13.012 Marine Hydrodynamics for Ocean Engineers
Fall 2004 Quiz #1

Student name: ______________________________________

This is a closed book examination. You are allowed 1 sheet of 8.5" x 11" paper with notes.

For the problems in Section A, fill in the answers where indicated by ____________, or in the provided space. When a list of options is presented ([…],[…],[….] etc), circle all the options (all, none, one or more) that apply.

Use the following constants unless otherwise specified:

Gravity: $g = 10\ m/s^2$	water density:	$\rho_w = 1000\ kg/m^3$	kinematic viscosity: $\nu_w = 1x10^{-6}\ m^2/s$
	Seawater density:	$\rho_w = 1025\ kg/m^3$	kinematic viscosity: $\nu_{sw} = 1x10^{-6}\ m^2/s$
	Air density:	$\rho_a = 1\ kg/m^3$	kinematic viscosity: $\nu_a = 1x10^{-5}\ m^2/s$

Assume the fluid is incompressible unless otherwise defined.
Give all answers in SI units (kg, m, s). All numerical answers MUST have the proper units attached.

Part A (30%):

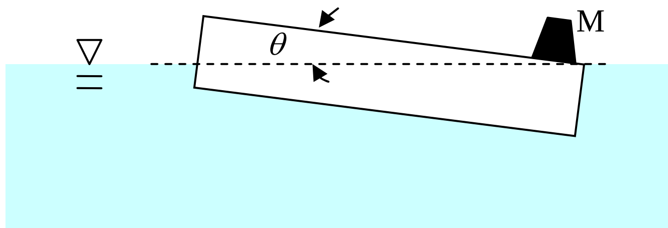
1) A 1 m^3 block of aluminum, specific gravity 2.7, is tethered to a piece of cork, specific gravity 0.24. The volume of cork required to keep the block neutrally buoyant in seawater is _______________ and the volume required in fresh water is _______________. Assume both the aluminum and cork are fully submerged and that the weight of the tether is negligible.

2) The velocity field $\vec{V} = 4xy\,\hat{i} + 10y^2\,\hat{j} + C\,zy\,\hat{k}$ is valid for C = _____________. The vorticity in this flow field is $\omega =$ ______________. This flow field is [irrotational][rotational].

3) The two *linearized* boundary conditions at the free surface for linear progressive free-surface gravity waves are _________________ and __________________. (give mathematically).

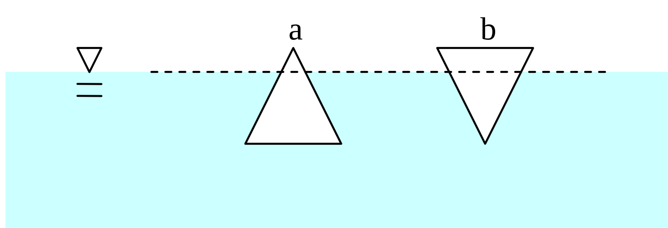
4) The linear free surface dispersion relationship is given by $\omega^2 =$ ________________. This relationship holds for [shallow] [intermediate] [deep] [all of the above] waves. The shallow water form of the dispersion relation is $\omega^2 =$ ________________. This holds for H/λ [greater than] [less than] _________________.

5) When a *M=5kg* weight is placed at the end of a uniform density floating wooden beam (*3 m* long, with a *10cm* x *10 cm* cross section), the beam tilts at an angle θ such that the upper right corner of the beam is exactly at the surface of the water (as shown below). The angle θ =_____________ and the specific gravity of the log is _________________.



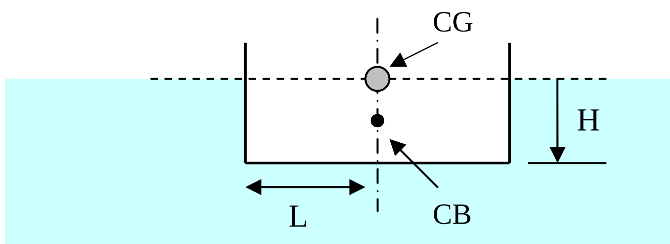
6) A progressive linear free surface gravity wave train is propagating in a tank from left to right. This wave train is generated by a wave paddle at one end. The deep water wave has frequency $\omega = 1 rad/s$. The time it takes for the front edge of the wave packet to reach the far end of the tank, 100 meters away, is ______________ seconds.

7) When floating in water ($\rho = 1000 kg/m^3$), an equilateral triangular body (SG = 0.9) with a long length into the board is more stable in position [a] [b].



