

DOF                      # of bodies                      # of full pinned joints



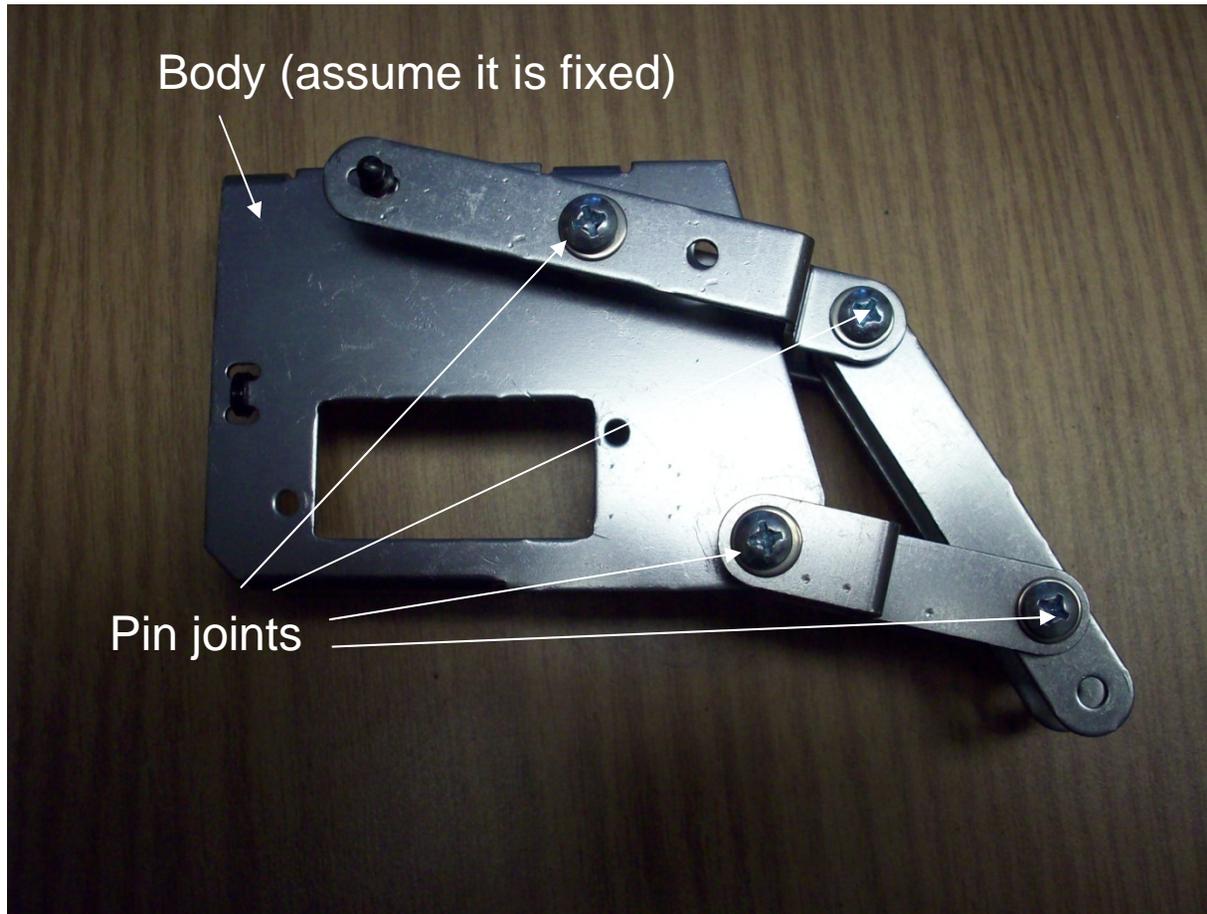
- Grübler criterion (to have a single DOF)

$$2j - 3n + 4 = 0$$

# Mobility Criteria in 2D

- Count the rigid bodies
- Every rigid body has 3 DOF
- For the “joints”
  - Consider how many degrees of freedom each joint removes
  - Sum up the degrees of freedom removed
- Consider the possibility of redundancy of certain “joints”
- Do the arithmetic

