2.007 Design and Manufacturing I
Spring 2009

2.007 –Design and Manufacturing I

Gears: Strength, Gear Trains,…

Presented by Dan Frey on 19 MAR 2009

Today's Agenda

- Some discussion of homework #3
- Gears
  - Selection of gears
  - Failure of gears
  - Strength
  - Gear trains
- Springs

4) (25 total) Consider a set of spur gears of 24 pitch and 20° pressure angle and either 12, 24, or 48 teeth.

A) (10 points) Make a sketch of a of 24 pitch gear with a 1 inch pitch diameter and about a 20° pressure angle. You can use Solidworks and can employ the DXF file on the 2.007 web site (24tooth.dxf) as a starting point.
B) (5 points) A 12 tooth gear is mated with the 48 tooth gear. Make a drawing of the two gears with their pitch diamters tangent. You can use Solidworks and can employ the DXF files on the 2.007 web site (12tooth.dxf and 48tooth.dxf) A torque of 1 Nm is applied to the 12 tooth gear and the 48 tooth gear is prevented from rotating. Estimate the separation forces between the gears. Indicate on the drawing where the forces are applied and where a reaction force is generated to maintain the gears in static equlibrium.
C) Using only 24 pitch gears with 12, 24, and 48 teeth and ¼ inch face width, design a compound spur gear train for a ratio of 32:1 Sketch the train to scale. (Note: there are many different solutions possible, you should make design decisions and explain your rationale.)
D) Imagine the gear train from (part C) is driven by a servo motor with the manufacturer's specifications shown to the right. Estmate the factor of safety for the gear train assuming the largest load applied to it will be the stall torque of the servomotor. You can assume the gears are molded from Delrin.

Interference

- Contact with the non-involute part of the gear
- In practice, avoided by under-cutting
- Weakens the tooth, esp. on gears with few teeth (pinions)

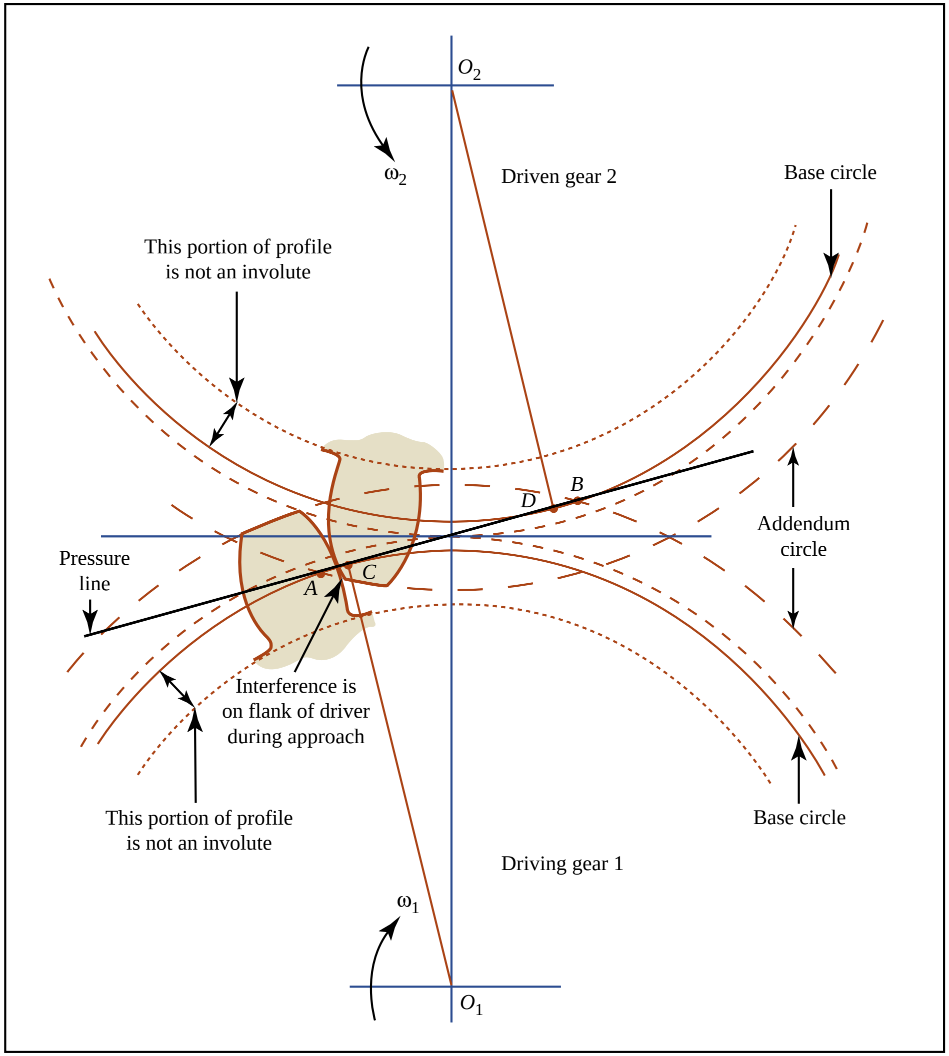
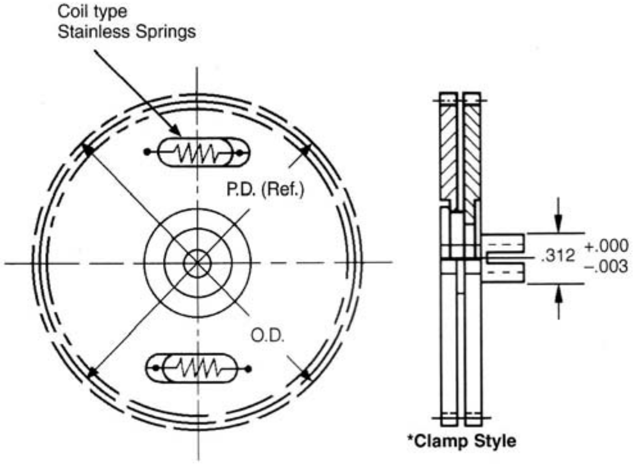


Figure by MIT OpenCourseWare.

From Shigley and Mischke

Backlash

- There is some space between the driving pair of gears and the next pair behind and ahead
- When gears reverse direction, that space allows relative motion
- Can be eliminated at some cost in parts and efficiency



An “anti-backlash” gear
from WM Berg

Gear Selection

- Pitch
- Face width
- Material
- Pressure angle
- # of teeth
- Hub style, bore, etc.

