2.016 Handout
October 27, 2005

Review on Complex numbers:

Complex Notation:

$z = x + iy$; $\text{Re}\{z\} = x$, $\text{Im}\{z\} = y$

$Q = f(x)$ is equivalent to $Q = f(\text{Re}\{z\})$

Properties of e^{iz}:

$$\sin z = \frac{e^{iz} - e^{-iz}}{2i} \qquad \cos z = \frac{e^{iz} + e^{-iz}}{2}$$

$$\sinh z = \frac{e^{z} - e^{-z}}{2} \qquad \cosh z = \frac{e^{z} + e^{-z}}{2}$$

$$\sinh(iz) = i\sin(z) \qquad \sin(iz) = i\sinh(z)$$

$$\cosh(iz) = \cos(z) \qquad \cos(iz) = \cosh(z)$$

$$e^{iz} = \cos z + i \sin z$$

$$z = x + iy = r(\cos\theta + i\sin\theta) = re^{i\theta}$$

$\left|e^{i\alpha}\right| = 1$; for α = Real Constant

$$e^{z} = e^{x+iy} = e^{x}e^{iy} = e^{x}\cos y + ie^{x}\sin y$$

Complex potential makes wave forces on bodies even easier to solve:

Wave Potential: $\phi(x,z,t) = -\frac{A\omega}{k} e^{kz}\cos(kx - \omega t)$

Real part of a complex Potential: $\phi(x,z,t) = \text{Re}\left\{-\frac{A\omega}{k} e^{kz} e^{i(kx-\omega t)}\right\}$

OR

Wave Potential: $\phi(x,z,t) = \frac{A\omega}{k} e^{kz}\sin(kx - \omega t)$

Real part of a complex Potential: $\phi(x,z,t) = \text{Re}\left\{\frac{A\omega}{k} e^{kz}\, ie^{i(kx-\omega t)}\right\}$

Total Potential: $\Phi_T(x,z,t) = \frac{A\omega}{k} e^{kz} e^{i(kx-\omega t)} = \frac{A\omega}{k} e^{kz}\left\{\cos\left(kx - \omega t\right) + i\sin\left(kx - \omega t\right)\right\}$

This total potential satisfies Laplace's Equation as well.