

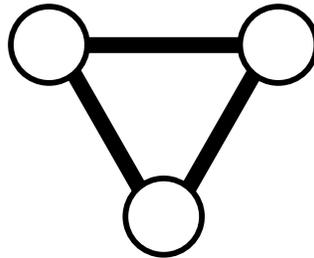


2.016 Hydrodynamics
Prof. Alex Techet
Fall 2005

HW #2
Out: September 20, 2005
Due: September 27, 2005

Problem 1: Buoyancy (3 pts.)

An AUV is designed as shown below. It has three, hollow, cylindrical canisters arranged in an upside down triangle. The cylindrical canisters are constructed out of aluminum, specific gravity of 2.7, and have a wall thickness of 1 cm, length 2 meters, and an inner diameter 14 cm. What is the maximum load (kg) this vehicle can carry without sinking, ignoring the additional weight and buoyancy of the struts? (Think neutrally buoyant!) Assume the vehicle will be operating in SALT water ($\rho = 1025 \text{ kg} / \text{m}^3$).



AUV Design.

Problem 2: Basic Fluids & Math (3 pts.)

- Although fluids, such as water, are really made up of discrete molecules, we can describe their behavior by differential equations by virtue of the _____ hypothesis.
- Discuss the difference between a pathline and a streamline. Under what conditions are they the same?

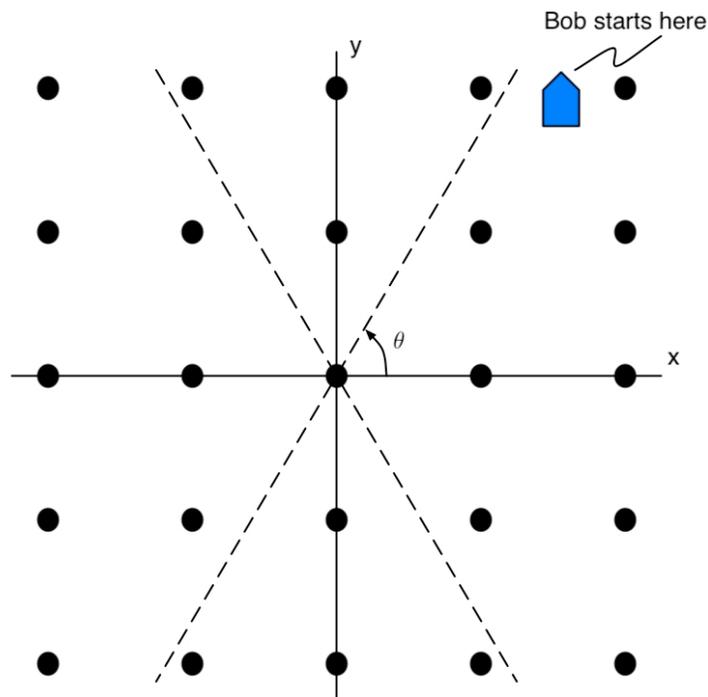
c) Show by expanding and collecting terms that $(\vec{V} \cdot \vec{\nabla})\vec{V} = \frac{1}{2}\vec{\nabla}(\vec{V} \cdot \vec{V})$ for irrotational

flow. (Hint: $\vec{\nabla} \times \vec{V} = \left(\frac{\partial w}{\partial y} - \frac{\partial v}{\partial z}\right)\hat{i} + \left(\frac{\partial u}{\partial z} - \frac{\partial w}{\partial x}\right)\hat{j} + \left(\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}\right)\hat{k} = 0\hat{i} + 0\hat{j} + 0\hat{k}$ for

irrotational flow.)

Problem 3: Velocity Field (8 pts.)

- a) A velocity field is given by $\vec{V} = (x^2 - y^2)\hat{i} - 2xy\hat{j}$. Is this a valid velocity field for an incompressible fluid? (Hint: It must satisfy the Continuity Equation.)
- b) Is the flow steady?
- c) Sketch arrows depicting the velocity vectors at each point on the graph below. (Hint: There is no flow across the dashed lines.)
- d) Sketch the streamlines on your graph.
- e) Determine the angle θ . (Recall, streamlines are everywhere tangent to the flow, so $\frac{dy}{dx} = \frac{v}{u}$ along a streamline.)
- f) Is the flow irrotational? (Hint: $\vec{\nabla} \times \vec{V} = \left(\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}\right)\hat{k}$ for this 2-D flow.)
- g) Sketch “Bob the Fluid Blob” at four different points along his path through this flow. Sketch his acceleration vector for each point.
- h) If we assume that pressure is the only force acting on Bob, then where is the pressure the highest?



Problem 4: Control Volume Analysis (3 pts.)

The largest artery in the body is the one that supplies blood to the legs. As it comes down the body it splits into a Y-junction, as shown in the figure below. Blood with specific gravity of 1.05 is pumped into the junction at speed $V_1 = 1.5$ m/s. The diameter of the entrance is $d_1 = 20$ mm. The two branches have diameter $d_2 = 15$ mm and $d_3 = 12$ mm. If the mass flow rates at stations 2 and 3 are equal, find the velocities V_2 and V_3 .

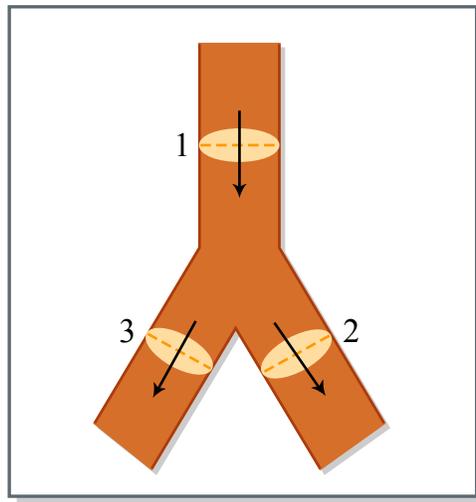


Figure by MIT OCW.

Problem 5: Bernoulli's Equation (3 pts.)

- Write down Bernoulli's Equation for steady flow.
- Under what conditions does this equation hold?

Fluid passes through a fan placed in a constant diameter duct shown in the figure below. Assume the density is constant.

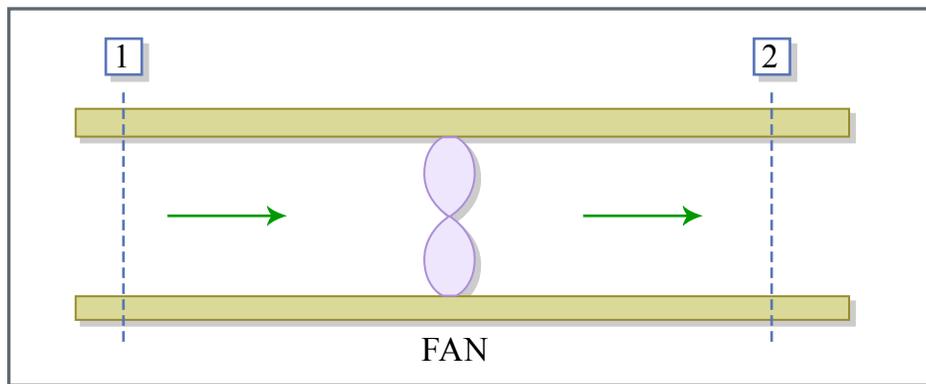


Figure by MIT OCW.

- Is the volume flow rate at station 1, the same as at station 2? Why?
- Can Bernoulli's equation be applied between stations 1 and 2? Why?