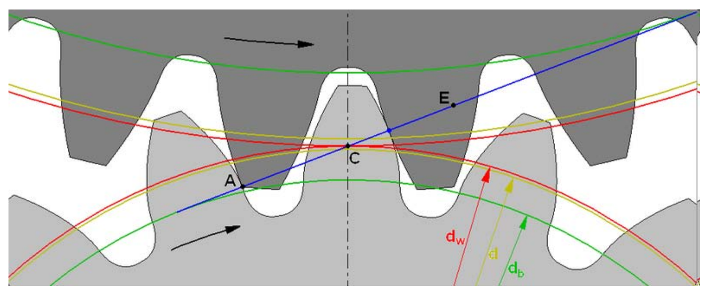
2.007 Design and Manufacturing I
Spring 2009

2.007 –Design and Manufacturing I

Gears: Terminology, Geometry, Gear Trains, Strength



Presented by Dan Frey on 17 MAR 2009

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Today's Agenda

- Distribute homework #3
- Gears
  - Applications
  - Types
  - Terminology / nomenclature
  - Congugate action
  - Involute curve
  - Analysis & design

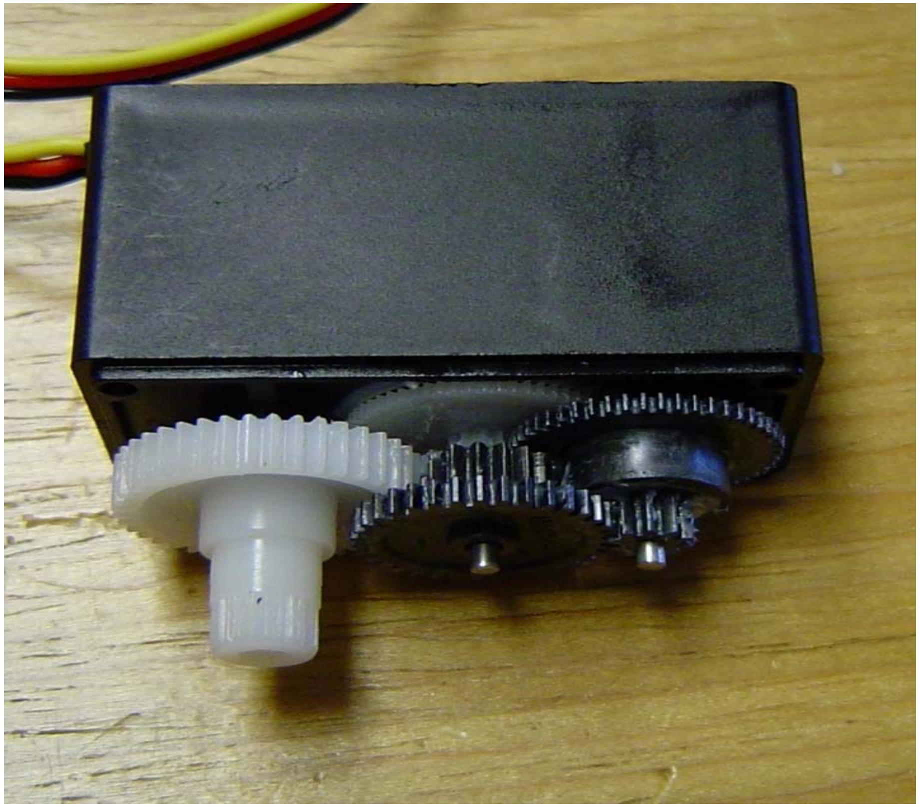
Applications of Gears

Images removed due to copyright restrictions. Please see
http://mossmotors.com/Graphics/Products/Schematics/SPM-027.gif
http://www.nmm.ac.uk/collections/displayRepro.cfm?reproID=A6263

Sobel, Dava, *Longitude*

Spur Gears

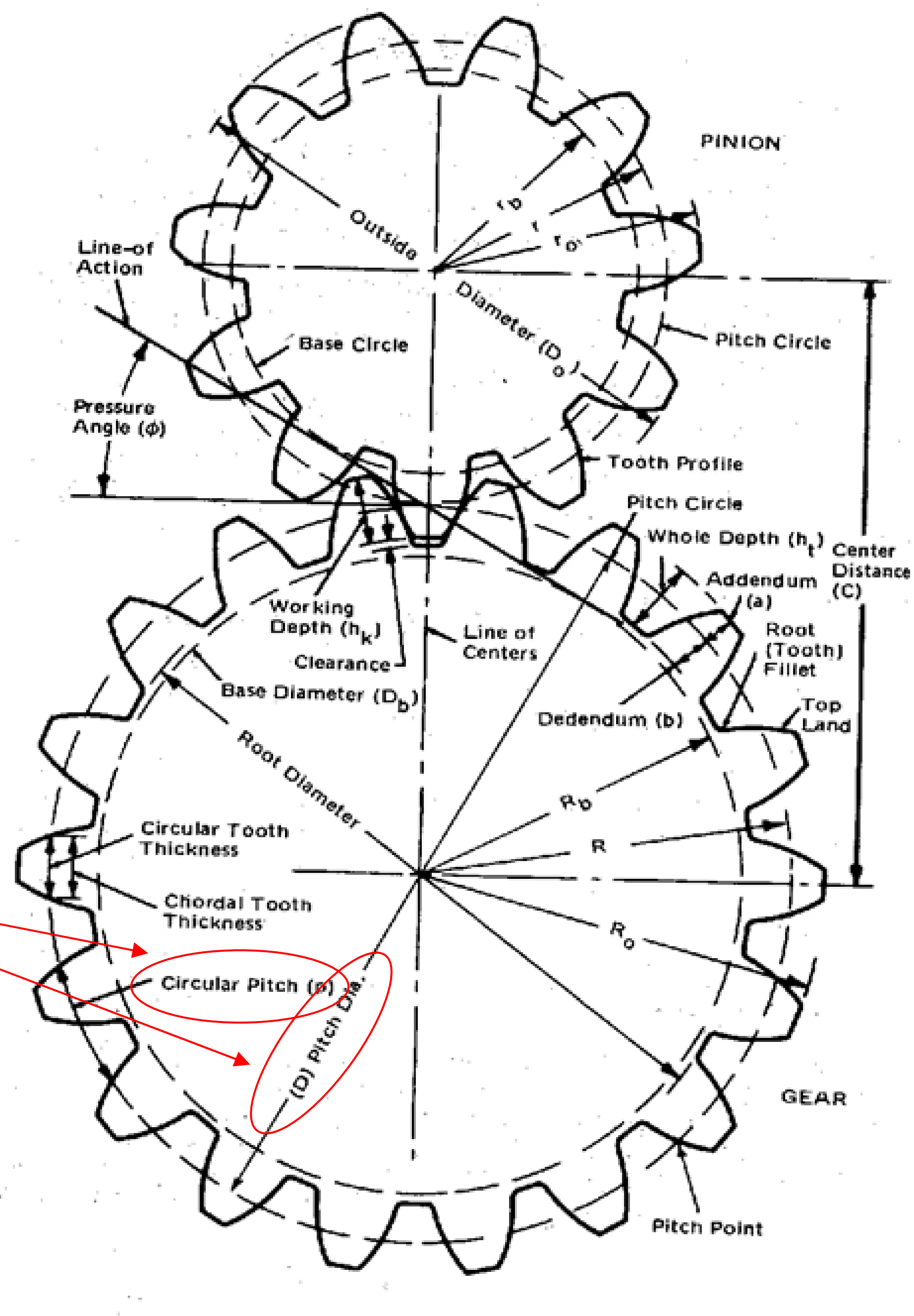
- Transmit motion between parallel shafts
- Teeth are parallel to the axis of rotation
- This is the simplest kind of gear we'll consider and most of today is dedicated to them



Gear Terminology

Diametral pitch (teeth per inch)
# of teeth on a gear with a
1 inch pitch diameter

Easily confused



Source: Fig. 1.1 in "Gears." *Design and Application of Small Standardized components Data Book 757*. Stock Drive Products, 1983. Accessed September 18, 2009. Courtesy of Stock Drive Products/Sterling Instrument.

Other Types of Gears

Helical

Image courtesy of perlmonger on Flickr.

Bevel

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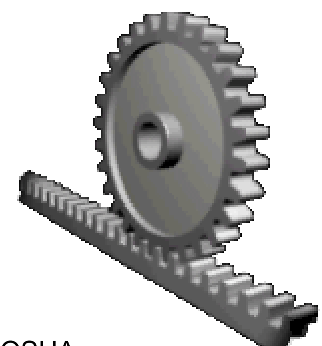
Worm

Rack



Courtesy OSHA.

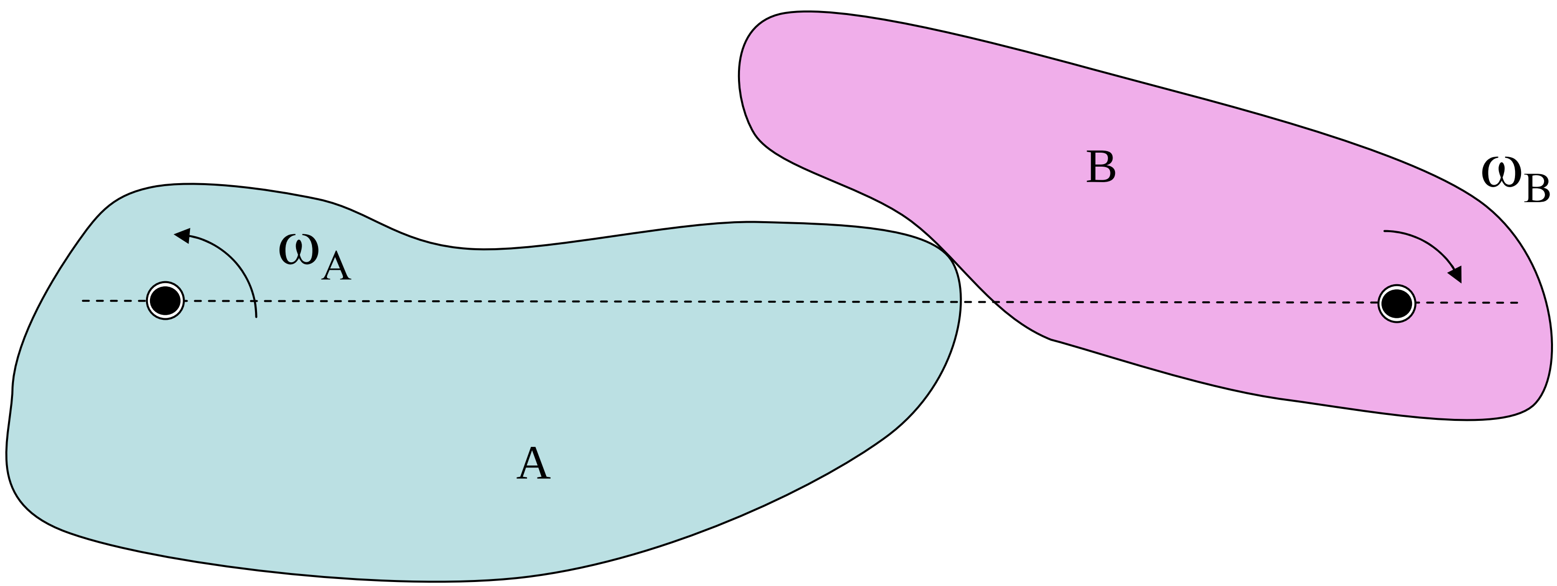
Early Gears

Drawings of waterwheels and gears removed due to copyright restrictions.

