2.019 Design of Ocean Systems

Lecture 11

Drift and Slowly-Varying Loads (I)

March 11, 2010

Drift (or Mean) Forces/Moments

- Wind: Steady (and unsteady) drag forces/moments
- Current: Steady drag forces/moments
- Waves: Nonlinear wave loads including steady and unsteady loads

Important for design of mooring system for station keeping !!

Wind Drag Forces/Moments

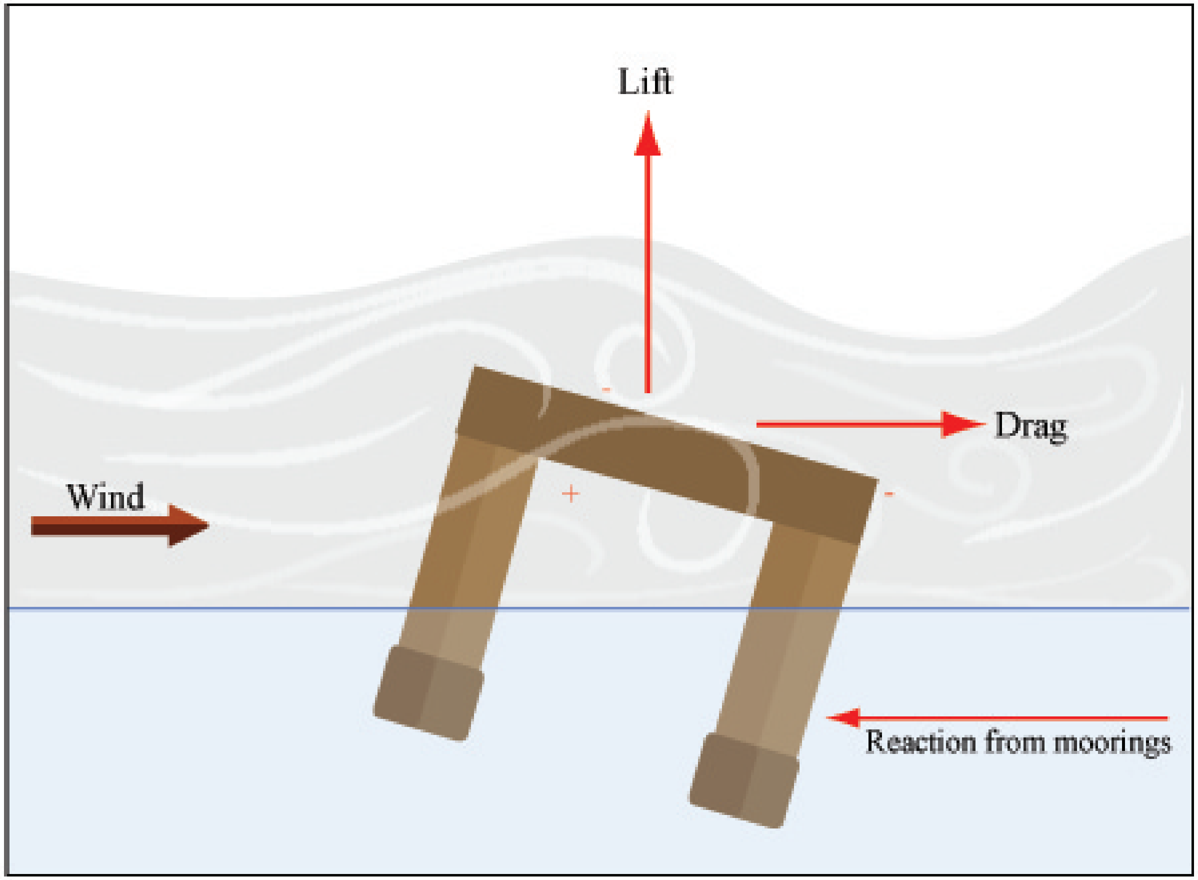


Image by MIT OpenCourseWare.