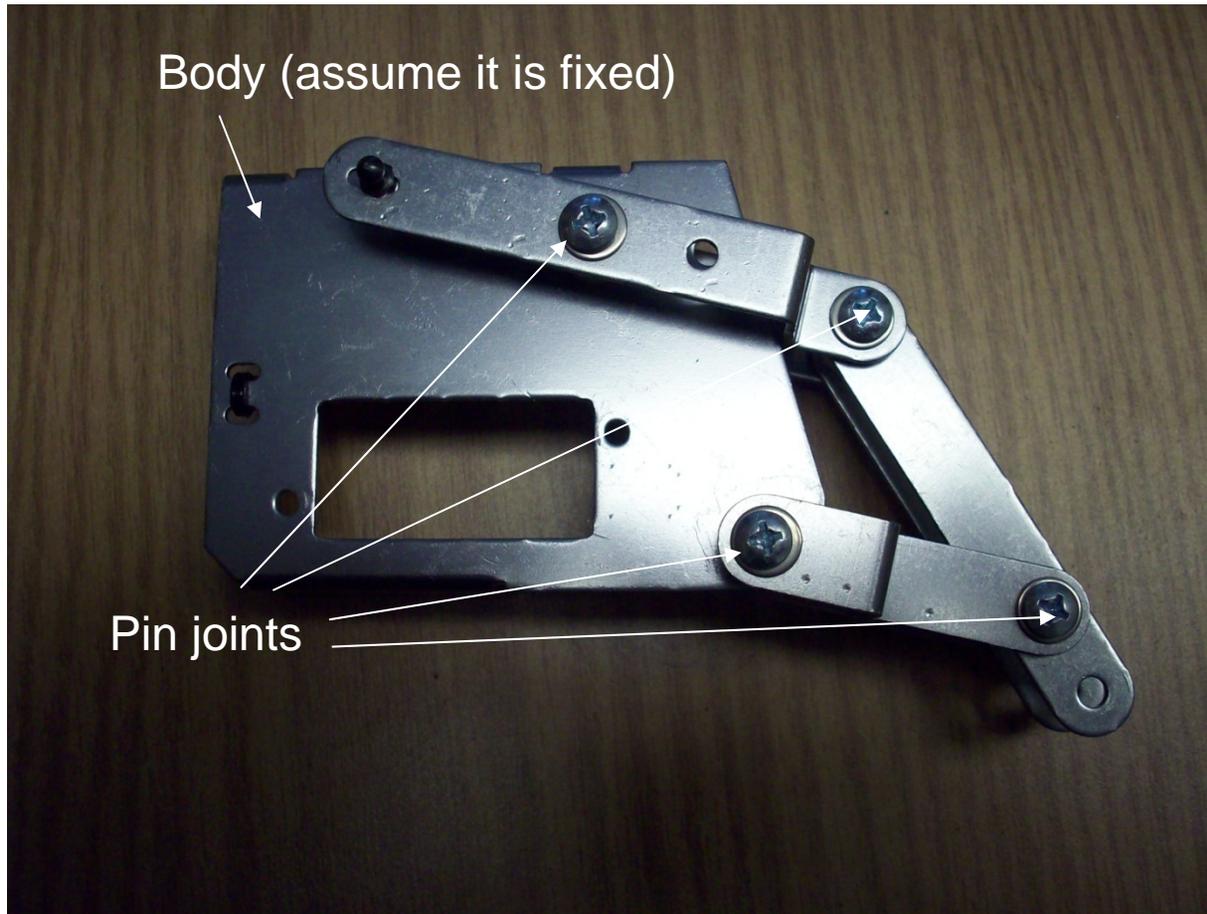
# Practice Together



How many degrees of freedom?

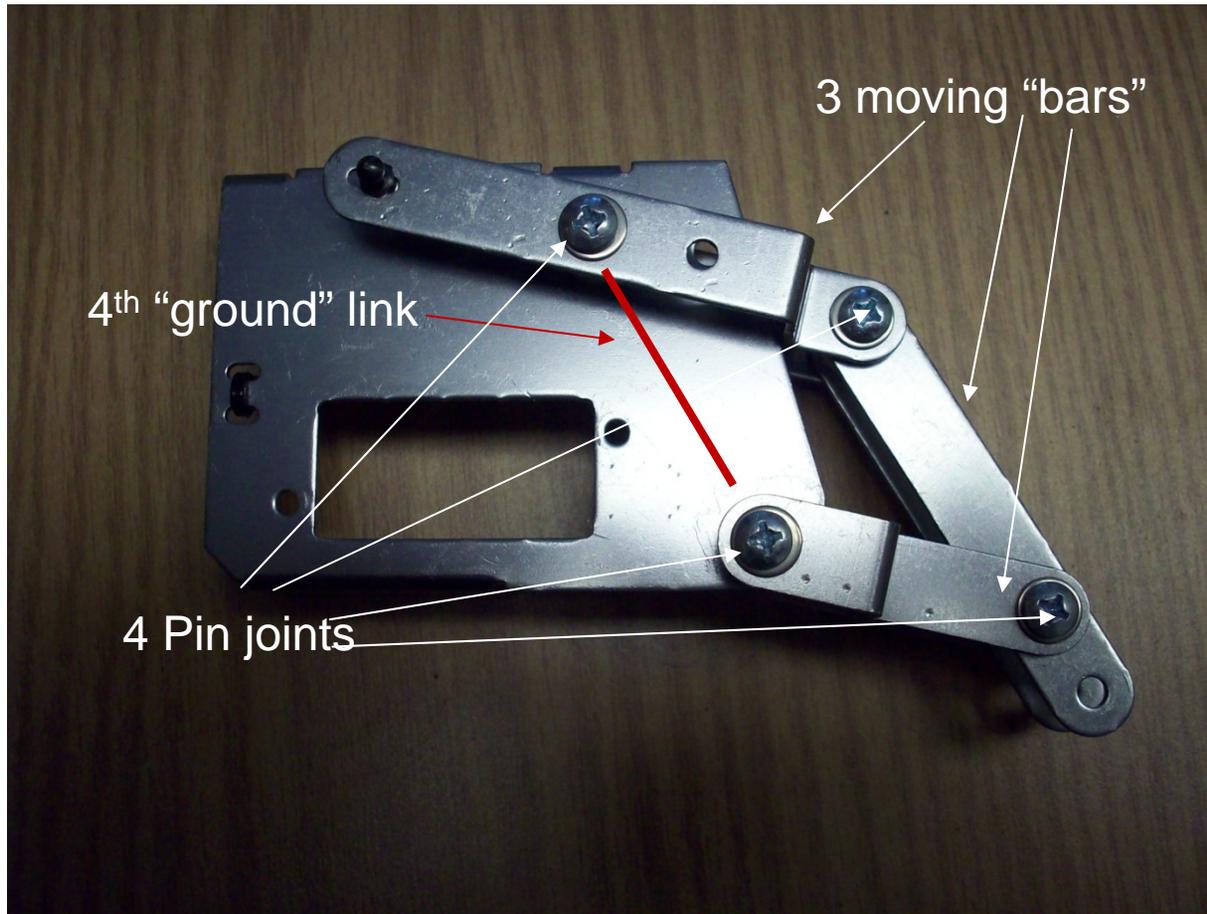
# Practice

The answer is one degree of freedom. Three bodies capable of motion in the plane each with 3 DOF. Four full pin joints each remove 2 DOF.  $3*3-4*2=1$



How many degrees of freedom?