Roman watermills at Barbegal
300AD

Application for powering textile machinery
18th century

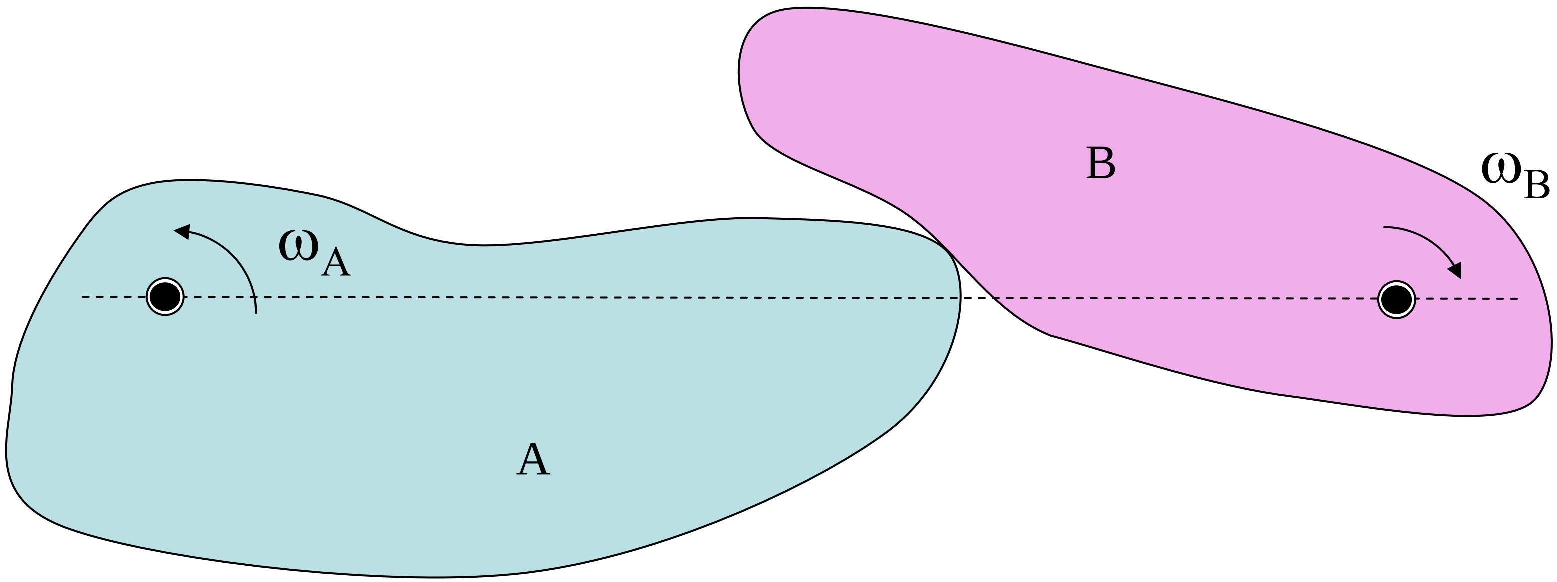
Conjugate Action



Let's say ω_A is a known. How can we determine ω_B?

Let's say ω_A is a constant with time. Can we synthesize a shape of body B so that ω_B is also constant with time?

Pitch Point

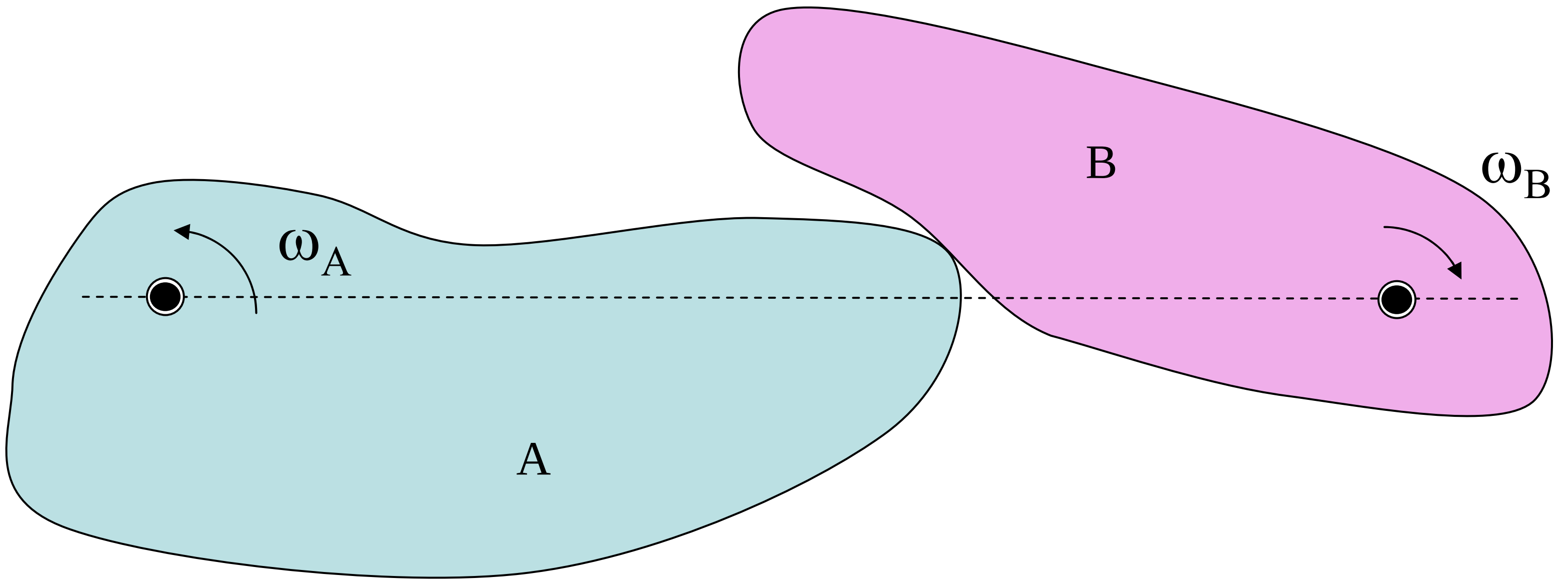


What is the line of action?

What is the pitch point?

What are the relationships among these?

Sliding and Rolling



When one body is driving another, do the surfaces slide, roll, or both?

What is the relationship to the pitch circles?

How could you determine this?

Rack Cutting

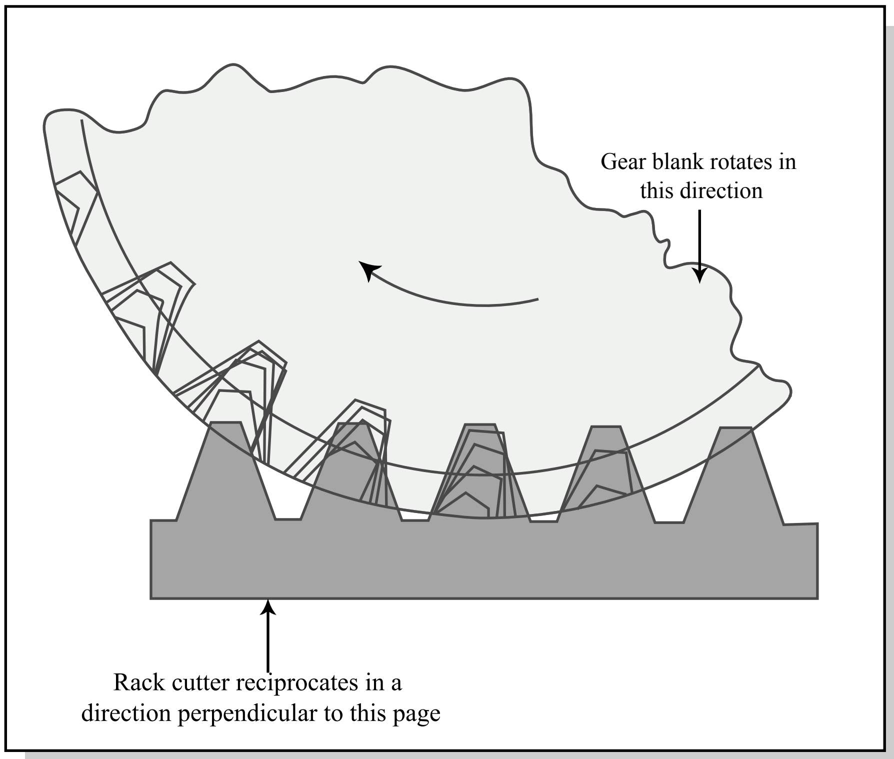
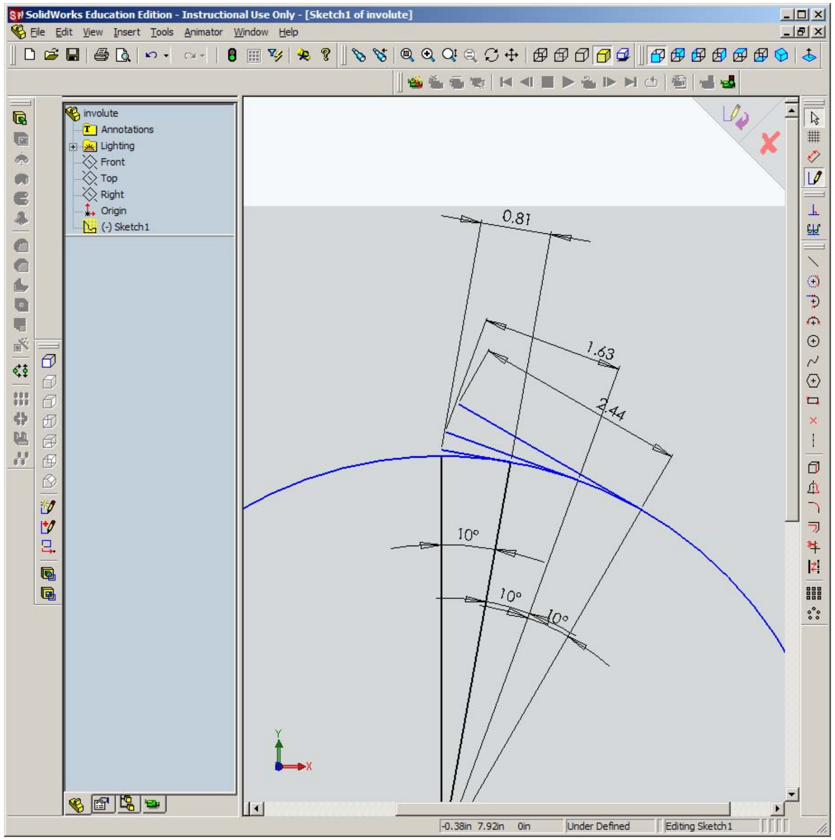


Figure by MIT OpenCourseWare.