Spur Gears

24, 32, 48, and 64 Pitch   1/8" Bore   AGMA Quality 4
Cold Rolled Steel and Brass   20˚ Pressure Angle

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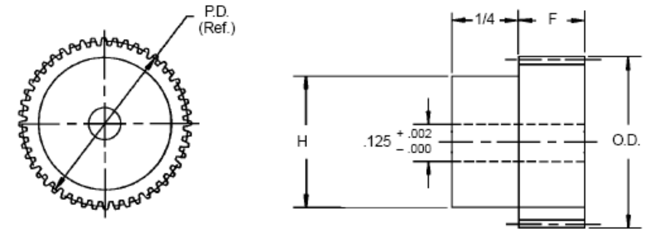

COLD ROLLED STEEL C12L14 OR C12L15 WITH SELENIUM	BRASS ALLOY 360					
STOCK NUMBER	STOCK NUMBER	NO OF. TEETH	PITCH DIA.	OUTSIDE DIA.	H	F
24 PITCH (.1309)						
PX24S-8	PX24B-8	8	.333	.416	.208	1/4
PX24S-9	PX24B-9	9	.375	.458	.250	
PX24S-10	PX24B-10	10	.417	.500	.291	
PX24S-12	PX24B-12	12	.500	.583	.375	
PX24S-16	PX24B-16	16	.666	.750	.542	
PX24S-18	PX24B-18	18	.750	.833	.625	
—	PX24B-22	22	.916	1.000	.792	
32 PITCH (.0981)						
PX32S-10	PX32B-10	10	.312	.375	.218	1/4
PX32S-11	PX32B-11	11	.344	.406	.250	
PX32S-12	PX32B-12	12	.375	.437	.281	
PX32S-14	PX32B-14	14	.438	.500	.343	
PX32S-15	PX32B-15	15	.469	.531	.375	
PX32S-16	PX32B-16	16	.500	.562	.406	
PX32S-18	PX32B-18	18	.563	.625	.468	
PX32S-20	PX32B-20	20	.625	.688	.532	
	PX32B-24	24	.750	.813	.656	
48 PITCH (.0654)						
PX48S-14	PX48B-14	14	.292	.333	.229	1/8
PX48S-15	PX48B-15	15	.312	.353	.250	
PX48S-16	PX48B-16	16	.333	.375	.271	
PX48S-18	PX48B-18	18	.375	.417	.312	
PX48S-24	PX48B-24	24	.500	.542	.437	
PX48S-32	PX48B-32	32	.666	.708	.604	
	PX48B-36	36	.750	.792	.687	
	PX48B-40	40	.833	.875	.770	
64 PITCH (.0490)						
PX64S-15	PX64B-15	15	.234	.265	.187	1/8
PX64S-16	PX64B-16	16	.250	.281	.203	
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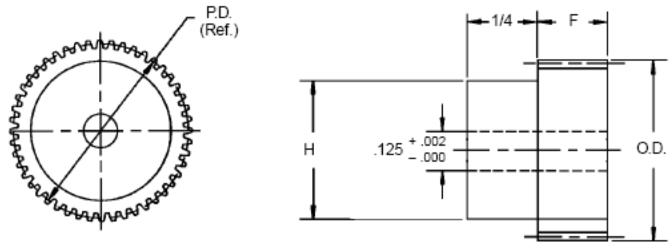
—	PX64B-24	24	.375	.406	.328	
—	PX64B-40	40	.625	.656	.578	
—	PX64B-48	48	.750	.781	.703	

Spur Gears

24, 32, 48, and 64 Pitch 1/8" Bore AGMA Quality 4
Cold Rolled Steel and Brass 20° Pressure Angle



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Berg Manufacturing "The Mark of Quality" **1-800-232-BERG**

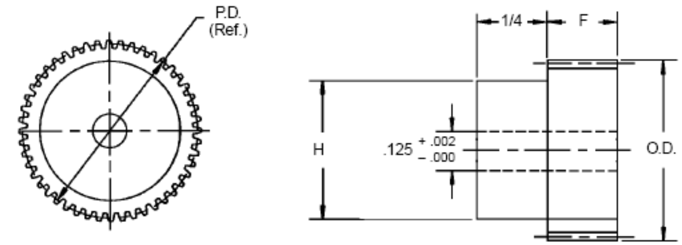
You call up the number 1-800-232-BERG and ask that, for a special application, you want a 48 pitch spur gear, but with a pitch dia of 0.32 inches. They will probably say:

1. OK, no problem
2. OK, but it will cost a lot
3. No, this is not technically possible

I'd say "3". A 48 pitch gear of 1 inch pitch dia has 48 teeth. The requested gear has 0.32*48=15.4 teeth. Integer s are better for the number of teeth.

Spur Gears

24, 32, 48, and 64 Pitch 1/8" Bore AGMA Quality 4
Cold Rolled Steel and Brass 20° Pressure Angle



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You call up the number 1-800-232-BERG and ask that, for a special application, you want a 48 pitch spur gear, but with a pitch dia of half the smallest one in the catalog. They will probably say:

1. OK, no problem
2. OK, but it will cost a lot
3. OK, but it will be weak
4. No, this is not technically possible

I'd say both "2" and "3". A 48 pitch gear is listed with 14 teeth. Half the dia will give a pinion with 7 teeth. It will be hard to make it and tricky to avoid lots of undercut.

Ways Gears Fail

Exceed endurance limit in bending

Exceed static yield stress in bending

"stripping"

Image courtesy of deltaMike at Flickr.

Exceed endurance limit in contact stress

Images removed due to copyright restrictions. Please see
http://materials.open.ac.uk/mem/mem_mf6.htm
http://www.hghouston.com/x/39_gearpit.html

"pitting"

Stress in Gears

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A Beam in Bending

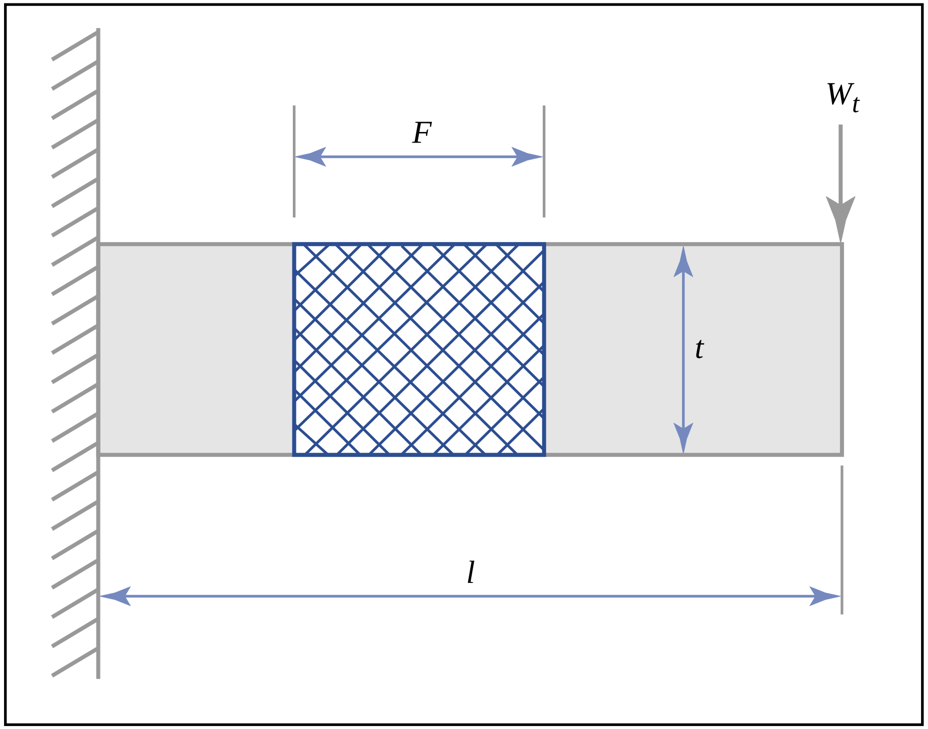


Figure by MIT OpenCourseWare.