# The “4 Bar” Mechanism



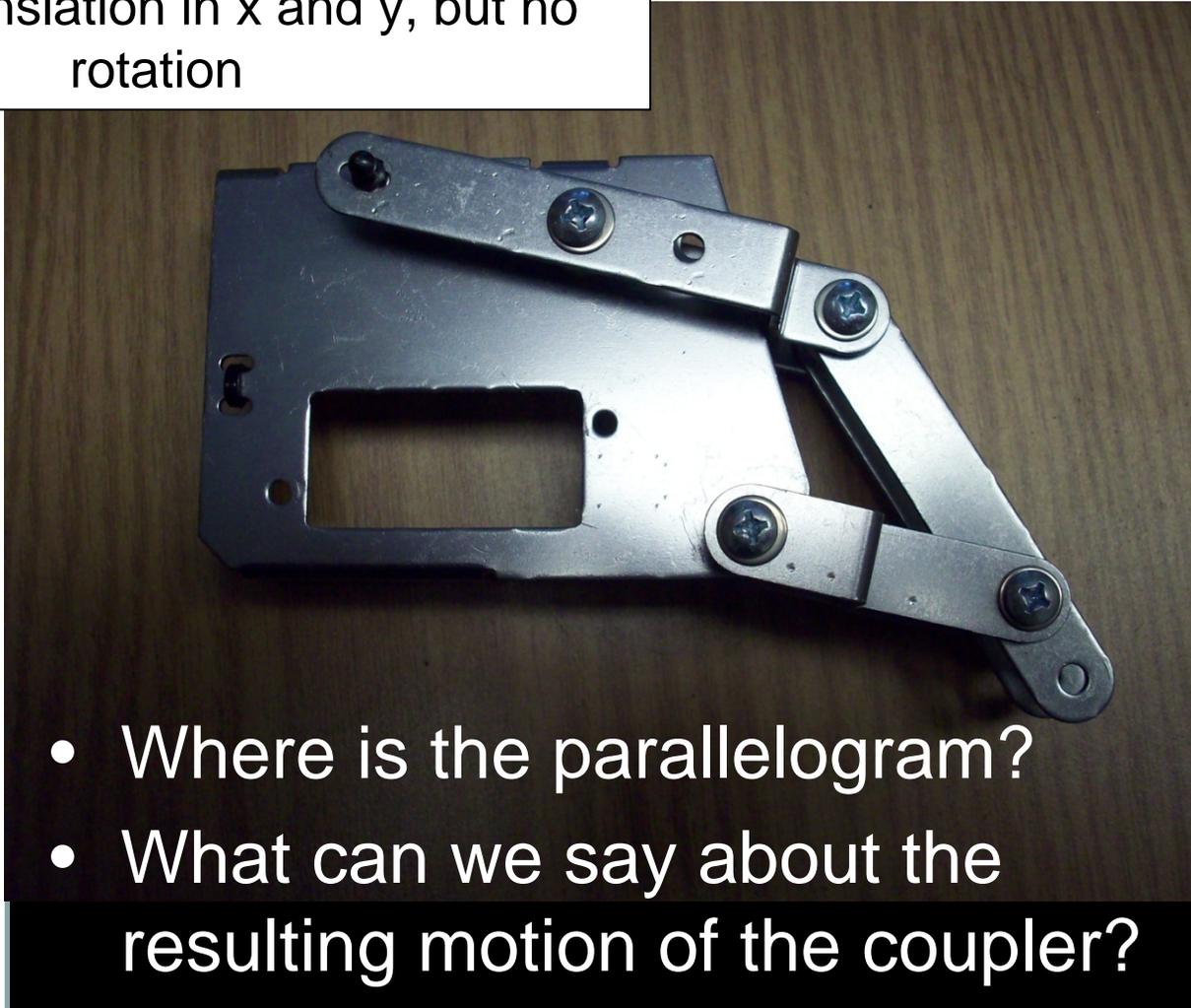
# Parallelogram Linkage



- Where is the parallelogram?
- What can we say about the resulting motion of the coupler?

# Parallelogram Linkage

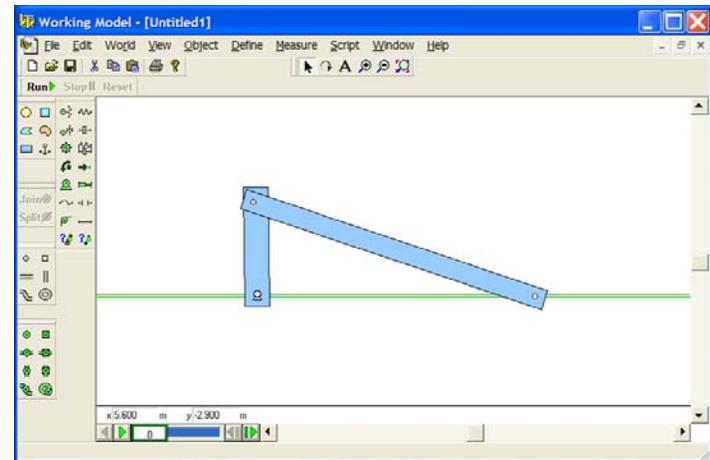
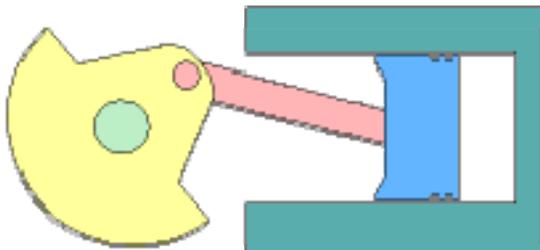
The resulting motion of the coupler exhibits translation in x and y, but no rotation



- Where is the parallelogram?
- What can we say about the resulting motion of the coupler?