- A way to get the relative motion you want
- Pick one shape as you wish
- Enforce the motion you want
- Cut away everything that interferes

Involute Profile

- How it is constructed
  - Demo
- Properties
  - Conjugate action
  - Allows design of whole sets of compatible gears
  - Conjugate action not sensitive to center distance variations



More Gear Terminology

From
Shigley and Mischke

Image removed due to copyright restrictions. Please see
http://commons.wikimedia.org/wiki/File:Gear_words.png

This geometry is not an involute.

Pressure Line

- Where the teeth contact, the surface normal defines a pressure line
- The force transmitted acts along this line
- The pressure line always includes the point of tangency between the pitch circles
- With the involute gear profile, the pressure line is constant

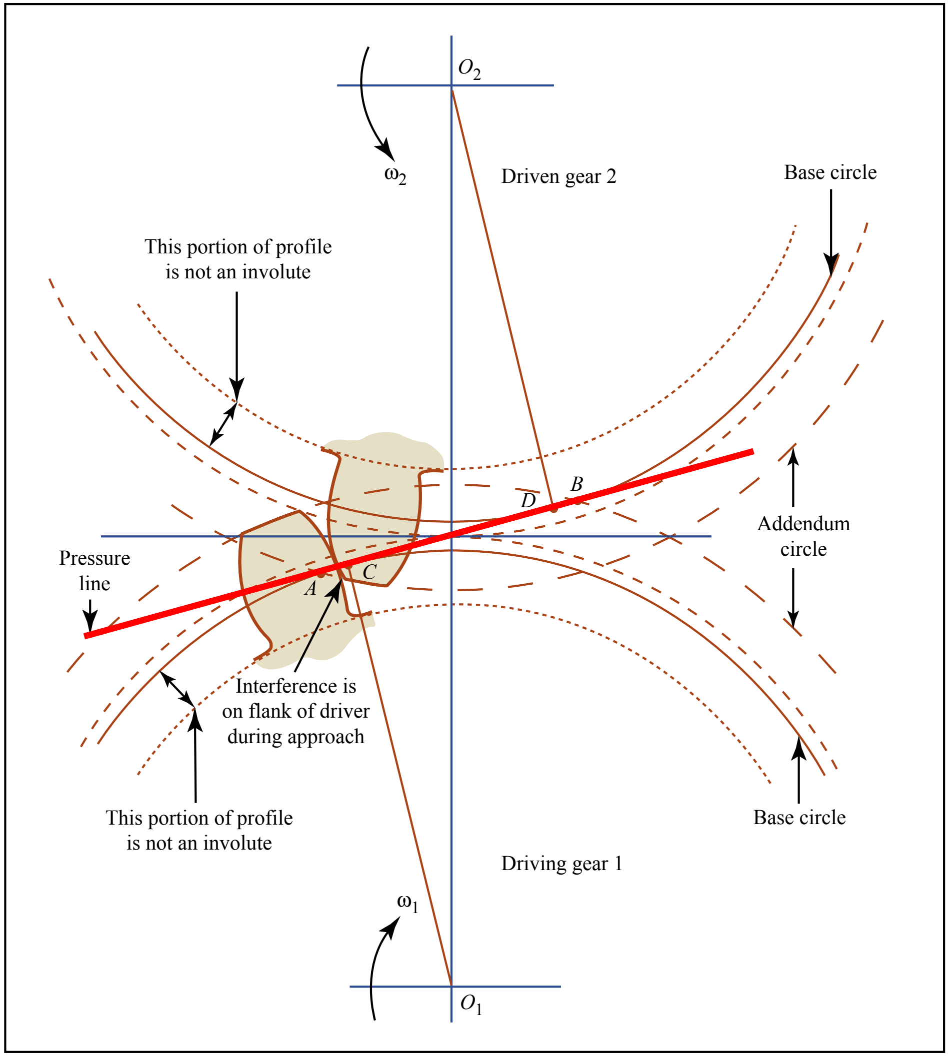
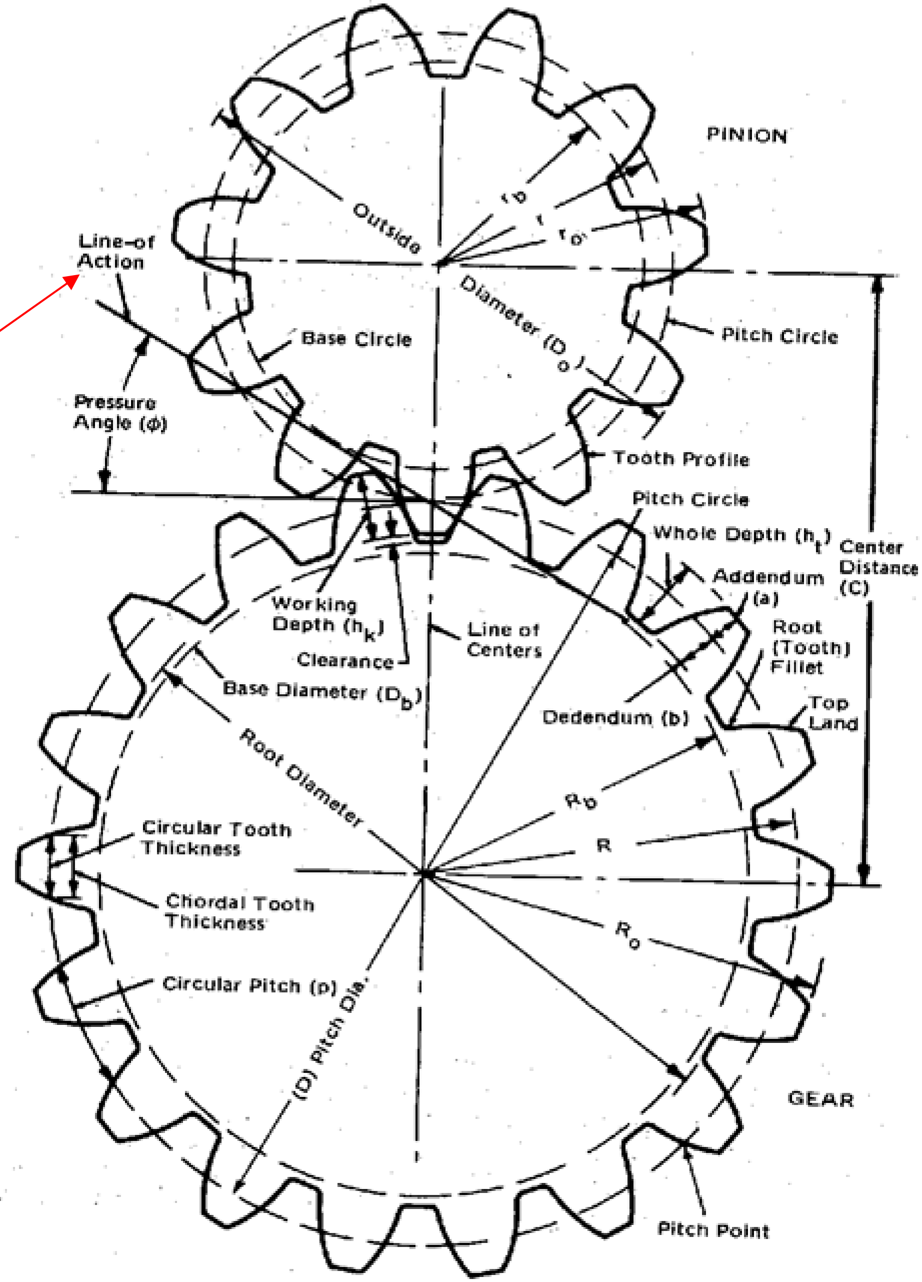


Figure by MIT OpenCourseWare.

From Shigley and Mischke

Gear Terminology

“Line of action” & “pressure line” & “generating line” are all synonymous



Source: Fig. 1.1 in “Gears.” *Design and Application of Small Standardized components Data Book 757*. Stock Drive Products, 1983. Accessed September 18, 2009. Courtesy of Stock Drive Products/Sterling Instrument.

Pressure Angle

- The pressure line acts at some angle to the tangent of the pitch circles
- This angle can be chosen by the designer
- It affects
  - Separation forces
  - Tooth shape

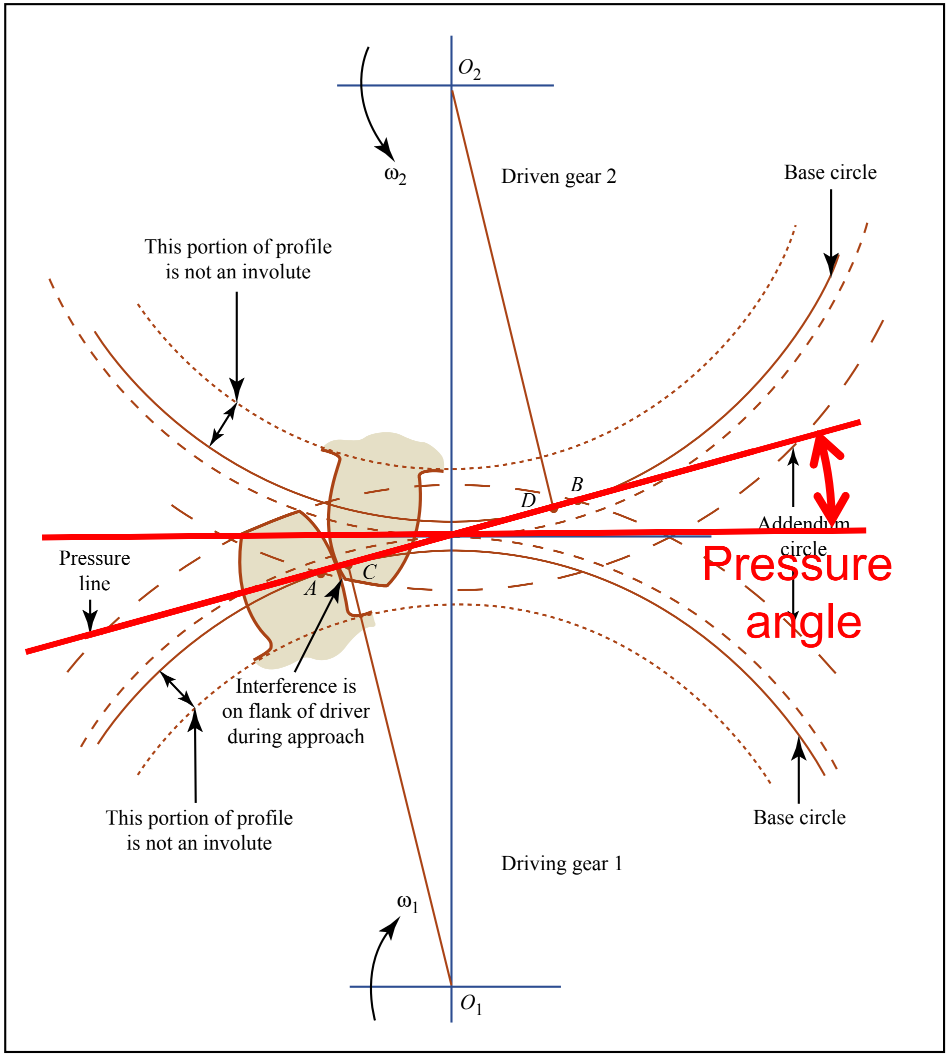


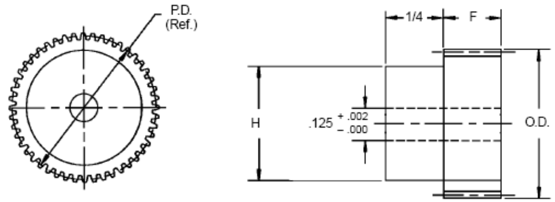
Figure by MIT OpenCourseWare.