$$\sigma = \frac{M}{I/c} = \frac{6W_t l}{Ft^2}$$

Concept Question

- In selecting a gear of one inch pitch diameter, we are choosing between 48 and 24 pitch gear teeth. The effect on torque that can be transmitted before bending failure of the teeth is

1. Around a factor of 10
2. Around a factor of 4
3. Around a factor of 2
4. Less than a factor of 2

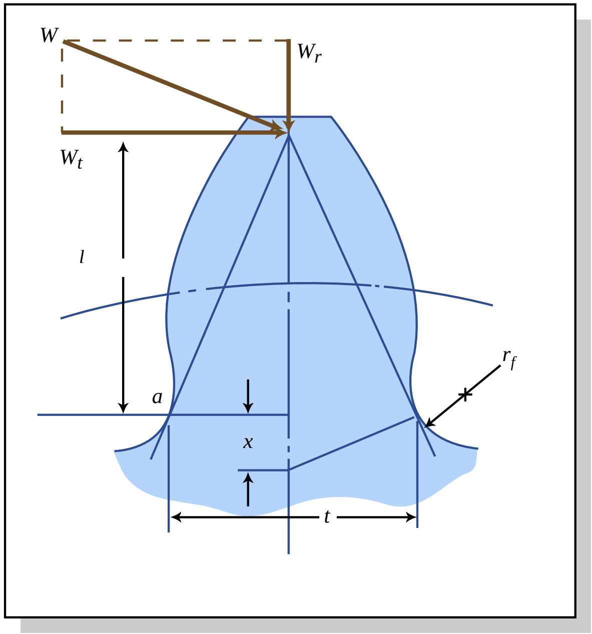


Figure by MIT OpenCourseWare.

Answ= 3: Model the gear as a beam. Its depth goes down by a factor of 2 which by itself raises stress by a factor of 4. But the length of the beam also drops by a factor of 2 and that reduces stress linearly.

Strength of Gears

- Any good catalog will have a formula and tables
- What factors must enter the equation?
  - –
  - –
  - –
- Where do the teeth wear the most?

Gear wear the most where they experience sliding motion the most. That's away from the pitch circle.

Gear Reference Guide

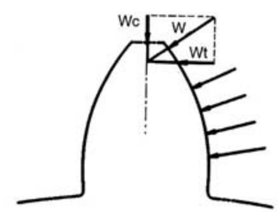
GEAR TOOTH STRENGTH

Many factors must be considered when designing a gear train. The information listed on this pag should be used as a general guideline for your application. If more critical strength calculation i required W.M. Berg suggests that you consult our engineering department or any one of the many gea handbooks that are readily available.

When a gear train is transmitting motion, it is save to assume that all of the load is being carried b one tooth. This is because as the load approaches the end of the tooth, where the bending force woul be the greatest, a second tooth comes into mesh to share the load. Simple results can be obtaine from the Lewis bending strength equation.

$$W_t = \frac{SFY}{D.P.}$$

W_t = Maximum transmitted load (lbs or N)
S = Maximum bending tooth stress (taken as 1/3 of the tensile strength) See Table C on Page 5
F = Face width of gear (in. or mm)
D.P. = Diametral Pitch = 1/module (for equation only)
Y = Lewis Factor (See Table)



NOTE: The maximum bending tooth stress (S) is valid for well lubricated, low shock applications. For high shock, poorly lubricated applications, the safe stress could be as low as .025S. If your design calls for an unfriendly environment for gears, you might want to lower S to assure a reasonable amount of gear life.

LEWIS FACTOR - Y

NO. OF TEETH	14 1/2° INVOLUTE	20° INVOLUTE
10	0.176	0.201
11	0.192	0.226
12	0.210	0.245
13	0.223	0.264
14	0.236	0.276
15	0.245	0.289
16	0.255	0.295
17	0.264	0.302
18	0.270	0.308
19	0.277	0.314
20	0.283	0.320
22	0.292	0.330
24	0.302	0.337
26	0.308	0.344
28	0.314	0.352
30	0.318	0.358
32	0.322	0.364
34	0.325	0.370
36	0.329	0.377
38	0.332	0.383
40	0.336	0.389
45	0.340	0.399
50	0.346	0.408
55	0.352	0.415
60	0.355	0.421
65	0.358	0.425
70	0.360	0.429
75	0.361	0.433
80	0.363	0.436
90	0.366	0.442
100	0.368	0.446
150	0.375	0.458
200	0.378	0.463
300	0.382	0.471
RACK	0.390	0.484

The Lewis Formula

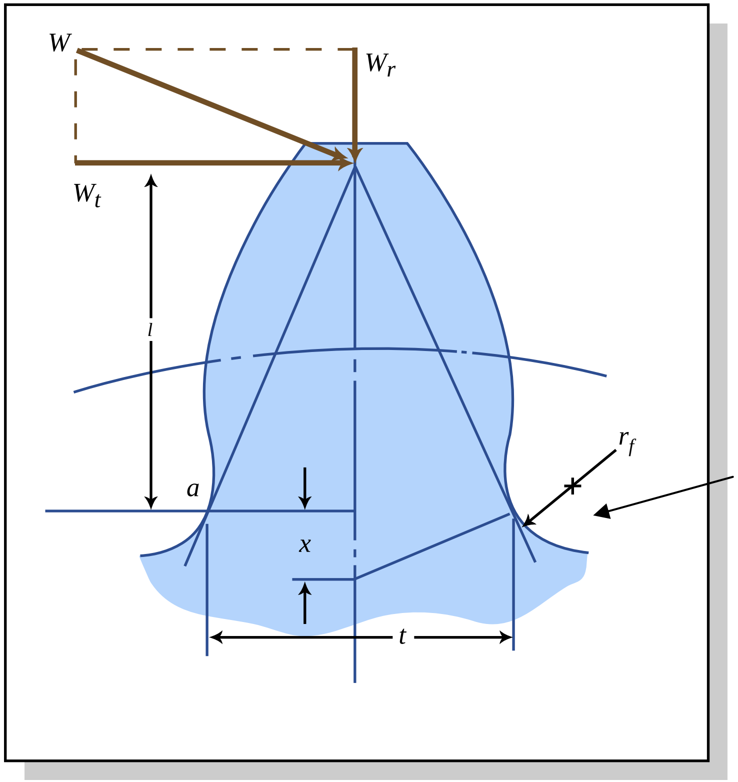


Figure by MIT OpenCourseWare.

$$\sigma = \frac{W_t P}{F y}$$

Diametral pitch (teeth/inch)

Face width

“Lewis form factor”

Low form factor → High stress

Point of max stress due to bending

Or Use a Canned Tool

Please see “Spur Gear Tooth Strength” at http://www.wmberg.com/tools/

Discussion Questions

- I glued the third stage teeth of this servo together
- Now I will apply a load to the output shaft (up to 10lbs)
- What’s going to happen?