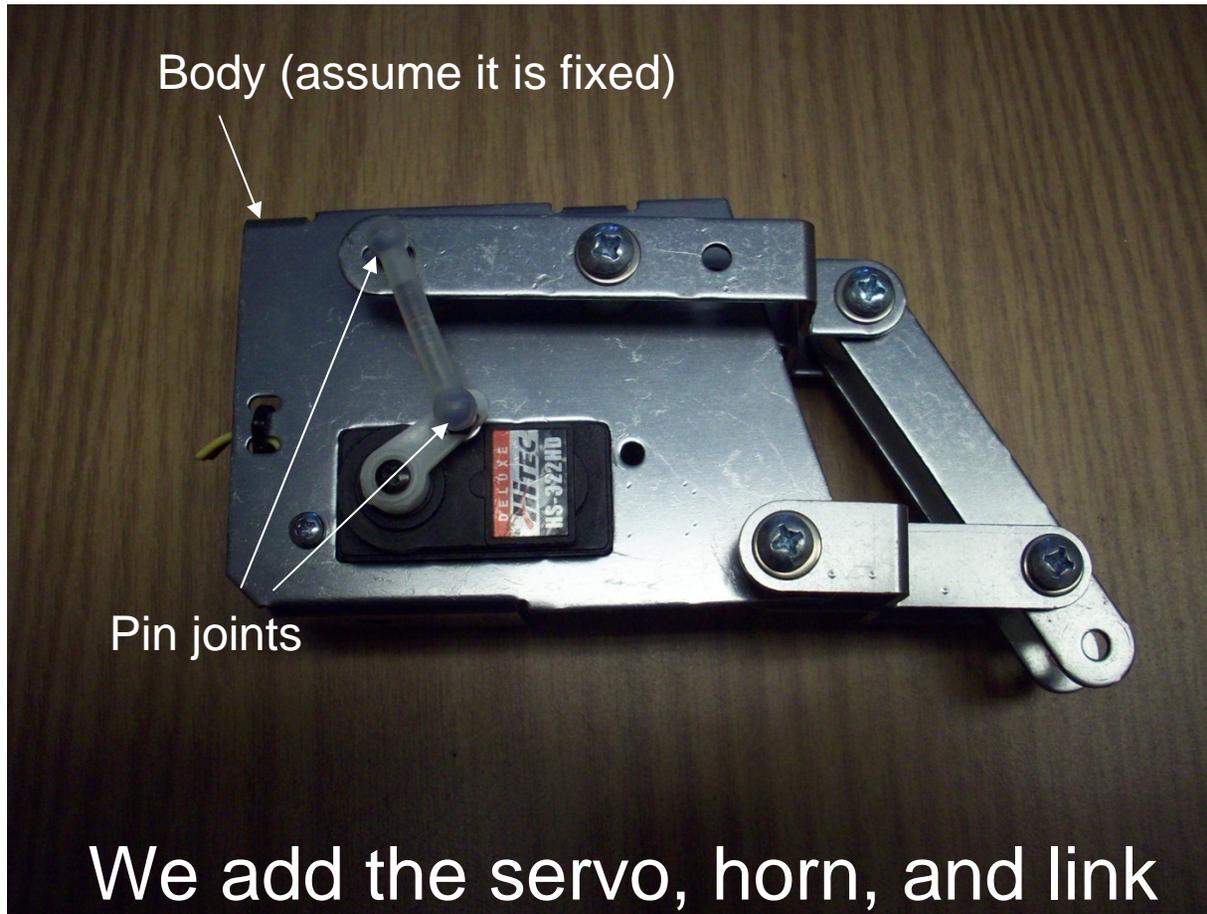
# Slider-Crank Mechanism

- Kinematically the same (in the limit) as a 4 bar mechanism with a very long output crank



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# More Practice Together



We add the servo, horn, and link  
NOW how many degrees of freedom?

# More Pract

The answer is one degree of freedom.  
Two more bodies capable of motion in the plane each with 3 DOF. Three more full pin joints each remove 2 DOF.  
 $2*3-3*2=0$  so no net change

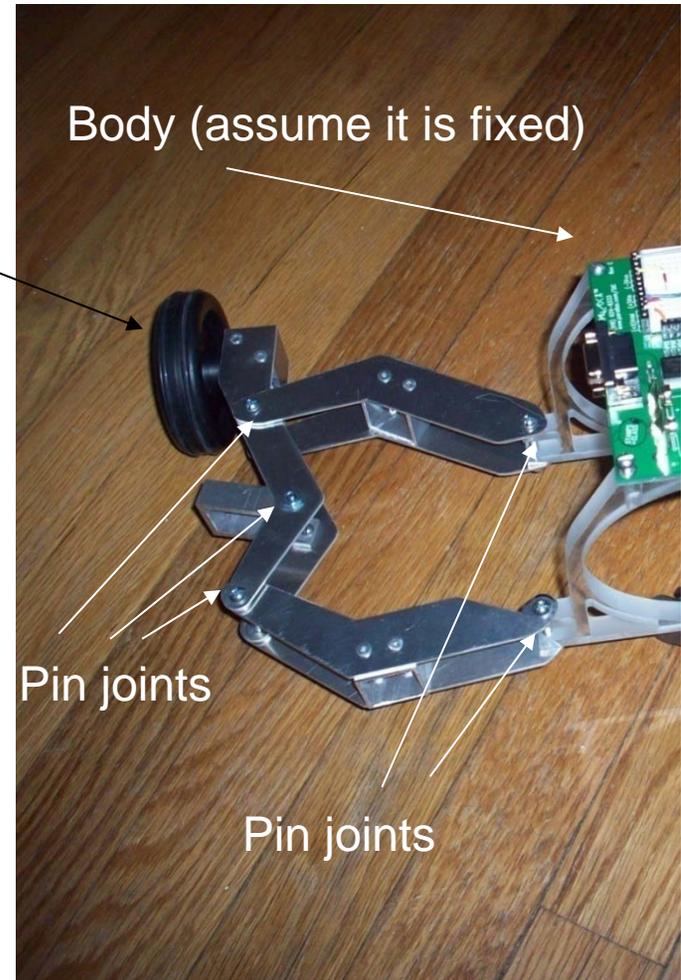


# Now You Do It

- How many degrees of freedom does this mechanism possess?

- 1) 1 DOF
- 2) 2 DOF
- 3) 3 DOF
- 4) 4 DOF
- 5) 0 DOF

Assume the wheel is fixed to its shaft and suspended over the table for the purpose of this question



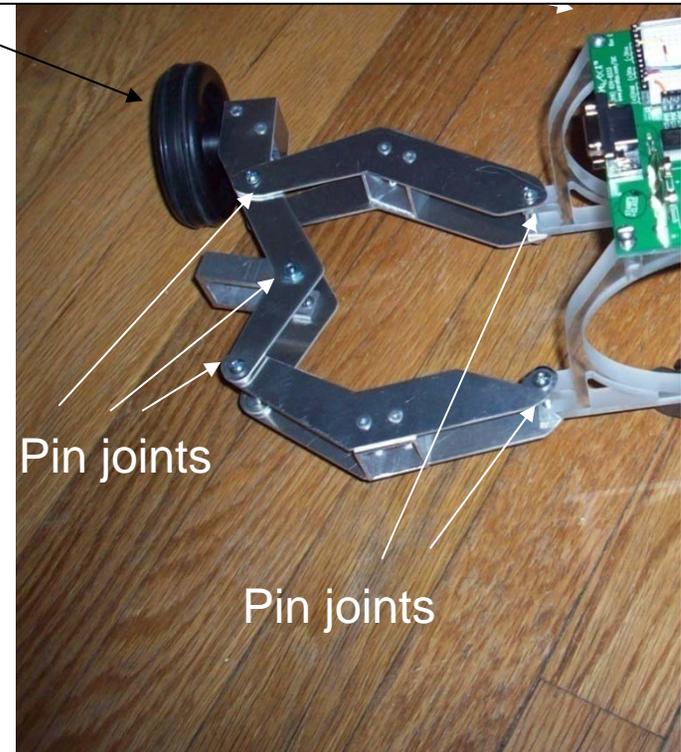
# Now You

- How many degrees of freedom does this mechanism possess?