From Shigley and Mischke

Concept Question

Spur Gears

24, 32, 48, and 64 Pitch 1/8" Bore AGMA Quality 4
Cold Rolled Steel and Brass 20° Pressure Angle



B 9

COLD ROLLED STEEL C12L14 OR C12L15 WITH SELENIUM	BRASS ALLOY 360					
STOCK NUMBER	STOCK NUMBER	NO OF. TEETH	PITCH DIA.	OUTSIDE DIA.	H	F
24 PITCH (.1309)						
PX24S-8	PX24B-8	8	.333	.416	.208	
PX24S-9	PX24B-9	9	.375	.458	.250	
PX24S-10	PX24B-10	10	.417	.500	.291	
PX24S-12	PX24B-12	12	.500	.583	.375	1/4
PX24S-16	PX24B-16	16	.666	.750	.542	
PX24S-18	PX24B-18	18	.750	.833	.625	
—	PX24B-22	22	.916	1.000	.792	
32 PITCH (.0981)						
PX32S-10	PX32B-10	10	.312	.375	.218	
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PX32S-14	PX32B-14	14	.438	.500	.343	
PX32S-15	PX32B-15	15	.469	.531	.375	1/4
PX32S-16	PX32B-16	16	.500	.562	.406	
PX32S-18	PX32B-18	18	.563	.625	.468	
PX32S-20	PX32B-20	20	.625	.688	.532	
	PX32B-24	24	.750	.813	.656	
48 PITCH (.0654)						
PX48S-14	PX48B-14	14	.292	.333	.229	
PX48S-15	PX48B-15	15	.312	.353	.250	
PX48S-16	PX48B-16	16	.333	.375	.271	
PX48S-18	PX48B-18	18	.375	.417	.312	1/8
PX48S-24	PX48B-24	24	.500	.542	.437	
PX48S-32	PX48B-32	32	.666	.708	.604	
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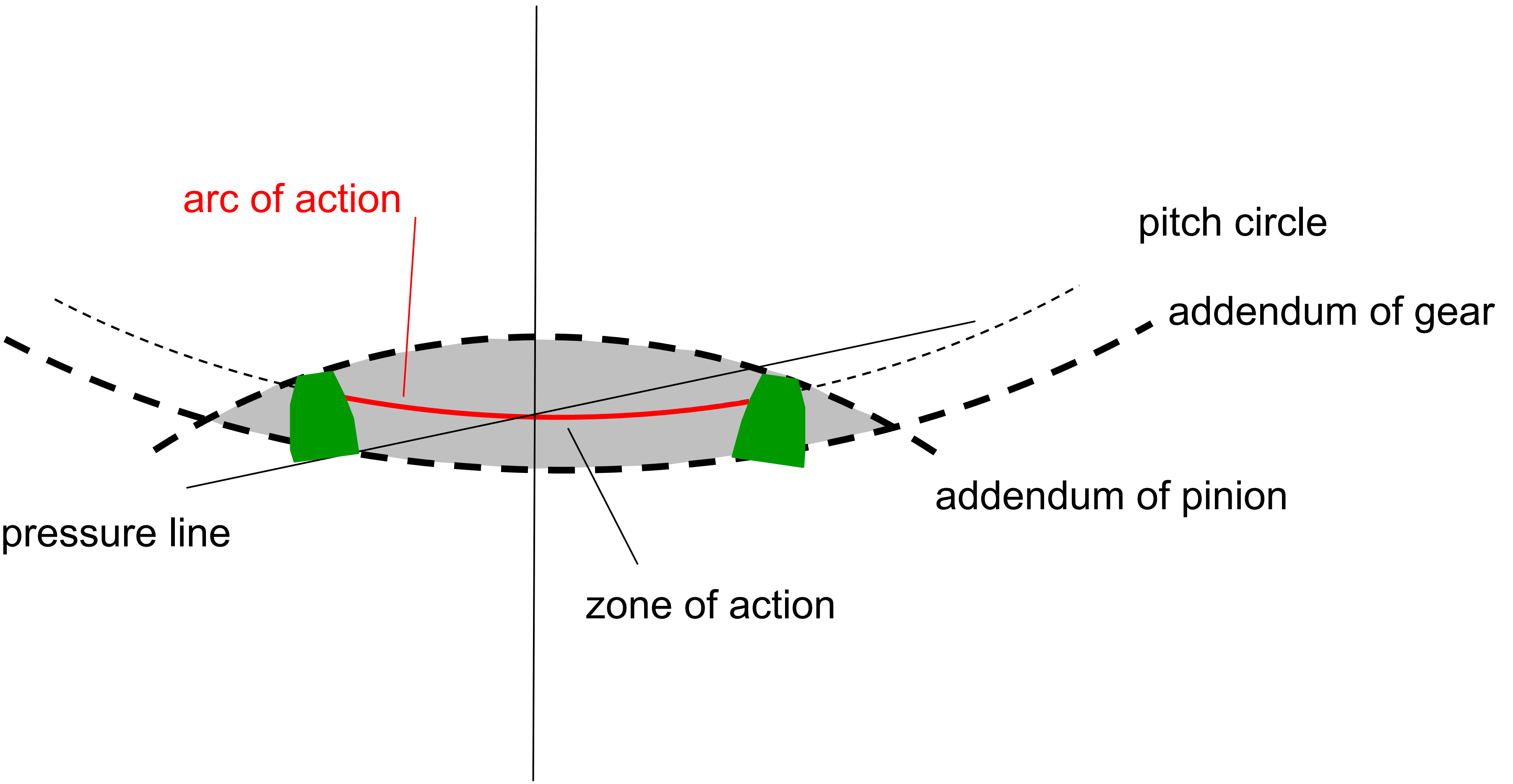
—	PX64B-48	48	.750	.781	.703	

Berg Manufacturing "The Mark of Quality" **1-800-232-BERG**

A pair of gears are mated. One is driven at a set torque, the other is regulated at a set speed. The gears are the ones circled. What is the ratio of the separation forces and the total force on the bearing?

1. << 0.3
2. About 0.3
3. About 0.5
4. >> 0.5

Contact Ratio



contact ratio = length of arc of action / pitch = average number of teeth engaged

Interference

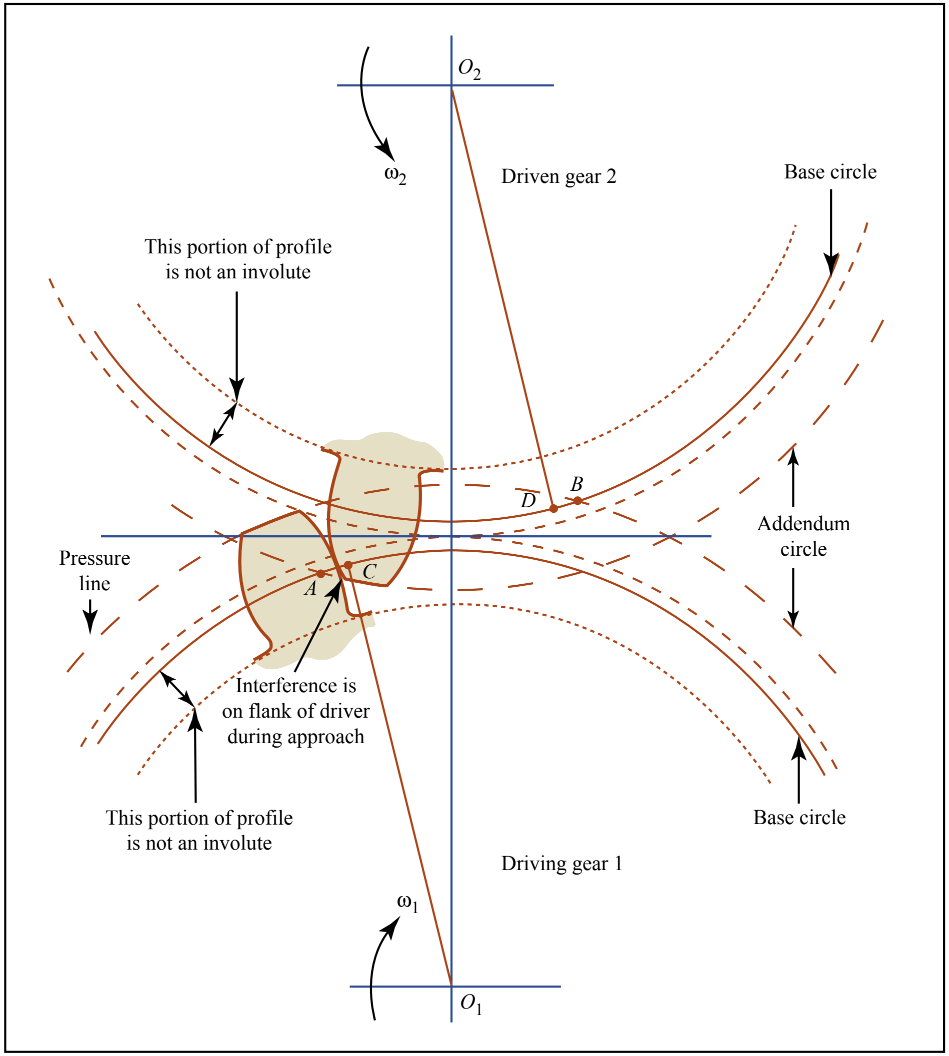
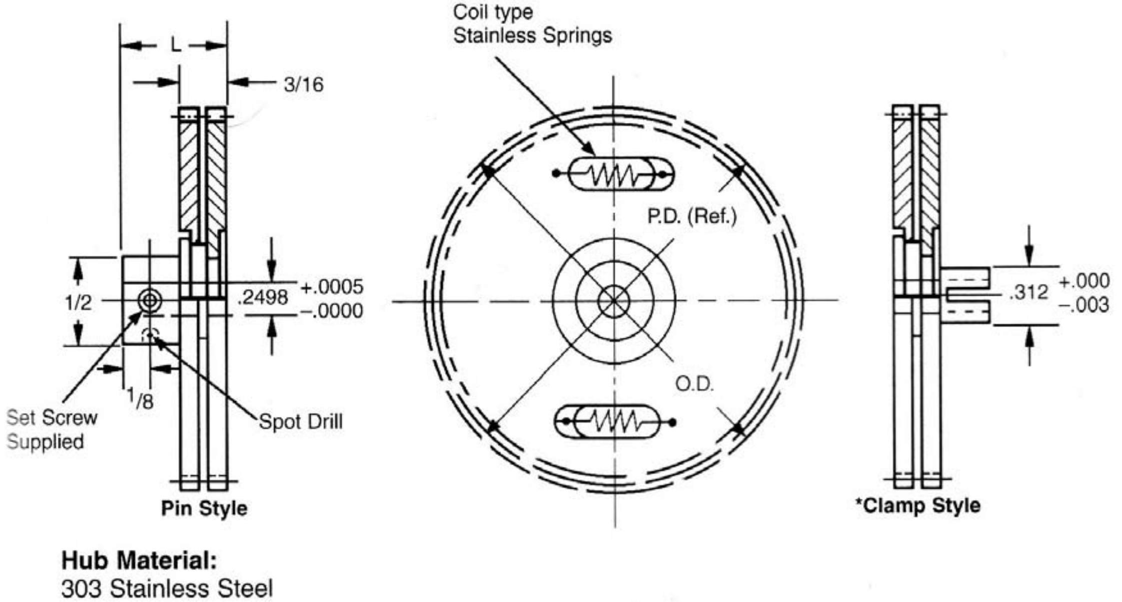