The gears nearest the output shaft will fail since they experience the highest loads. The mode for Delrin gears is most probably in bending or “stripping”. I would estimate that the smaller gear (black in the photo) will fail rather than the white one since it has a narrower base and so a lower Lewis factor.

Concept Question

- For a gear to provide the highest strength at a fixed diameter, we prefer

1. High pressure angle
2. Low pressure angle
3. It doesn't matter much

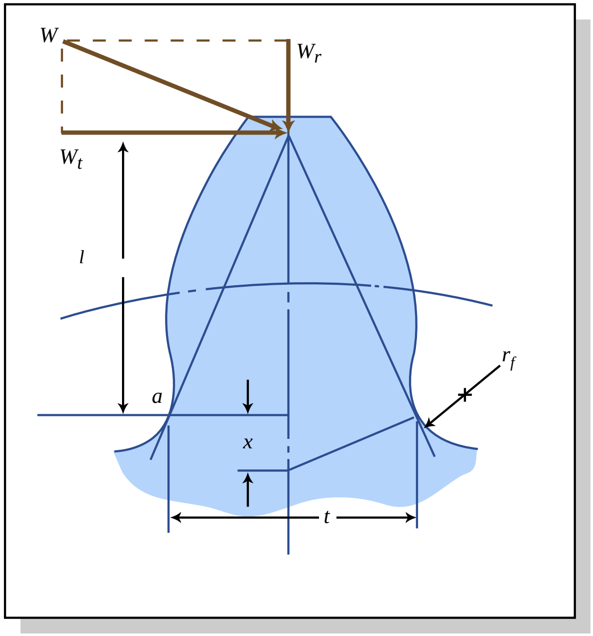
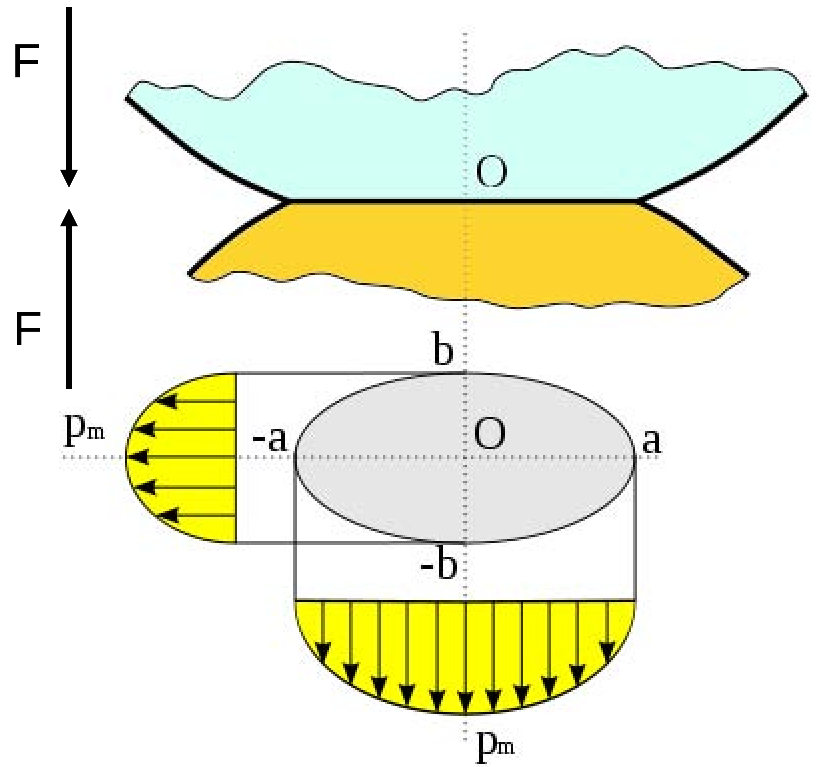


Figure by MIT OpenCourseWare.

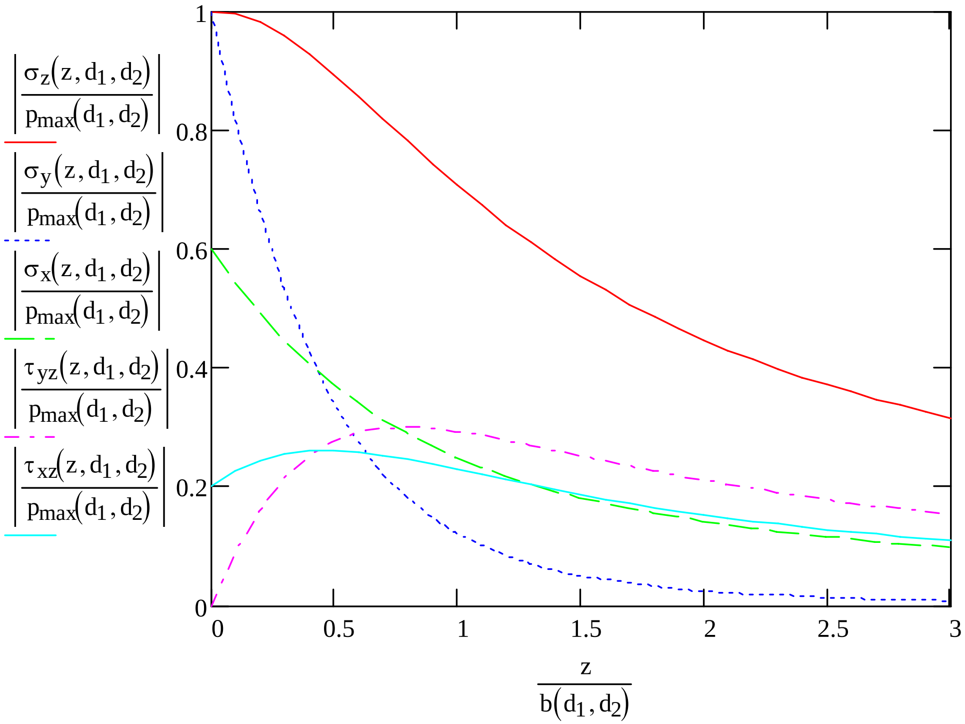
Answer =1. High pressure angle will help raise the Lewis factor and lower stress. Comparing 14.5 deg and 20 deg pressure angle (back a couple slides), the difference is 10 to 20%, so not a negligible difference.

