

## Problem Set 5

### Problem 5-1. Casting

(a) I would like to make one bronze casting of my face, life size, and hollow. How would you do this?

*Solution:* I would consider using a lost-wax method. First make an approximation of my face in plaster (one that is slightly smaller than my real face). Then apply a layer of wax to this slightly smaller mold. Then have a sculptor sculpt the wax into my face. At this point, I have a wax positive of my face which is built upon a solid plaster core— that is, the wax itself is only a shell. (In a lost-wax method, the metal replaces the wax and so it is critical that the wax here is only a shell.) At this point, dip the wax positive in a slurry and build a plaster mold around it, adding venting holes and risers and making sure that the bottom of the mold is connected to the outside shell. After letting this dry, melt the wax out of the mold. Finally, pour the liquid metal into the mold and let it solidify. Break apart the exterior plaster, and shake apart the interior core, and what is left is a hollow casting.

(b) As I fill the mold, what two parameters of the sand casting operation effect the hydrostatic pressure in the mold cavity. What does this pressure determine?

*Solution:* The cross-sectional area of the part and the density of the poured metal determine the hydrostatic pressure in the mold. If the hydrostatic pressure in the mold exceeds the weight of the upper part of the mold (cope), then the cope will float, and the metal will ooze out of the mold. Therefore, it might be necessary to weigh down the cope to prevent it from floating.

### Problem 5-2. Riser

(a) What is the purpose of a riser in sand casting?

*Solution:* As the metal in the casting solidifies, it will shrink. The riser provides extra liquid material that will seep into these areas so as to prevent short shots.

(b) What are the design parameters for a riser, and what are the design constraints that you need to satisfy given your answer in (a)?

*Solution:* The design parameters are the riser's height and diameter. We would like for the riser to solidify last. If it solidifies early, then it will no longer be able to meet its functional requirement.

(c) Assuming you are using a sand casting process, determine the appropriate relationships between the design parameters for your riser that result in the longest solidification time.

*Solution:* Maximizing the  $V/A$  ratio of this cylinder maximizes the riser's solidification time. The volume and surface area of a cylinder with diameter  $d$  and height  $l$  are  $V(d, l) = \pi d^2 l / 4$  and  $A(d, l) = \frac{\pi d^2}{2} + \pi d l$  respectively, and so we would like to maximize the following equation :

$$C(d, l) = \frac{V(d, l)}{A(d, l)} = \frac{\pi d^2 l / 4}{(\pi d^2 + 2\pi d l) / 2} = \frac{2dl}{d + 2l}$$

Now fix a volume  $V'$  and solve for  $l$ . We are now considering the question, for a fixed volume, what is the best  $l$  that minimizes the surface area?

$$l = \frac{4V'}{\pi d^2}$$

We can substitute this back into our original equation:

$$C(d, l) = \frac{8V'd / \pi d^2}{d + (8V' / \pi d^2)} = \frac{8V'd}{\pi d^3 + 8V'}$$

Differentiate this function with respect to  $d$  but remember that  $V'$  is now just a constant.

$$\frac{\partial C(d, l)}{\partial d} = \frac{8V'(\pi d^3 + 8V') - 8V'd(3d^2)}{(\pi d^3 + 8V')^2}$$

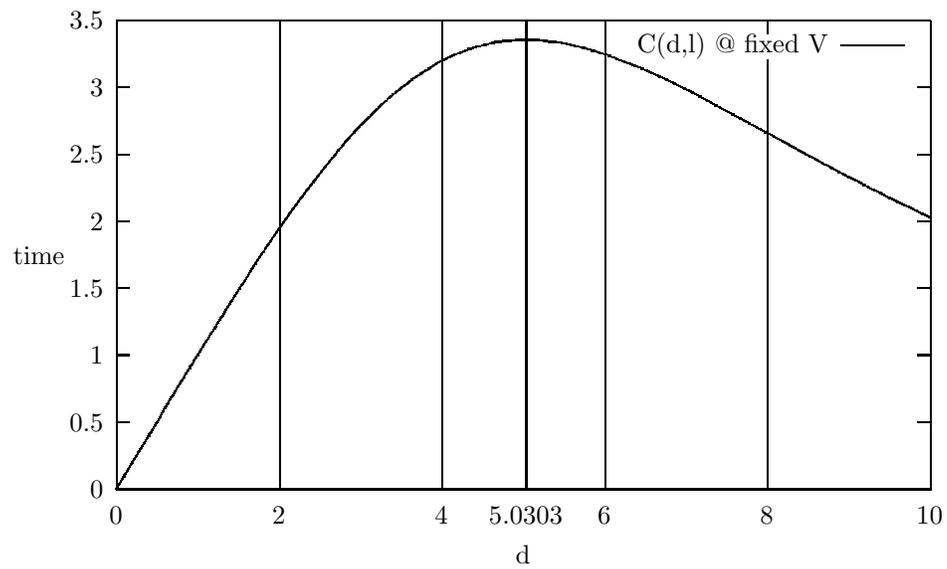
Now set the numerator to zero and solve (and make sure that the denominator is also not zero at this value).

$$\begin{aligned} 8V'(\pi d^3 + 8V') - 8V'd(3d^2) &= 0 \\ 8V'\pi d^3 + 64V'^2 - 24V'd^3 &= 0 \\ -16V'\pi d^3 &= -64V'^2 \\ d &= \left(\frac{4V'}{\pi}\right)^{1/3} \end{aligned}$$

Finally, substitute this back into our equation for  $l$ :

$$l = \frac{4V'}{\pi d^2} = \frac{4V'}{\pi \left(\frac{4V'}{\pi}\right)^{2/3}} = \left(\frac{4V'}{\pi}\right)^{1/3}$$

And so  $d = l$ ! The graph below illustrates this for a fixed riser with volume  $V = 100$ .



**Figure 1:** Solidification time for a fixed volume riser,  $V = 100$ . Notice that the optimum occurs at  $d = (400/\pi)^{1/3} = 5.0303$