Figure by MIT OpenCourseWare.

From Shigley and Mischke

Backlash

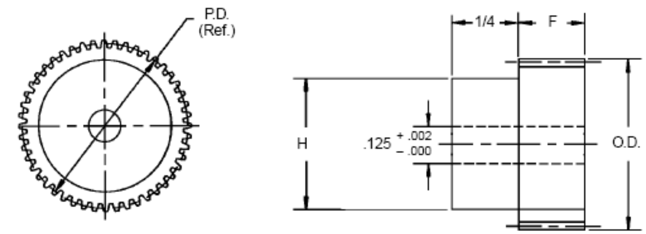

L
3/16
1/2
.2498
+.0005
−.0000
1/8
Set Screw
Supplied
Spot Drill
Pin Style
Coil type
Stainless Springs
P.D. (Ref.)
O.D.
.312
+.000
−.003
*Clamp Style
Hub Material:
303 Stainless Steel

Gear Selection

- Pitch
- Face width
- Material
- Pressure angle
- # of teeth
- Hub style, bore, etc.

Spur Gears

24, 32, 48, and 64 Pitch 1/8" Bore AGMA Quality 4
Cold Rolled Steel and Brass 20° Pressure Angle



B
9

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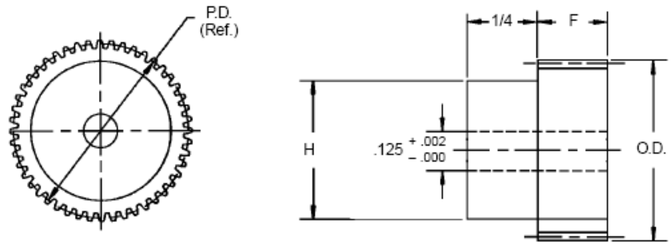
Spur Gears

24, 32, 48, and 64 Pitch   1/8" Bore   AGMA Quality 4
Cold Rolled Steel and Brass   20° Pressure Angle



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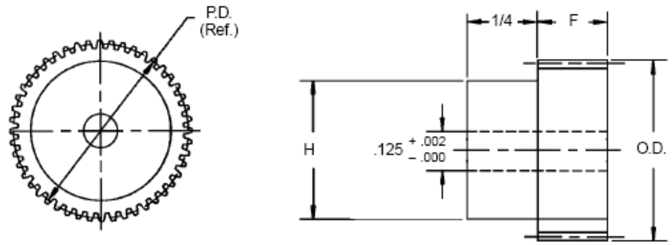
Berg Manufacturing "The Mark of Quality"   **1-800-232-BERG**

You call up the number 1-800-232-BERG and ask that, for a special application, you want a 48 pitch spur gear, but with a pitch dia of 0.32 inches. They will probably say:

1. OK, no problem
2. OK, but it will cost a lot
3. No, this is not technically possible

Spur Gears

24, 32, 48, and 64 Pitch   1/8" Bore   AGMA Quality 4
Cold Rolled Steel and Brass   20° Pressure Angle



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Berg Manufacturing "The Mark of Quality"   **1-800-232-BERG**

You call up the number 1-800-232-BERG and ask that, for a special application, you want a 48 pitch spur gear, but with a pitch dia of half the smallest one in the catalog. They will probably say:

1. OK, no problem
2. OK, but it will cost a lot
3. OK, but it will be weak
4. No, this is not technically possible

Ways Gears Fail

Exceed endurance limit in bending

Exceed static yield stress in bending

"stripping"

Image courtesy of deltaMike at Flickr.

Exceed endurance limit in contact stress

Images removed due to copyright restrictions. Please see
http://materials.open.ac.uk/mem/mem_mf6.htm
http://www.hghouston.com/x/39_gearpit.html

"pitting"

Stress in Gears

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http://courses.washington.edu/mengr356/daly/Gear_stress.pdf

A Beam in Bending

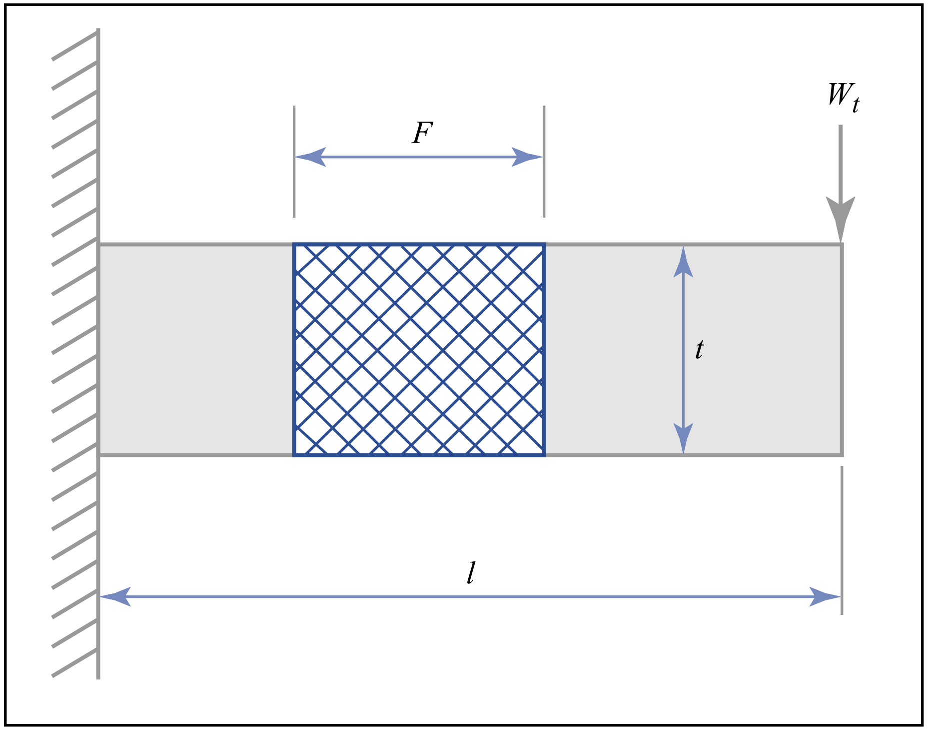


Figure by MIT OpenCourseWare.

$$\sigma = \frac{M}{I/c} = \frac{6W_t l}{Ft^2}$$

Concept Question

- In selecting a gear of one inch pitch diameter, we are choosing between 48 and 24 pitch gear teeth. The effect on torque that can be transmitted before <u>bending</u> failure of the teeth is

1. Around a factor of 10
2. Around a factor of 4
3. Around a factor of 2
4. Less than a factor of 2

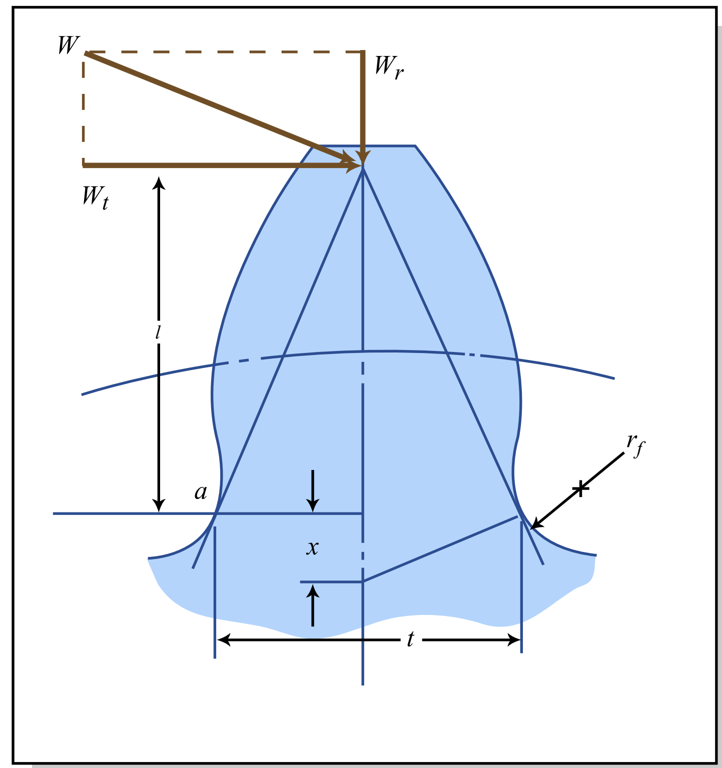


Figure by MIT OpenCourseWare.

Strength of Gears

- Any good catalog will have a formula and tables
- What factors must enter the equation?
  - –
  - –
  - –
- Where do the teeth wear the most?

Gear Reference Guide

GEAR TOOTH STRENGTH

Many factors must be considered when designing a gear train. The information listed on this pag should be used as a general guideline for your application. If more critical strength calculation i required W.M. Berg suggests that you consult our engineering department or any one of the many gea handbooks that are readily available.