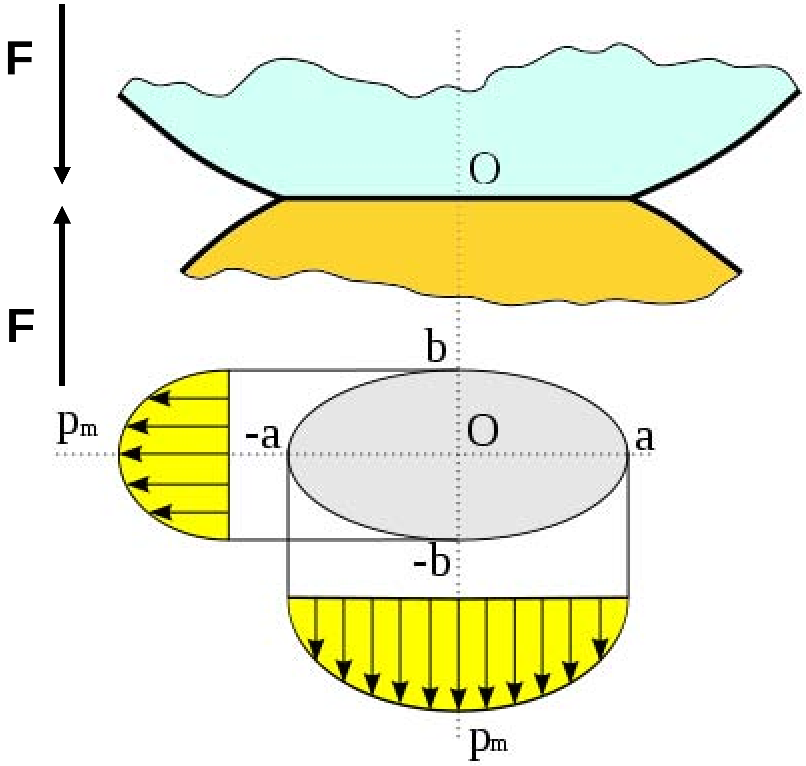
Contact Stress
(Hertzian Stress)



Contact Stress

Quantitative Characterization

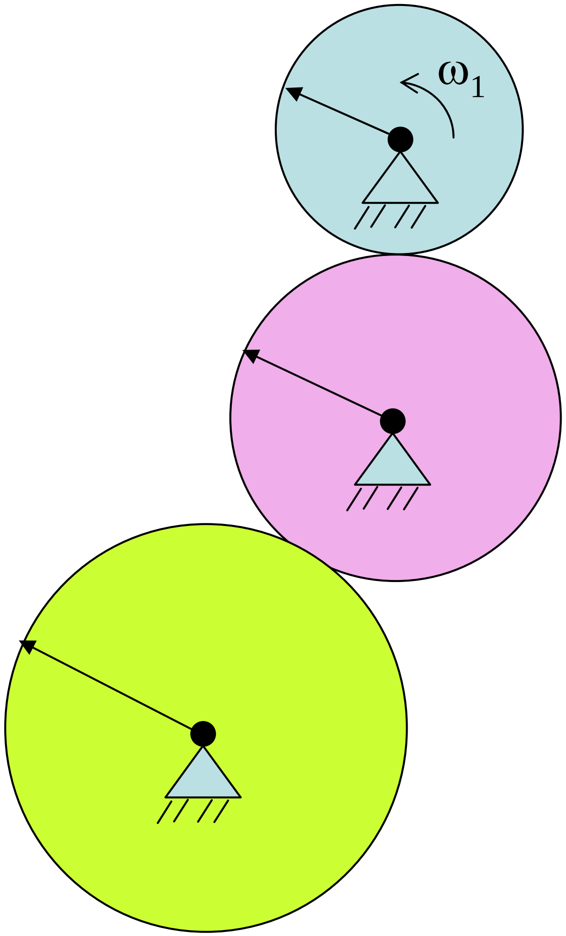




$$b(d_1,d_2) := \sqrt{\frac{2\cdot F}{\pi\cdot l}\cdot\frac{\dfrac{\left(1-\nu_1^{\,2}\right)}{E_1}+\dfrac{1-\nu_2^{\,2}}{E_2}}{\dfrac{1}{d_1}+\dfrac{1}{d_2}}} \qquad p_{max}(d_1,d_2) := \frac{2\cdot F}{\pi\cdot b(d_1,d_2)\cdot l}$$

Simple Gear Trains

- A “simple” gear train has only one gear on each shaft
- How does this arrangement behave?



The gears turn in alternating directions. Such an arrangement might be useful to get a motion the same direction as that of the servo but at a different rate. The speed of the driven (yellow) one is not a function of the dia of the middle (pink) one.

Compound Gear Trains

- A “compound” gear train has at least one shaft with multiple gears
- How does this arrangement behave?

Image from Wikimedia Commons, http://commons.wikimedia.org

The key thing is that the total reduction ratio is a product of the ratios of the two mating pairs.

Compound Helical Gears

- Note the difference in the handedness of the gears
- Another book shows the gear with the same handedness
- Which is better?

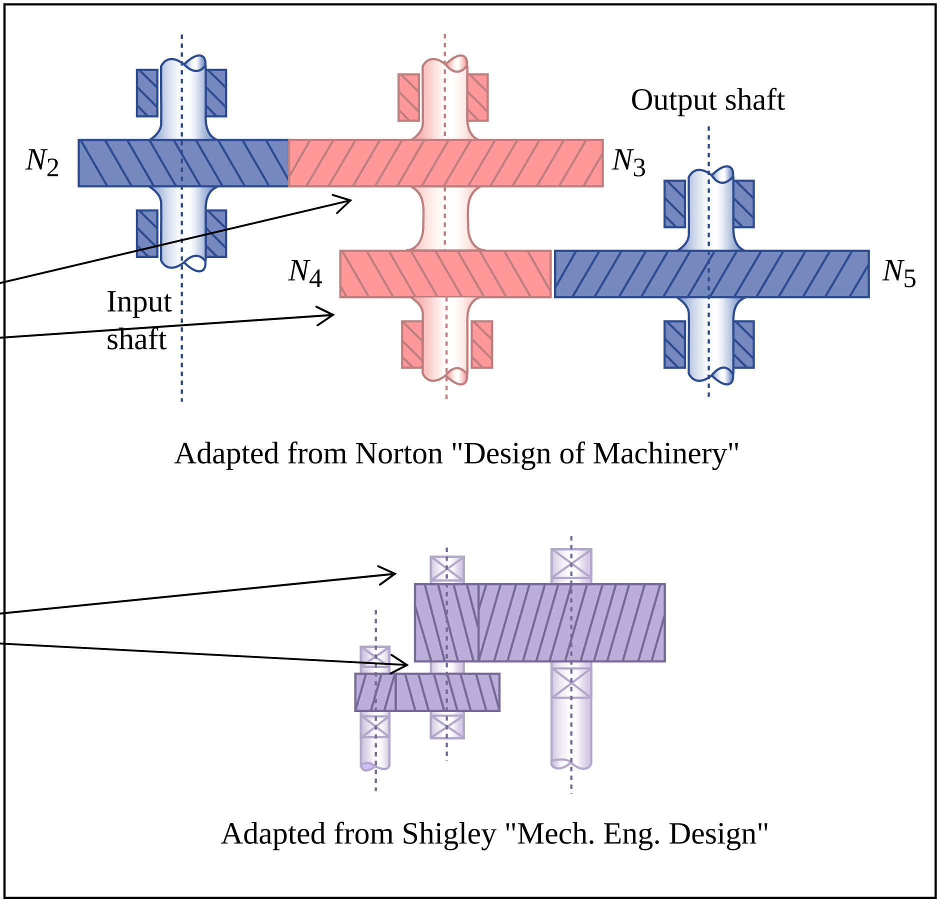


Figure by MIT OpenCourseWare.