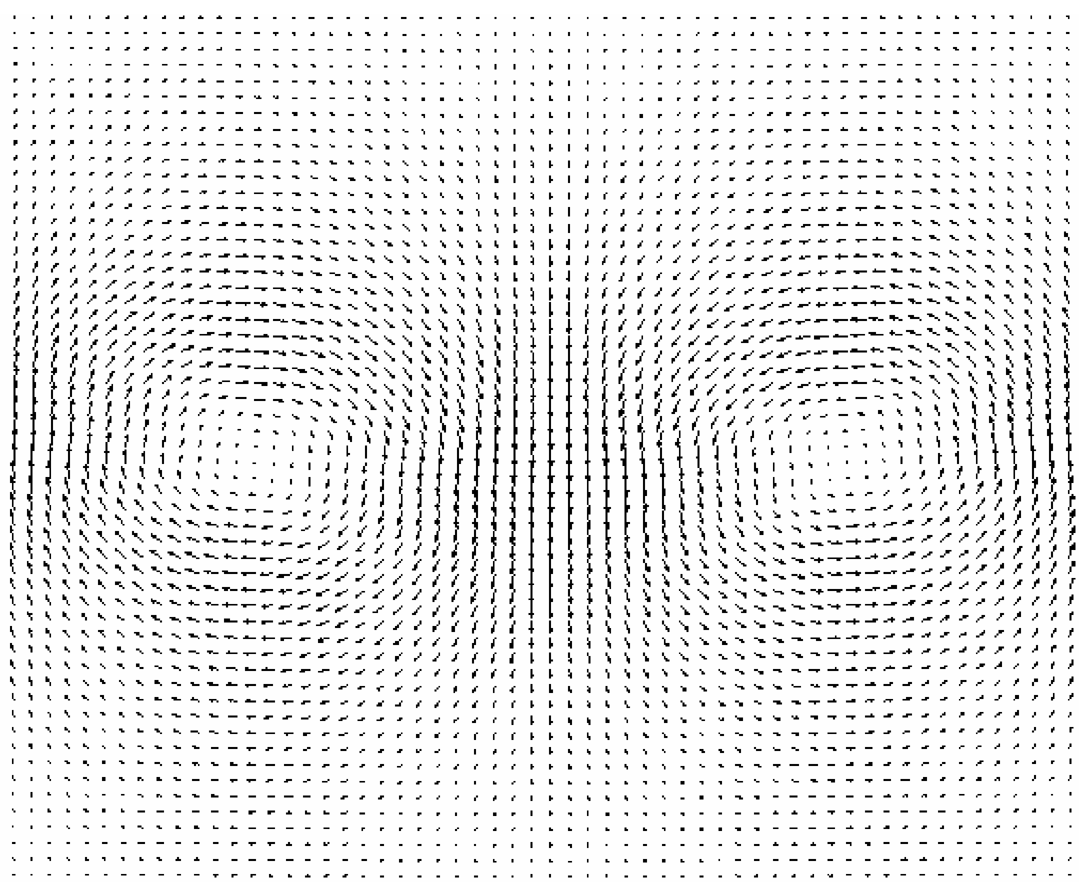
8) A deep water wave train with amplitude $a = 0.5m$ is propagating from left to right in a tank. The tank depth is 15 meters, and the wave frequency is 0.25 Hz. These waves are [linear] [non-linear][cannot determine] water waves. The appropriate dispersion relationship is given by ω^2 = _______________. The wavelength λ = _____________. Phase speed V_p = ______________. Group speed of the waves, V_g = _____________.

9) For the rectangular barge with width *2L* and vertical draft *H* shown below, the Metacentric Height $\overline{GM} =$ ____________________ for small tilt angles (in terms of *L* and *H*). This barge can only be stable if ___________ > ___________.



10) The conservation of mass equation depends on the assumption(s) of [constant density] [irrotationality] [inviscid fluid] [incompressibility] [Newtonian fluid] [matter cannot be created]. For incompressible flow, the density ρ satisfies the equation ________________________.

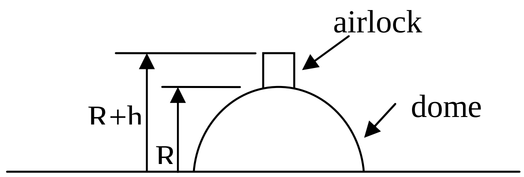
11) BONUS (5pts) Sketch *ten* streamlines on the plot below. Vectors represent the flow velocity.



FOR PART B PROBLEMS: BE SURE TO SKETCH THE PROBLEM, STATE YOUR ASSUMPTIONS AND SHOW YOUR WORK! USE THE PROVIDED EXAM BOOKLET. DON'T FORGET TO CHECK UNITS!