When a gear train is transmitting motion, it is save to assume that all of the load is being carried b one tooth. This is because as the load approaches the end of the tooth, where the bending force woul be the greatest, a second tooth comes into mesh to share the load. Simple results can be obtaine from the Lewis bending strength equation.

$$W_t = \frac{SFY}{D.P.}$$

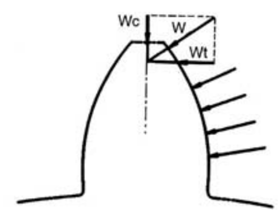
W_t = Maximum transmitted load (lbs or N)

S = Maximum bending tooth stress (taken as 1/3 of the tensile strength) See Table C on Page 5

F = Face width of gear (in. or mm)

D.P. = Diametral Pitch = 1/module (for equation only)

Y = Lewis Factor (See Table)



NOTE: The maximum bending tooth stress (S) is valid for well lubricated, low shock applications. For high shock, poorly lubricated applications, the safe stress could be as low as .025S. If your design calls for an unfriendly environment for gears, you might want to lower S to assure a reasonable amount of gear life.

LEWIS FACTOR - Y

NO. OF TEETH	14 1/2° INVOLUTE	20° INVOLUTE
10	0.176	0.201
11	0.192	0.226
12	0.210	0.245
13	0.223	0.264
14	0.236	0.276
15	0.245	0.289
16	0.255	0.295
17	0.264	0.302
18	0.270	0.308
19	0.277	0.314
20	0.283	0.320
22	0.292	0.330
24	0.302	0.337
26	0.308	0.344
28	0.314	0.352
30	0.318	0.358
32	0.322	0.364
34	0.325	0.370
36	0.329	0.377
38	0.332	0.383
40	0.336	0.389
45	0.340	0.399
50	0.346	0.408
55	0.352	0.415
60	0.355	0.421
65	0.358	0.425
70	0.360	0.429
75	0.361	0.433
80	0.363	0.436
90	0.366	0.442
100	0.368	0.446
150	0.375	0.458
200	0.378	0.463
300	0.382	0.471
RACK	0.390	0.484

The Lewis Formula

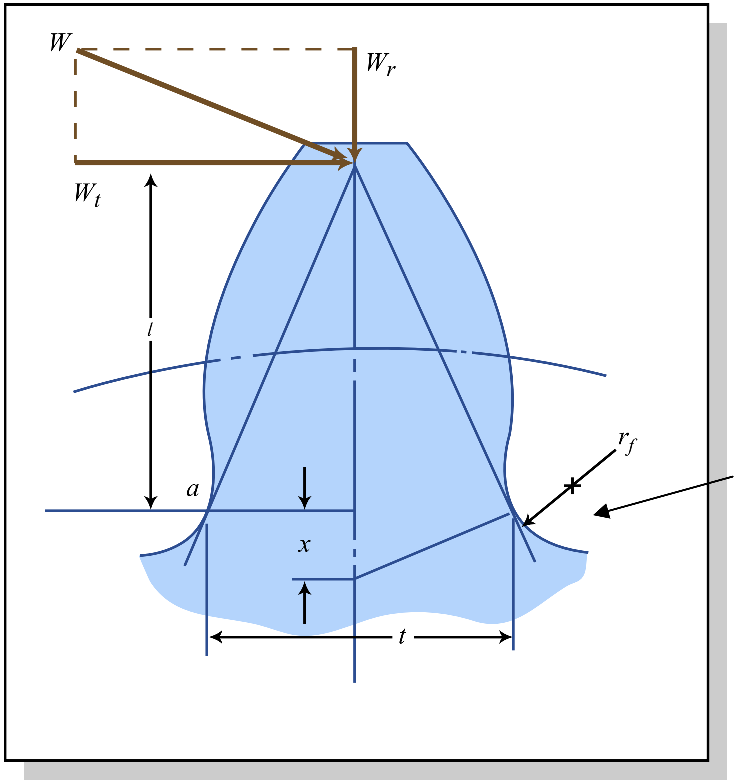


Figure by MIT OpenCourseWare.

$$\sigma = \frac{W_t P}{F y}$$

Diametral pitch (teeth/inch)

Face width

"Lewis form factor"

Low form factor → High stress

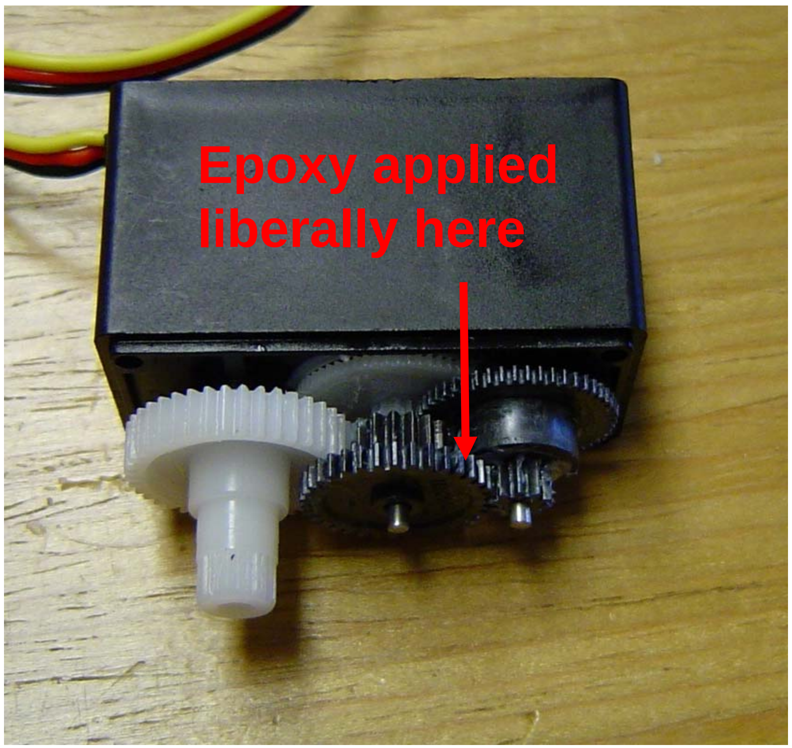
Point of max stress due to bending

Or Use a Canned Tool

Please see “Spur Gear Tooth Strength” at http://www.wmberg.com/tools/

Discussion Questions

- I glued the third stage teeth of this servo together
- Now I will apply a load to the output shaft (up to 10lbs)
- What's going to happen?



Concept Question

- For a gear to provide the highest strength at a fixed diameter, we prefer

1. High pressure angle
2. Low pressure angle
3. It doesn't matter much

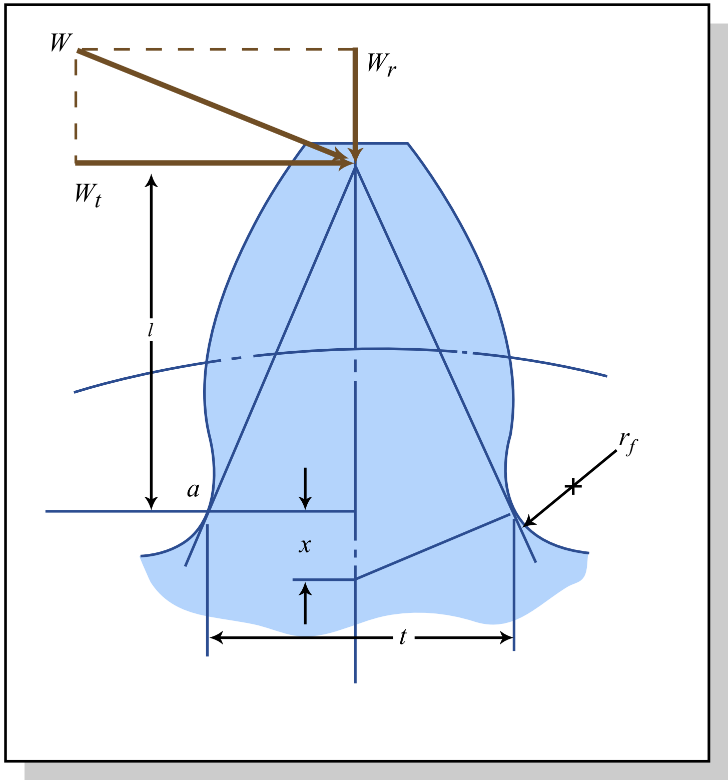
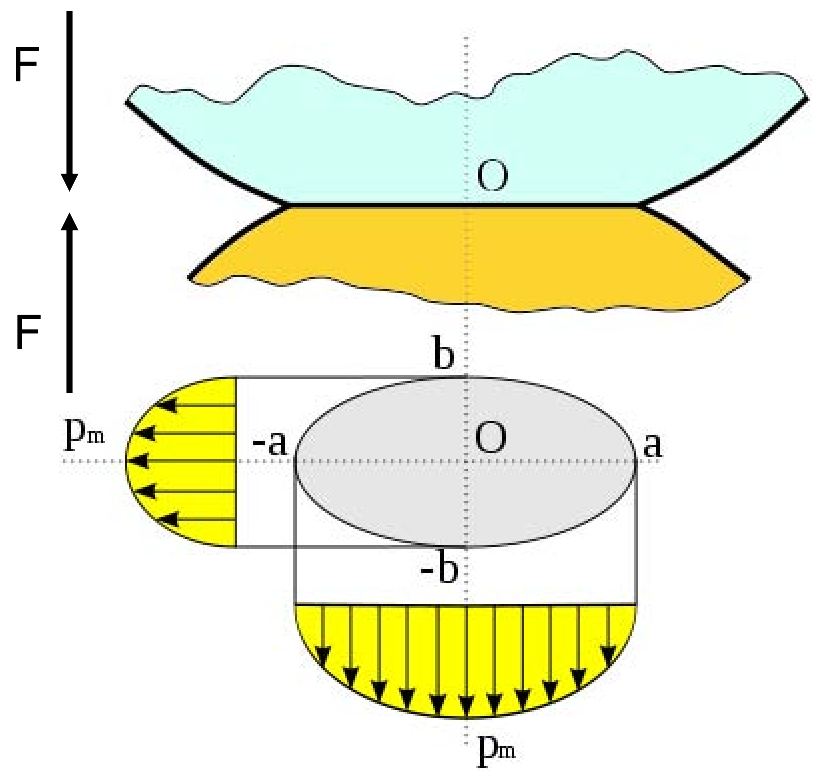
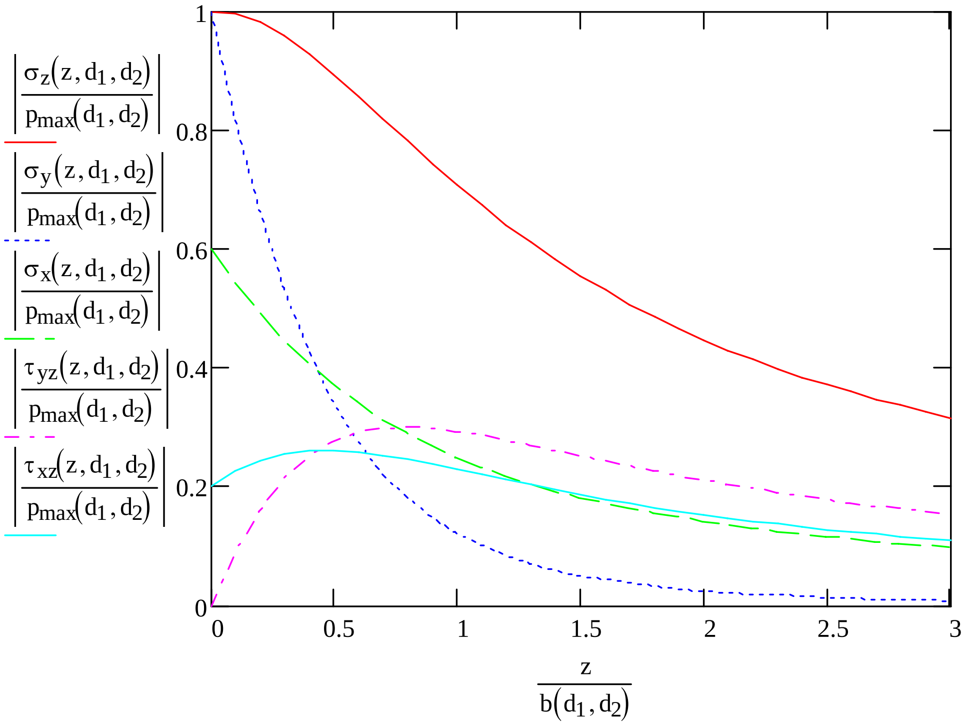