I'd argue for Shigley's arrangement. Its axial loads cancel because one is driving and one is driven.

Manual Transmissions

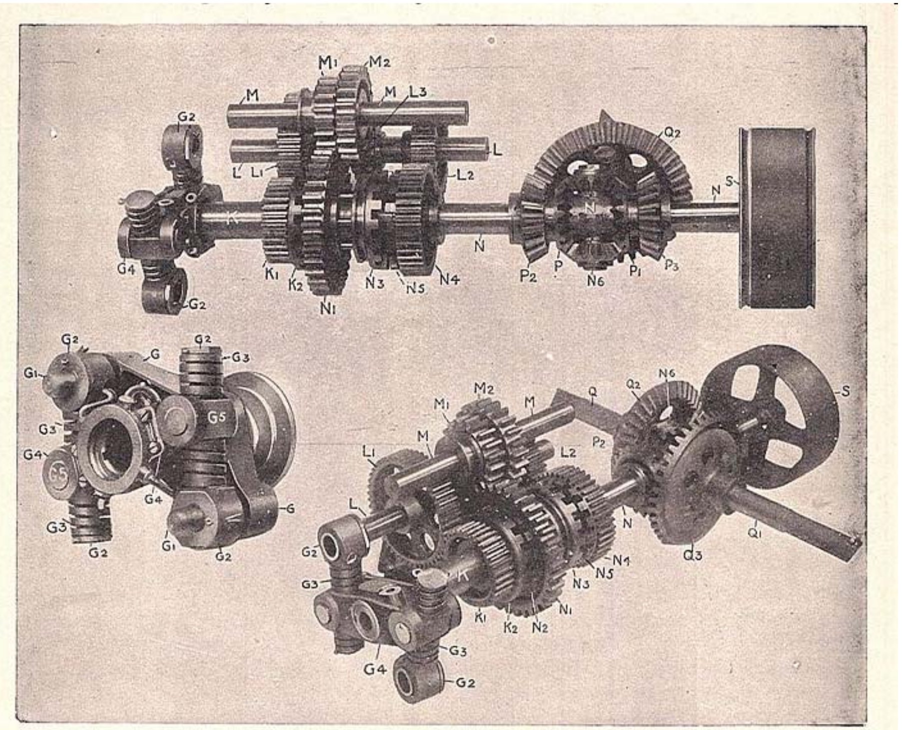


Fig. 56.—Albion Change Speed Gear.

Image from Wikimedia Commons, http://commons.wikimedia.org.
Please see http://mossmotors.com/Graphics/Products/Schematics/SPM-025.gif

If you find just two axles in a machine, does that nean there are just two stages?

No, you probably have a “reverting” arrangement with many compound gears sharing a single pair of shafts.

Discussion Questions

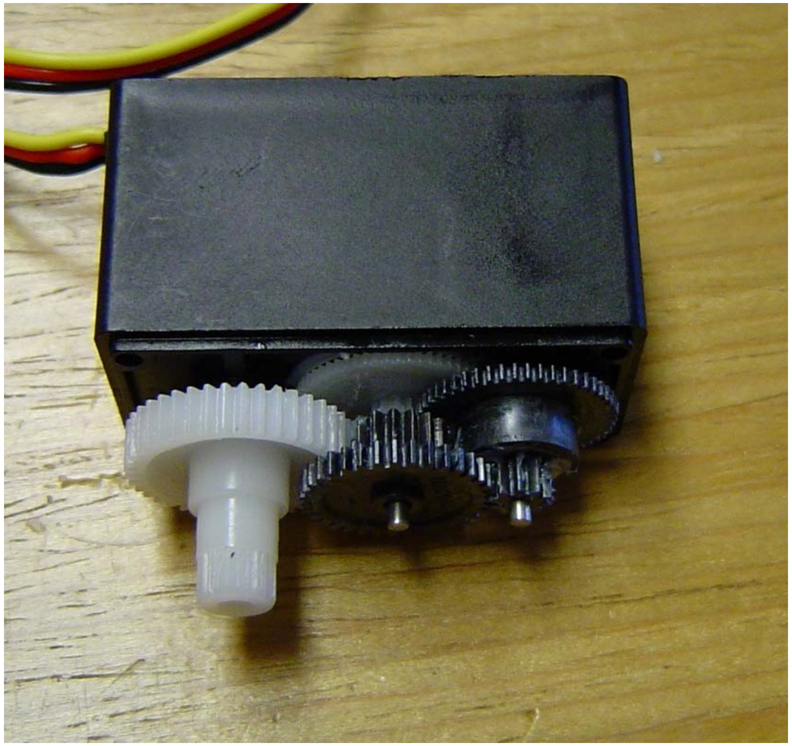
Image from Wikimedia Commons, http://commons.wikimedia.org

- Are there any disadvantage to a helical gear as compared to a spur gear?
- How can the disadvantages be remedied?
- Is a helical gear set stronger than a spur gear of the same diameter, pitch, face width, & material?

Yes, helical gears cause axial loads. They are also more expensive.
You can pair up helical gears to cancel the axial loads resulting in a “herring-bone” pattern. Helical gears are not really stronger, they are mostly quieter and smoother.

Concept Question

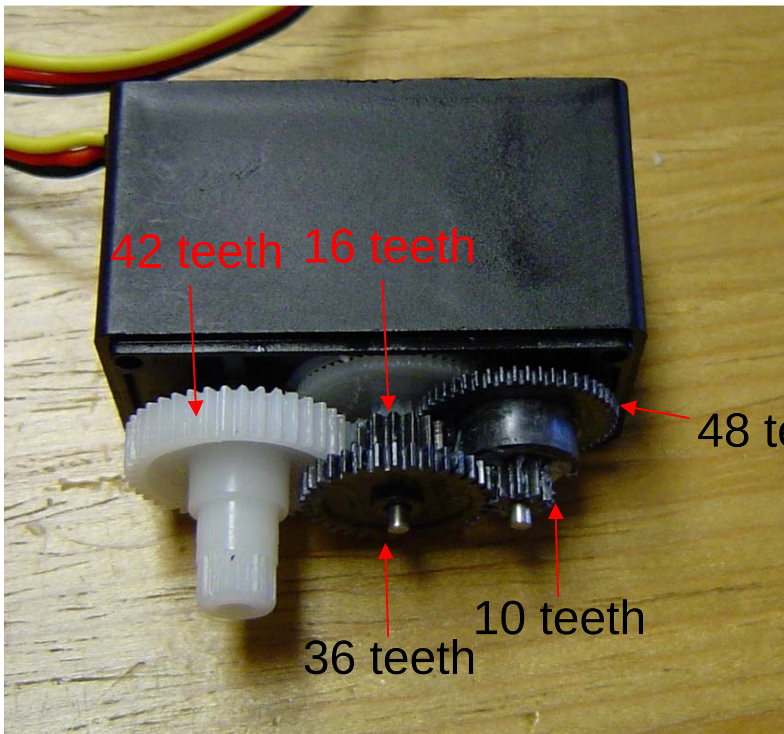
A compound gear train is formed of eight gears. As we proceed from the pinion on the electric motor to the gear on the output shaft, how do the pitch and face width vary?