- 1) 1 DOF
- 2) 2 DOF
- 3) 3 DOF
- 4) 4 DOF
- 5) 0 DOF

Assume the wheel is fixed to its shaft and suspended over the table for the purpose of this question

The answer is two degrees of freedom. Four bodies capable of motion in the plane each with 3 DOF. Five full pin joints each remove 2 DOF.  $4 \cdot 3 - 5 \cdot 2 = 2$ . One DOF is the desired motion of the link on which the wheel is attached. One DOF is the link in the middle that is free to spin for now until the pneumatic is attached.

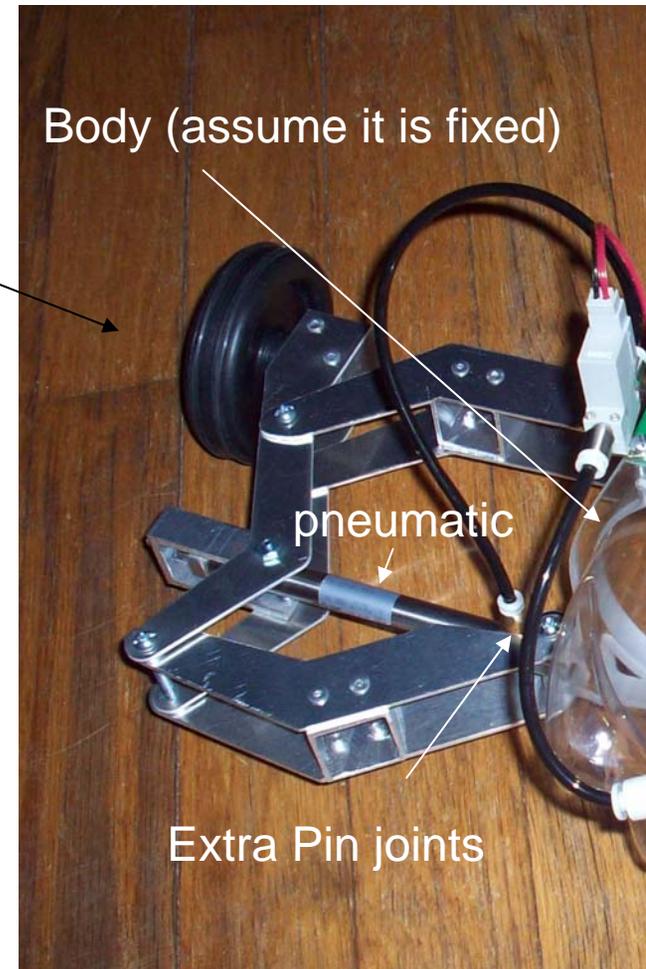


# Now You Do It Again

- How many degrees of freedom does this mechanism possess?

- 1) 1 DOF
- 2) 2 DOF
- 3) 3 DOF
- 4) 4 DOF
- 5) 0 DOF

Assume the wheel is fixed to its shaft and suspended over the table for the purpose of this question



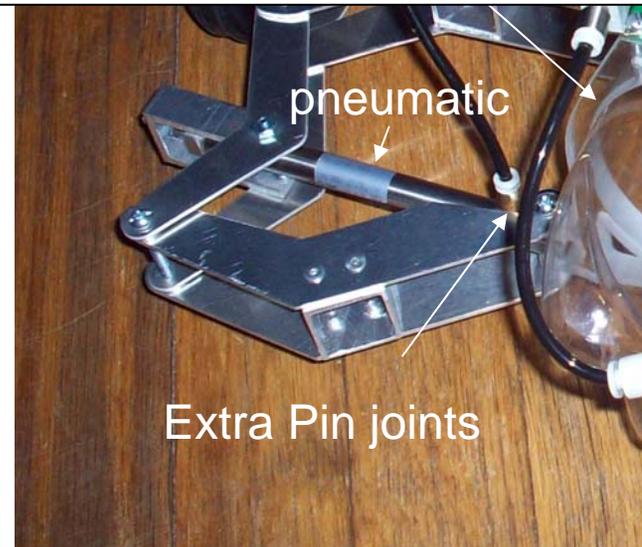
# Now You

- How many degrees of freedom does this mechanism possess?

- 1) 1 DOF
- 2) 2 DOF
- 3) 3 DOF
- 4) 4 DOF
- 5) 0 DOF

Assume the wheel is fixed to its shaft and is suspended over the table for the purpose of this question