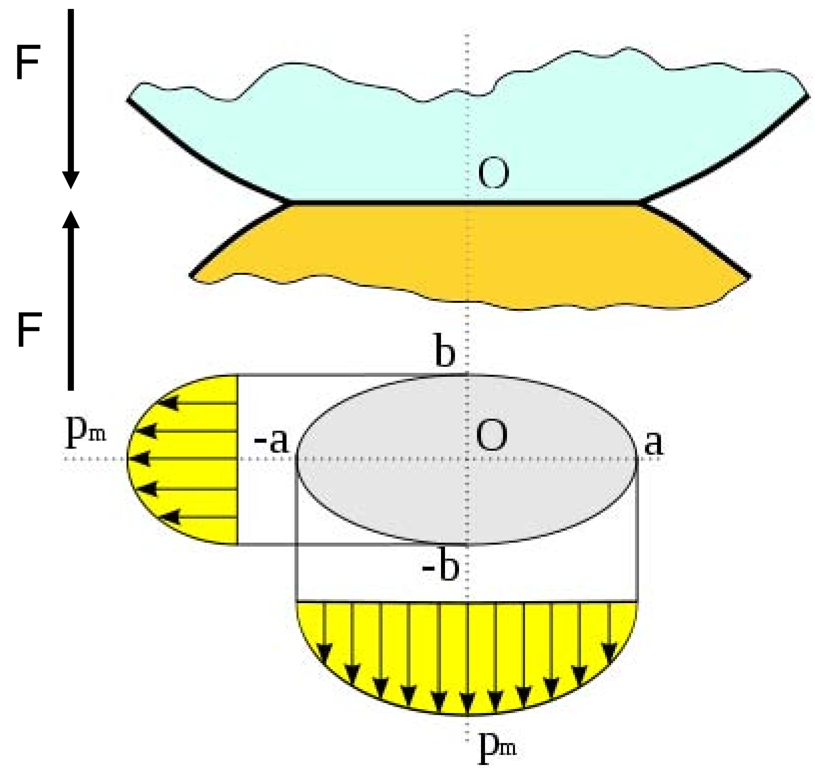
Figure by MIT OpenCourseWare.

Contact Stress
(Hertzian Stress)



Contact Stress
Quantitative Characterization

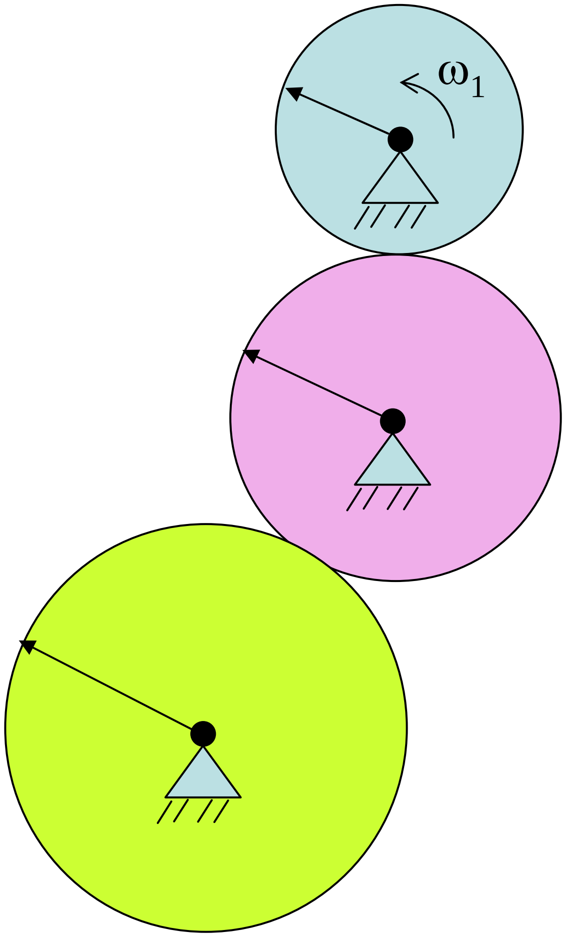




$$b(d_1,d_2) := \sqrt{\frac{2\cdot F}{\pi\cdot l}\cdot\frac{\frac{\left(1-\nu_1^{\,2}\right)}{E_1}+\frac{1-\nu_2^{\,2}}{E_2}}{\frac{1}{d_1}+\frac{1}{d_2}}}\qquad p_{max}(d_1,d_2) := \frac{2\cdot F}{\pi\cdot b(d_1,d_2)\cdot l}$$

Simple Gear Trains

- A "simple" gear train has only one gear on each shaft
- How does this arrangement behave?



Compound Gear Trains

- A "compound" gear train has at least one shaft with multiple gears
- How does this arrangement behave?

Image from Wikimedia Commons, http://commons.wikimedia.org

Manual Transmissions

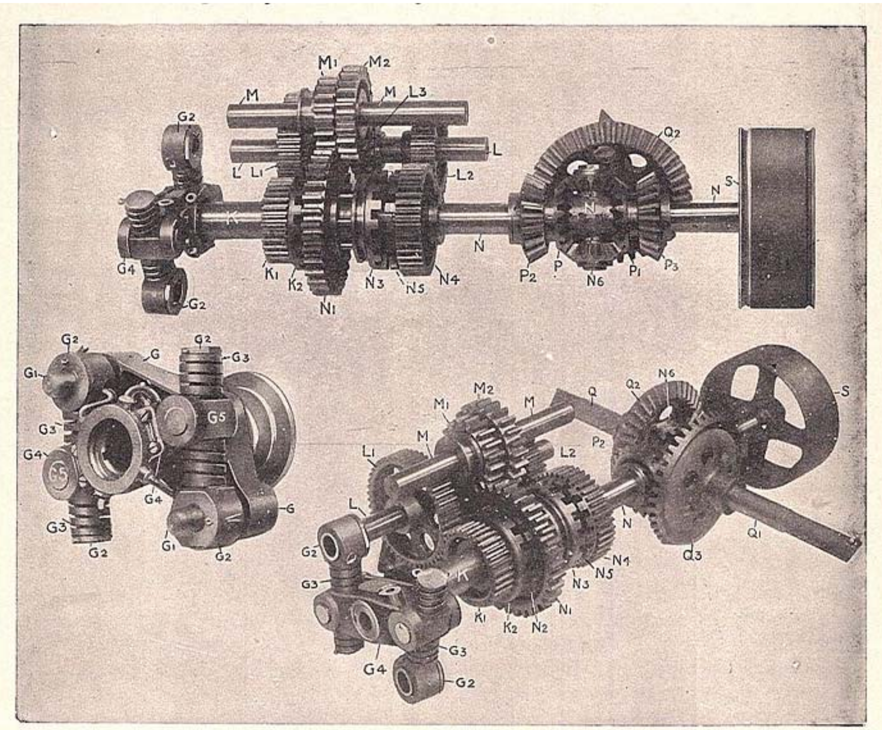


Fig. 56.—Albion Change Speed Gear.

Image from Wikimedia Commons, http://commons.wikimedia.org.
Please see http://mossmotors.com/Graphics/Products/Schematics/SPM-025.gif

If you find just two axles in a machine, does that mean there are just two stages?

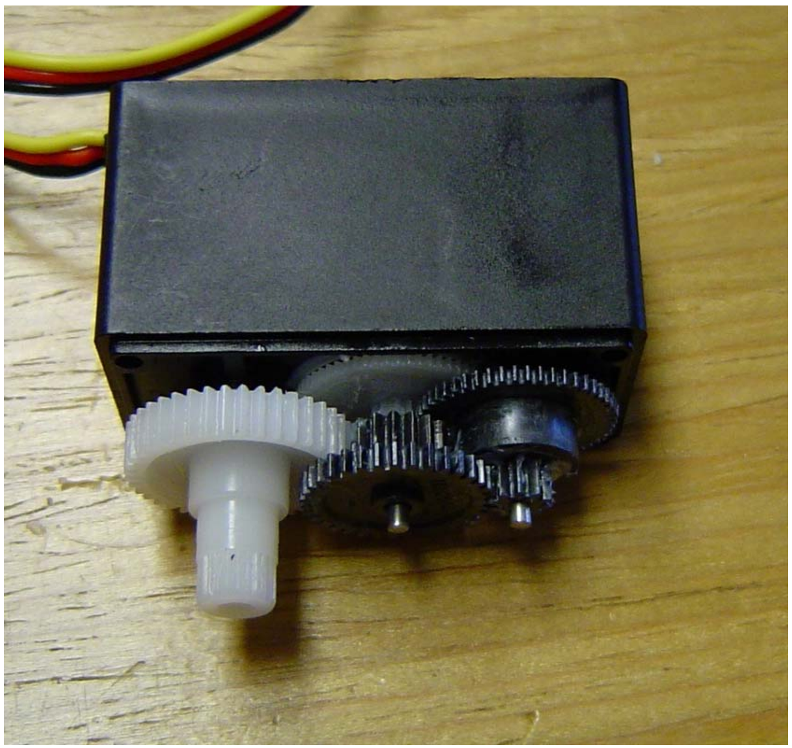
Discussion Questions

Image from Wikimedia Commons,
http://commons.wikimedia.org

- Are there any disadvantage to a helical gear as compared to a spur gear?
- How can the disadvantages be remedied?
- Is a helical gear set stronger than a spur gear of the same diameter, pitch, face width, & material?