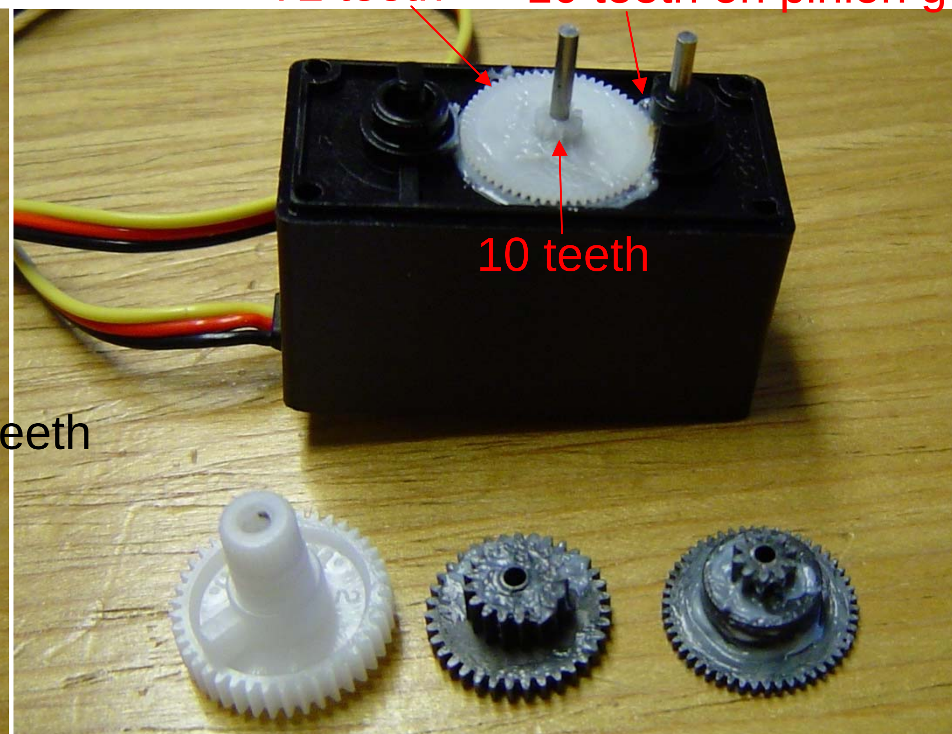


1. Pitch rises,
   face width rises
2. Pitch rises,
   face width falls
3. Pitch falls,
   face width rises
4. Pitch falls,
   face width falls

Discussion Questions

- How many stages in this device? 4 stages
- How do you suppose this number is chosen? Usually not much more than 10 to 1 ratios per stage are practical.
- Are the reduction ratios typically all nearly the same in all successive stages? No, less reduction ratio as we approach the output shaft.





Differentials

- Allows shafts to move at different speeds
- Applies same torque to both
- Slippage problem

Image removed due to copyright restrictions. Please see http://mossmotors.com/Graphics/Products/Schematics/SPM-027.gif

http://static.howstuffworks.com/flash/differential.swf

Planetary Gear Trains

- One or more of the gear axes are allowed to rotate
- aka “epicyclic”
- Used in
  - Power tools
  - Automatic transmissions
  - Gear boxes

Courtesy NASA.

Please also see
http://commons.wikimedia.org/wiki/File:Epicyclic_carrier_locked.png

http://i.i.com.com/cnwk.1d/i/ss/2007/0828_Driving_it/DSG_440.jpg

Analysis of Planetary Gear Trains

Speed of last gear (the ring) (usually fixed)

Speed of arm

$$e = \frac{n_L - n_A}{n_F - n_A}$$

Speed of first gear (the sun)

Gear ratio (teeth on sun / teeth on ring)

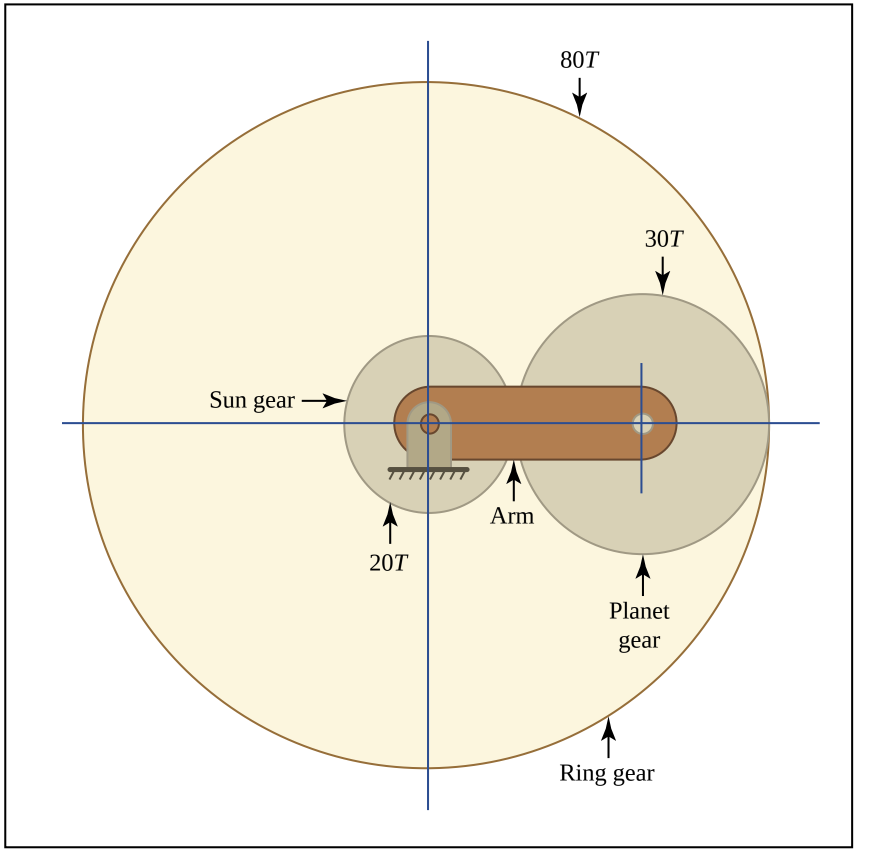


Figure by MIT OpenCourseWare.