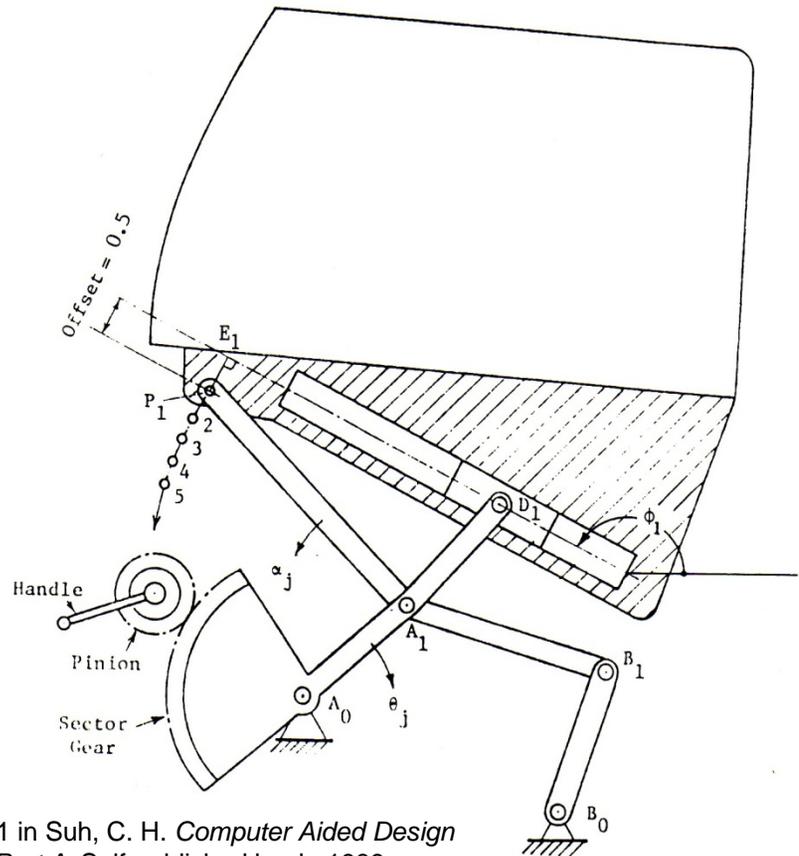
The answer is one degree of freedom. Two more bodies were added, the piston body and the piston rod. Three attachments exist. Sliding motion constrains the piston rod within the piston body (-2DOF). The piston body is attached with a full pin joint to the frame (-2DOF). Finally, the piston rod is fixedly attached to the pivot so that they move together (-3DOF)  $2*3-2-2-3=-1$ . One DOF is removed from a system that previously had 2 DOF.



# One More Time

- How many degrees of freedom does this mechanism possess?

- 1) 1 DOF
- 2) 2 DOF
- 3) 3 DOF
- 4) 4 DOF
- 5) 0 DOF

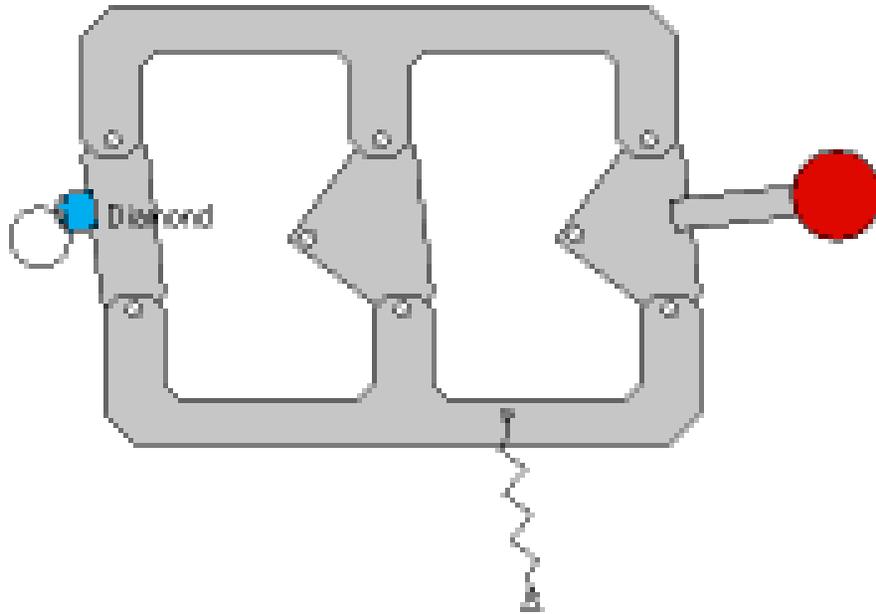


Source: Fig. 4-31 in Suh, C. H. *Computer Aided Design of Mechanisms Part A*. Self-published book, 1989. Courtesy of Dr. C. H. Suh. Used with permission.



# Redundancy

- How can I make sense of this?



5 bodies

8 pin joints

$15 - 16 = -1$

But it looks  
like +1 in the  
animation

Courtesy of Design Simulation Technologies, Inc.  
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<http://workingmodel.design-simulation.com/WM2D/simulationlibrary/advmechanisms.php>

# Degrees of Freedom in 3D

- How many degrees of freedom does this mechanism possess?

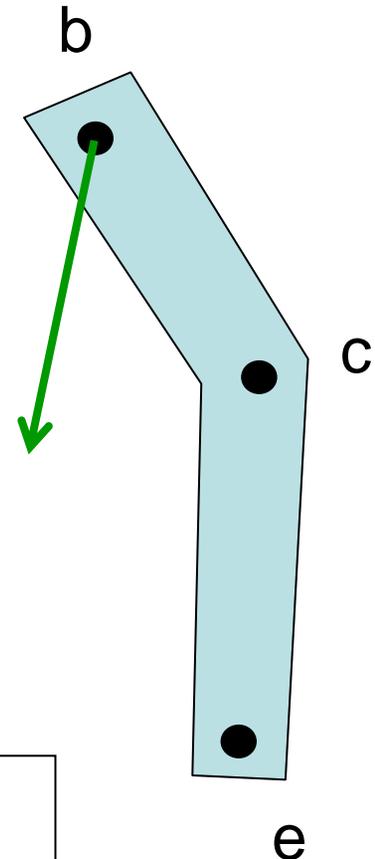
We won't emphasize  
3D mechanisms in this  
course

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<http://www.automotive-illustrations.com/img/suspensions/accord-rear-sus.jpg>

Rear suspension of a Honda Accord

# Velocity Analysis

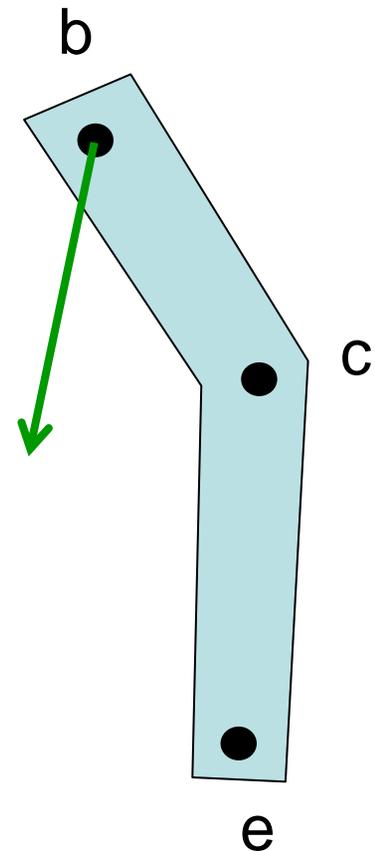
- This object is translating and is NOT rotating
- The velocity of point “b” is indicated by a green arrow
- What do we know about the velocity of point “e”?