Concept Question

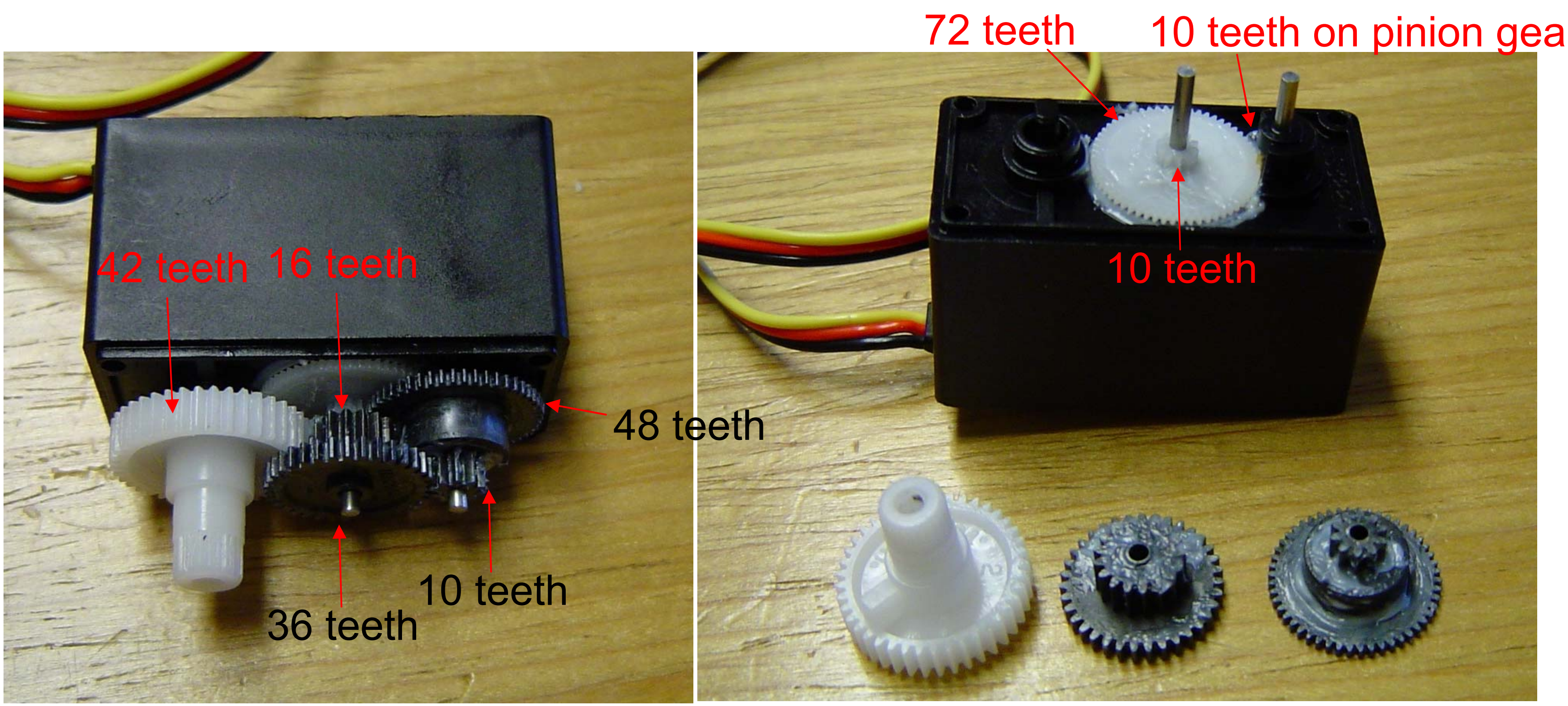
A compound gear train is formed of eight gears. As we proceed from the pinion on the electric motor to the gear on the output shaft, how do the pitch and face width vary?



1. Pitch rises,
   face width rises
2. Pitch rises,
   face width falls
3. Pitch falls,
   face width rises
4. Pitch falls,
   face width falls

Discussion Questions

- How many stages in this device?
- How do you suppose this number is chosen?
- Are the reduction ratios typically all nearly the same in all successive stages?



Differentials

- Allows shafts to move at different speeds
- Applies same torque to both
- Slippage problem

Image removed due to copyright restrictions. Please see
http://mossmotors.com/Graphics/Products/Schematics/SPM-027.gif

http://static.howstuffworks.com/flash/differential.swf

Next Steps

- Begin Homework #3
- Next lecture Thursday 19 March
  - CAD case study
- Spring break
- Lecture Tuesday 31 March
  - More gears, and also springs
- HW#3 due 7 April
- Quiz #2 on 16 April
- Impounding week 29 April to 1 May

Planetary Gear Trains

- One or more of the gear axes are allowed to rotate
- aka "epicyclic"
- Used in
  - Power tools
  - Automatic transmissions
  - Gear boxes

Courtesy NASA.

Please also see
http://commons.wikimedia.org/wiki/File:Epicyclic_carrier_locked.png

http://i.i.com.com/cnwk.1d/i/ss/2007/0828_Driving_it/DSG_440.jpg

Analysis of Planetary Gear Trains

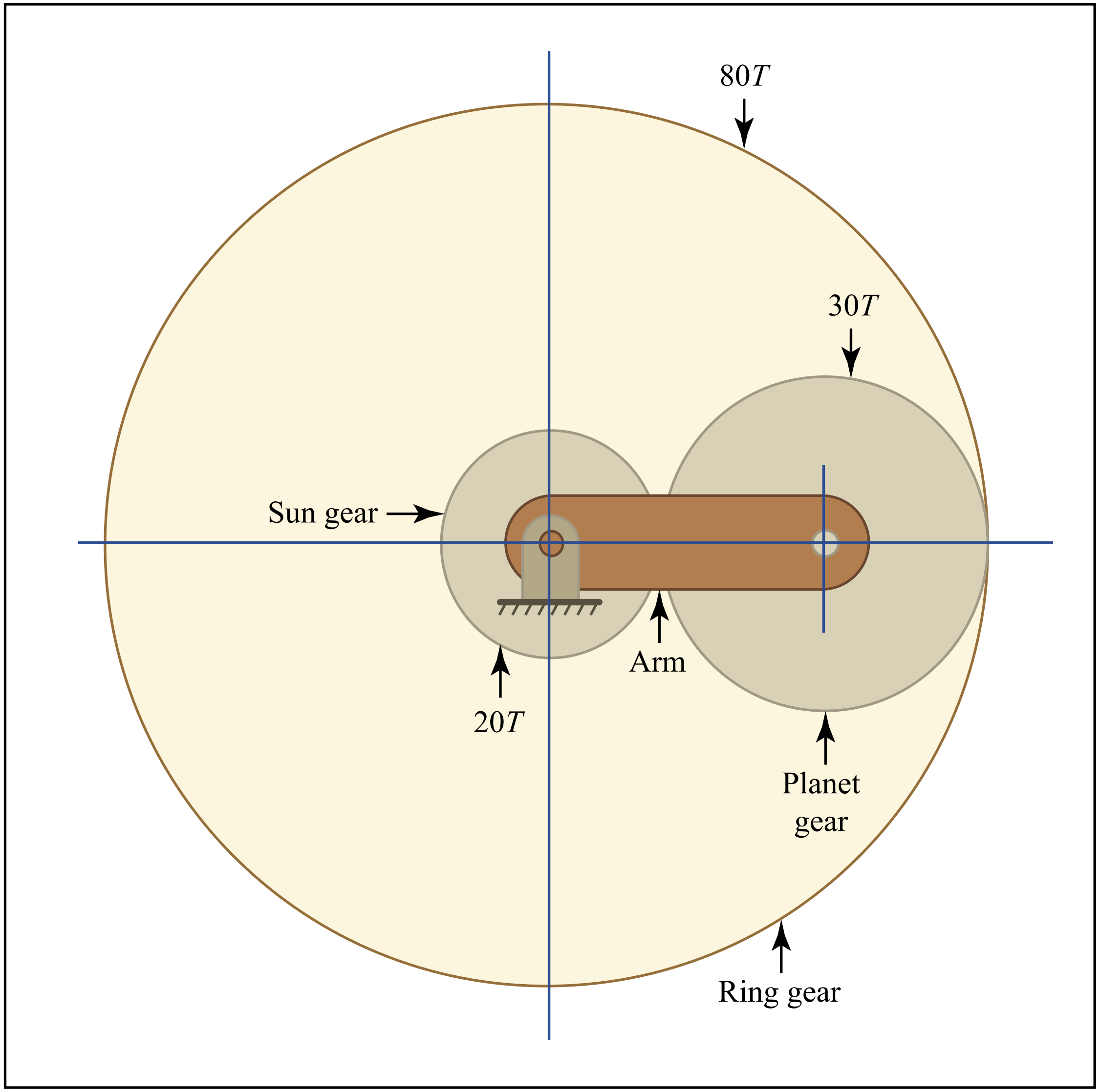


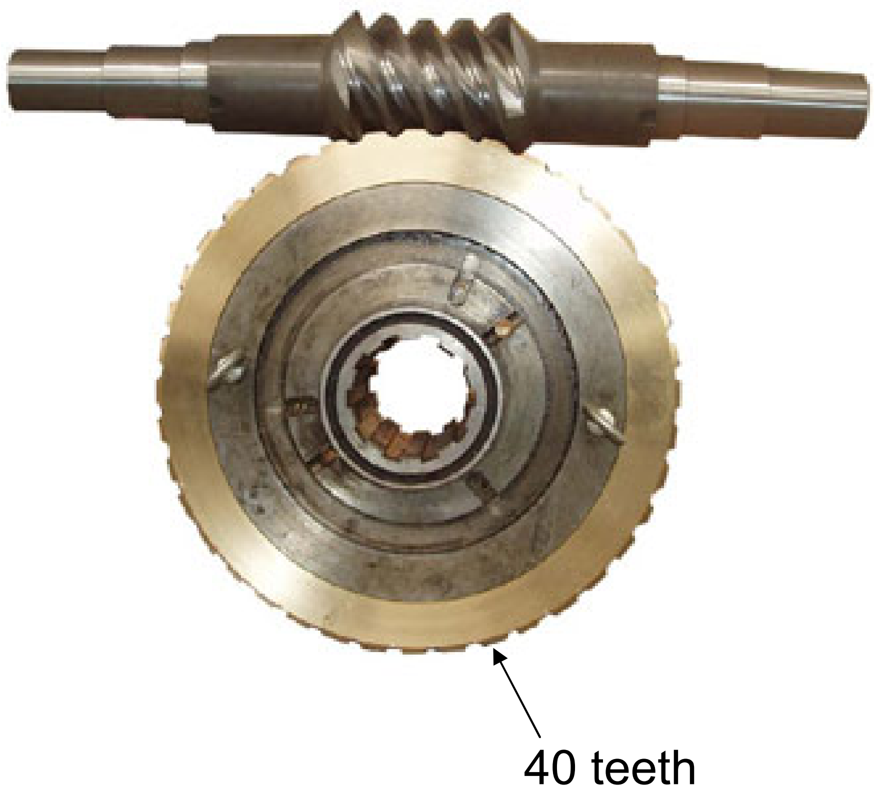
Figure by MIT OpenCourseWare.

Name That Gear

What type of worm gear set is this?

1) Single-enveloping, single threaded
2) Single-enveloping, multi-threaded
3) Double enveloping single threaded worm gear
4) Double enveloping multi- threaded

Follow up



What is the reduction ratio of this gear set?

1) 10:1
2) 20:1
3) 40:1
4) 80:1