Problem B.1: (35%)

A highly skilled diver is to be sent on a mission to repair a broken cable that supplies electricity to an underwater habitat. The cable has come un-plugged and is tangled about a nearby rock formation. The habitat is located 100m under the water, and is shaped as a hemispherical dome (radius $R=15m$) with a cylindrical airlock chamber (height $h=5m$, radius $r=h/2$) at the very top of the dome.



a) To release the diver into the water the airlock must be filled with water and pressurized to ambient ocean pressure at the depth of the hatch on top of the airlock. Determine the magnitude of pressurization required inside the airlock to release a diver into the water from the underwater habitat.

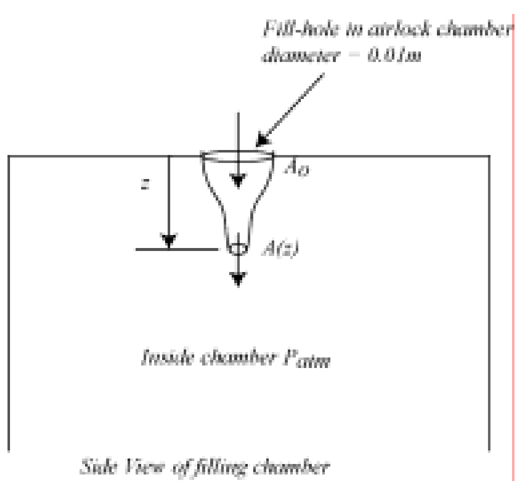
b) If the airlock chamber did not pressurize to ambient ocean pressure, but stayed at atmospheric pressure, what is the force that would be required to open the hatch (upwards in to the ocean)? The hatch has the same radius as the airlock cylindrical chamber.

The airlock fill valve is a simple circular hole in the top of the airlock, as shown below. When the fill valve is open water pours into the airlock. At some depth, *z*, below the fill the water spout has area *A(z)*.

c) Determine area *A(z)* of the incoming water spout at some depth below the opening.

d) Calculate how long it will take (time in seconds) the airlock chamber to completely fill

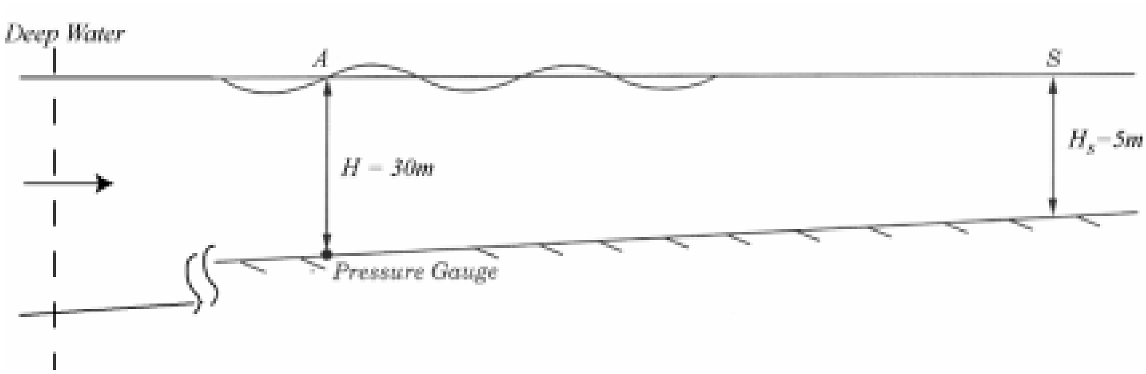
   Assume that the chamber is kept at atmospheric pressure while it fills and the air is released as the water comes in so that the air does not compress. The diameter of the fill hole is $D_o = 0.01m$. The flow at the inlet and across *A(z)* is uniform.



Hint: Bernoulli's equation will come in *very* handy here. Consider a streamline along the middle of the spout and a reference point where the vertical velocity is zero.

PROBLEM B.2: (35%)

A plane progressive LINEAR wave travels from deep water towards shallow depth over a gently sloping bottom. To record the wave characteristics, a pressure gage is positioned on the bottom at station A where the depth is $H = 30\ m$.



The difference between maximum and minimum pressure measured is $6000\ N/m^2$. Measurements had a period of 10 *seconds*. The bottom slope is very gradual, and can be taken locally as constant, such that reflections and energy dissipation can be neglected and the wave period remains constant. For simplicity use $\rho = 1000 m^3 / kg;\ g = 10 m / s^2$.

a) Determine the wavelength, λ, amplitude, a, group velocity, V_g, at point A and point S.

b) Out in deep water the wave period and amplitude can be assumed to be the same as at point A. Determine the energy flux $\Im$ across a vertical plane in deep water.

c) Calculate the mean volume (mass) transport per unit width through the same vertical plane in deep water.

d) Calculate the maximum instantaneous horizontal volume flux at this point. (ie. depth integrated velocity).

e) Sketch the particle orbits at the two positions A and S.