Point e must be described by motions which are the superposition of the translation of b and rotation about b.

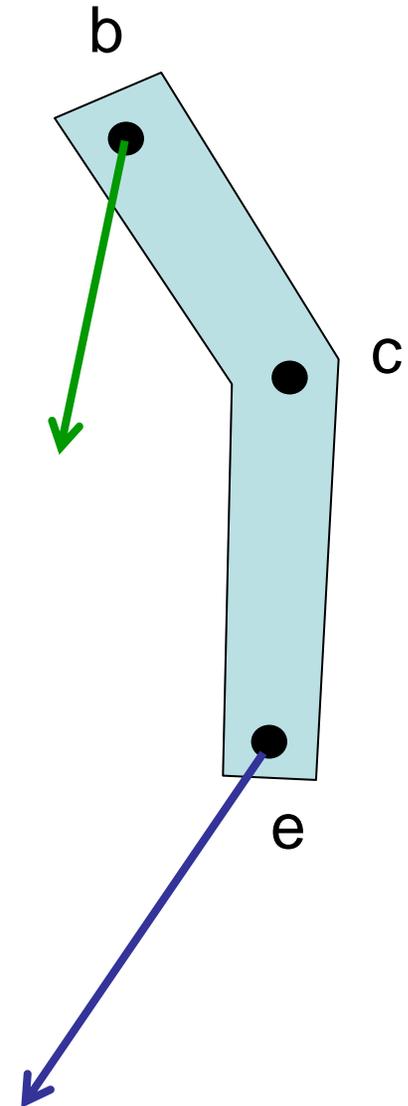
# Velocity Analysis

- This object is translating and ALSO rotating about “b” at rate  $\omega$
- The velocity of point “b” is indicated by a green arrow
- Can we construct the velocity of vector of point “e”?



# Velocity Analysis

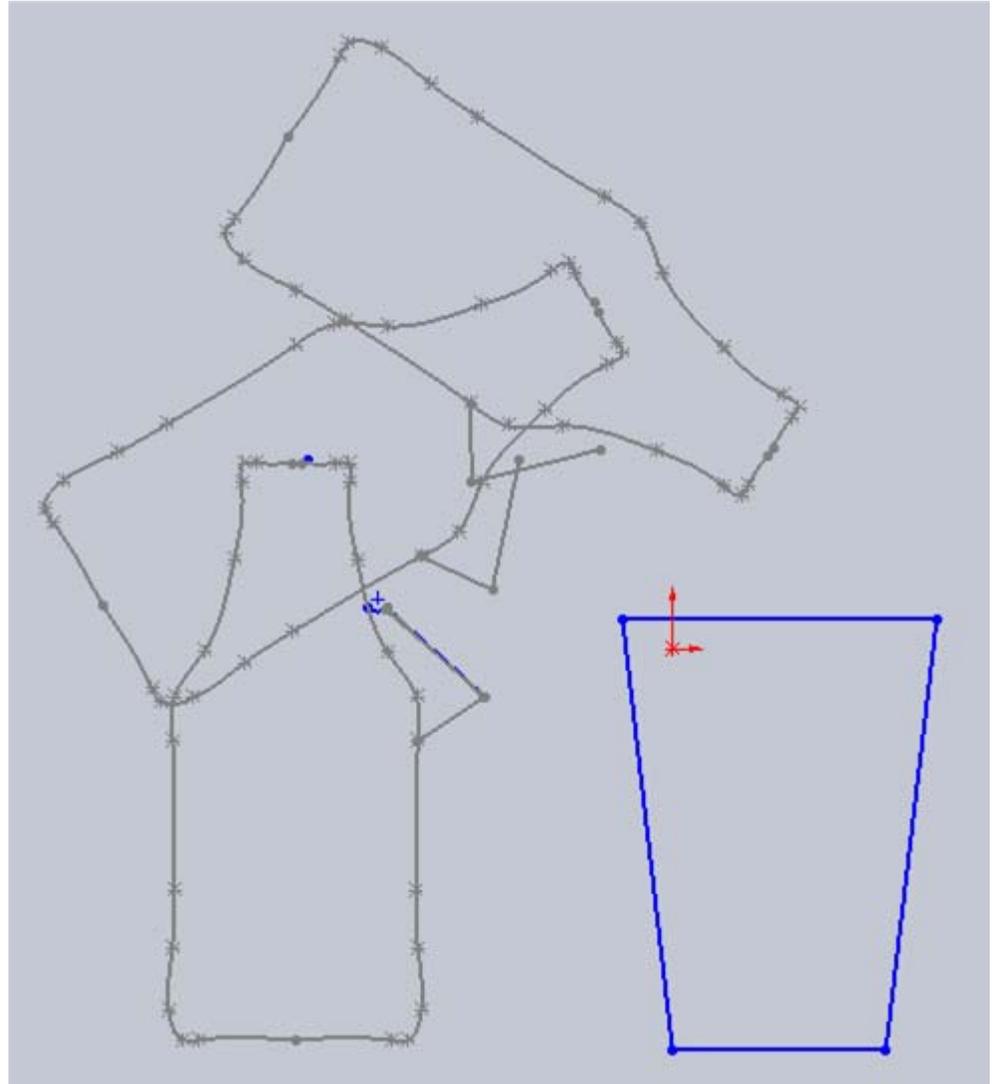
- This object is a rigid body
- The velocity of point “b” is indicated by a green arrow
- The velocity of point “e” is indicated by a blue arrow
- Can the vector “e” take any value, or are there some restrictions on values it might have?



Yes, the tip of the vector must lie on a line.