Types of Springs

- Extension springs place the metal on torsion
- Torsion springs place the metal in bending
- “Constant force” springs also place the metal in bending

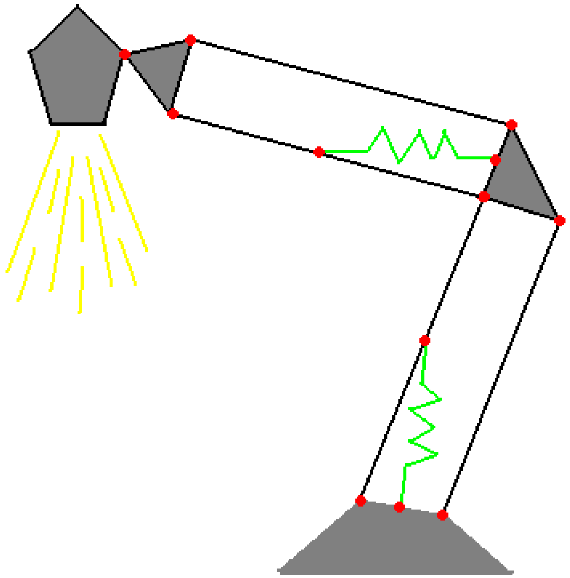
Volvo 1800ES

Rubber as a Spring

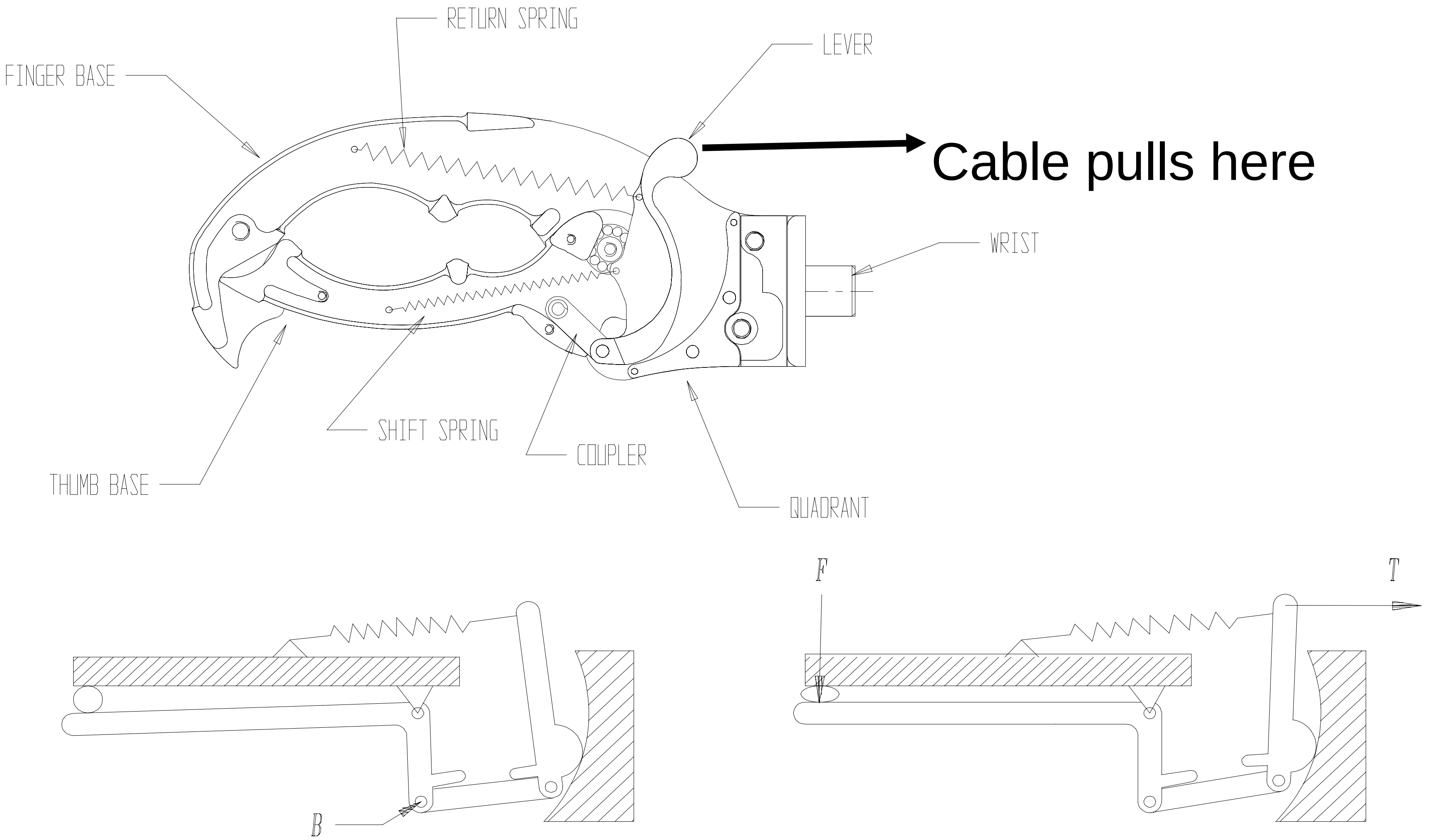
Image removed due to copyright restrictions. Please see
http://www.doitpoms.ac.uk/tlplib/optimisation-biomaterials/modulus_strength.php

Springs to Counterbalance

- Let’s say there’s a static load tending to force your mechanism away from a desired state
- Springs within your mechanism can be used to counterbalance the load
- Less intertia than a “counterweight”



Gears & Springs in Mechanisms



Courtesy of TRS Inc. Used with permission.

Next Steps

- Begin Homework #3
  - Please don’t spend more than 6 hours on it
- Spring break
- Lecture Tuesday 31 March
  - Microcontrollers
- HW#3 due 7 April
- Quiz #2 on 16 April
- Impounding week 29 April to 1 May

Updated Schedule

R 19 MAR	Gears – Strength, gear trains	module
Spring Break (23-27 March)		
T 31 MAR	Microcontrollers	Integration of design
R 2 APR	Lab time	
T 7 APR	Sensors	Demonstration of integrated machine
R 9 APR	Lab time	
T 14 APR	Belts / chains / cams	Design iteration
R 16 APR	Quiz #2	
T 21 APR	No Lecture -- Patriot's Day	Fabrication of improved machine
R 23 APR	Lab time	
T 28 APR	Optimization	Demonstration of improved machine
R 30 APR	Professional ethics	
T 5 MAY	Review of festivity procedures	Final refinements Design Festivity at Johnson Rink IN THE EVENING (6 May is first round, 7 May is finals)
R 7 MAY	No Lecture because of design festivity (although the festivity is in the evenings)	
T 12 MAY	A review of this year's machines	Debriefs with section Locker cleanup and recycling
R 14 MAY	Course summary / feedback	

Name That Gear

We really didn't discuss this, so I think it's not testable. But the "enveloping" feature is the curvature of the faces to accomplish more tooth to tooth contact. You should be able to see this gear is multi-threaded because the thread on the worm gear extends across such a long distance as it moves 180 deg across the face. There are clearly two threads interleaved.

What type of worm gear set is this?

1) Single-enveloping, single threaded
2) Single-enveloping, multi-threaded
3) Double enveloping single threaded worm gear
4) Double enveloping multi- threaded

Follow up

40 teeth

What is the reduction ratio of this gear set?

1) 10:1
2) 20:1
3) 40:1
4) 80:1

We really didn't discuss this, so I think it's not testable. But when the worm turns once two gear teeth go by since there are two threads on the worm. Therefore it takes 20 turns of the worm to cause one turn of the output shaft.