# 3 Position Synthesis

- Say we want a mechanism to guide a body in a prescribed way
- Pick 3 positions
- Pick two attachment points
- The 4 bar mechanism can be constructed graphically



# A Proper Pour

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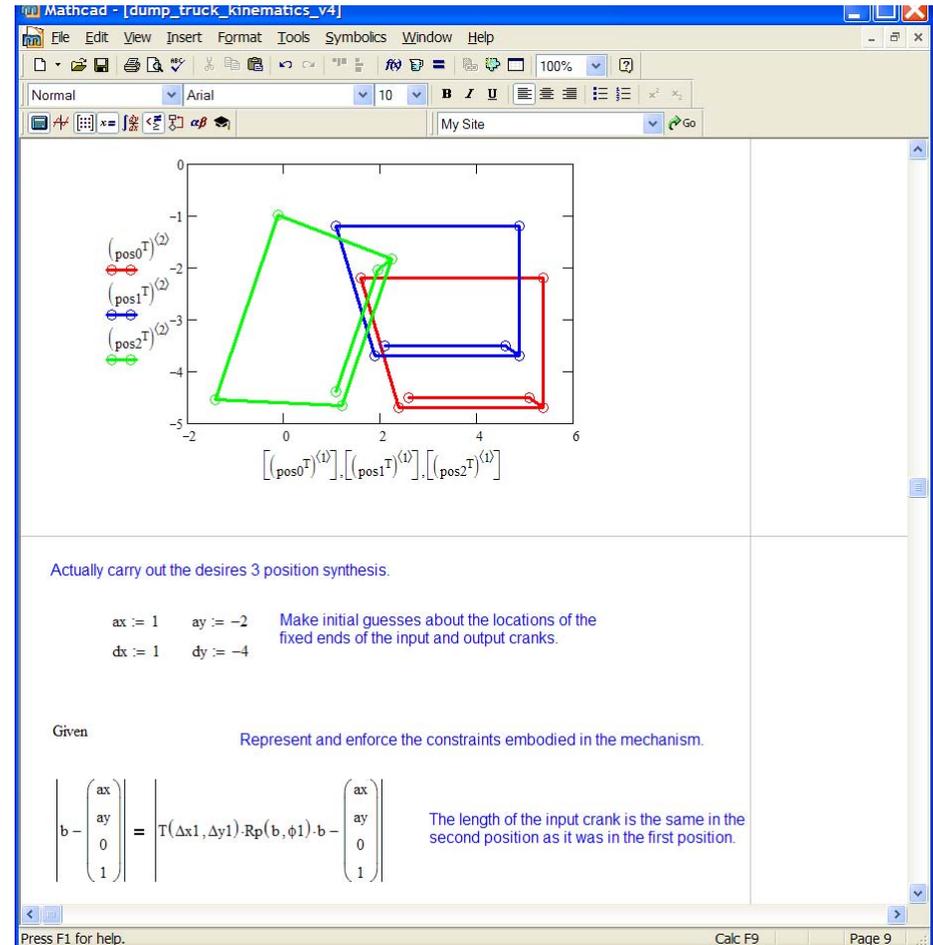
# Automating the Task

- Very simple mechanically
- Nicely compact
- Springs could allow the servo to be loaded uniformly on the up and down stroke

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# Computer Aided 3 Position Synthesis

- Express the same basic ideas from the graphical construction in mathematical form
- Computers can easily solve coupled systems of non-linear equations (well, usually they can)



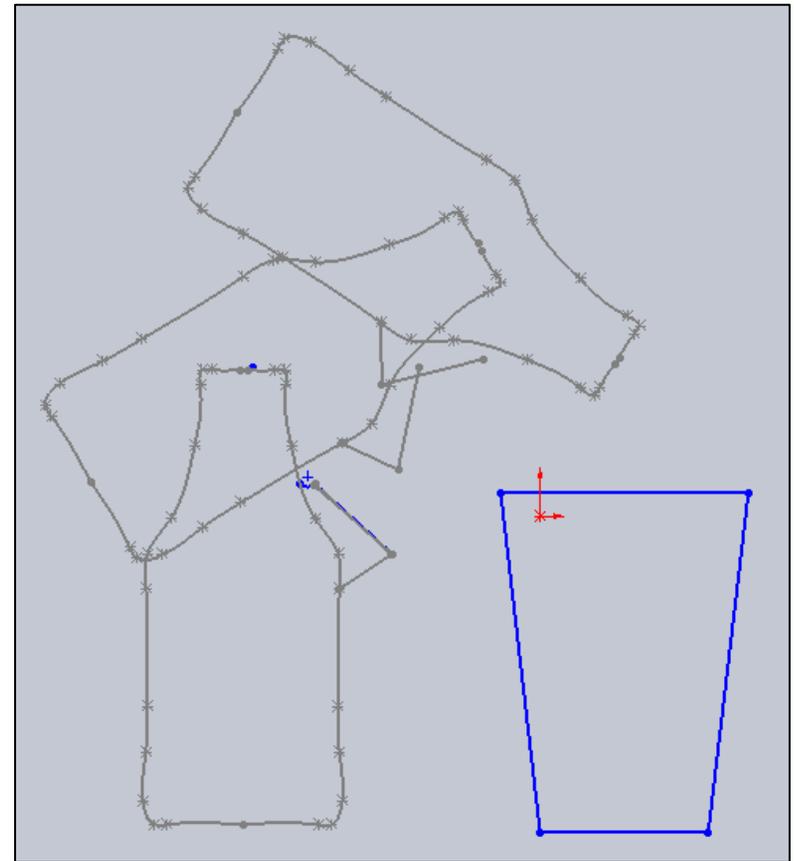
# Homogeneous Transformation Matrices

- How can we mathematically express the motions of a rigid body?
  - Translations
  - Rotations
  - Both
- Matrix multiplication will do the trick

$$\text{body}' := \begin{pmatrix} \cos(\theta) & -\sin(\theta) & 0 & \delta_x \\ \sin(\theta) & \cos(\theta) & 0 & \delta_y \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \cdot \text{body}$$

# Computer Aided Mechanism Synthesis: More Possibilities

- Specify 4 or 5 positions of the body
- Specify a path rather than body positions (if angle is not important)
- Maximize objective functions on linkage geometry



# Next Steps

- Thursday 26 FEB, 11 AM
  - Lecture on CAD
- Tuesday 3 MAR, 11 AM
  - Another lecture on mechanisms
- Homework #2 due 5 MAR by 11AM
- Quiz #1 on 10 MAR, 11AM-12:30