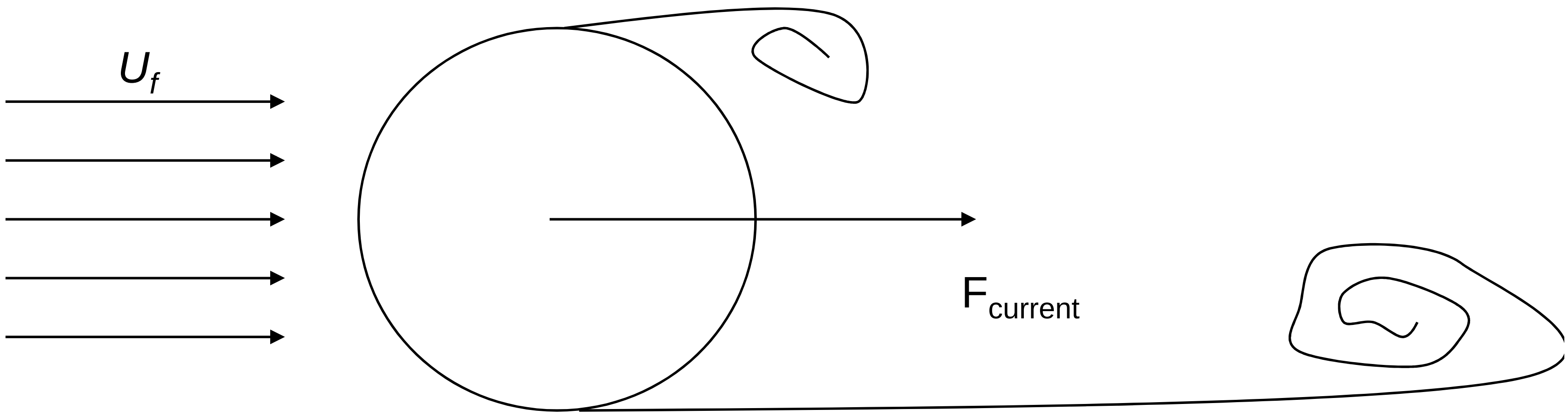
$$F_{wind} = \frac{1}{2}\rho_{air} C_s C_H V_{10}|V_{10}|S$$

$$M_{wind} = \sum F_{wind} \times h$$

C_s: shape coefficient (or Drag coefficient), C^s=1.0 for large flat surface
C_H: height coefficient, C_H=1.0 for FPSO
V_{10}: wind velocity at 10m above sea surface
S: Project area of the exposed surfaces in the vertical or the heeled condition
h: vertical distance between center of wind force and center of resistance (by mooring lines, etc)

Current Drag Forces and Moments

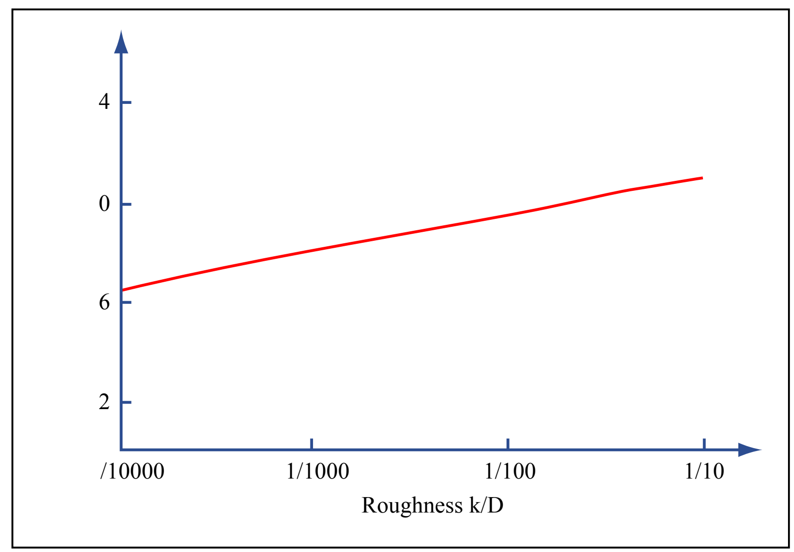
For normal incidence,



- Body is fixed:

$$F_{current} = \frac{1}{2}\rho_w C_D U_f |U_f| S$$

C_D: Drag coefficient
U_f: Current velocity
S: Project area of the exposed surfaces
ρ_w: water density



- Body moves at U_b:

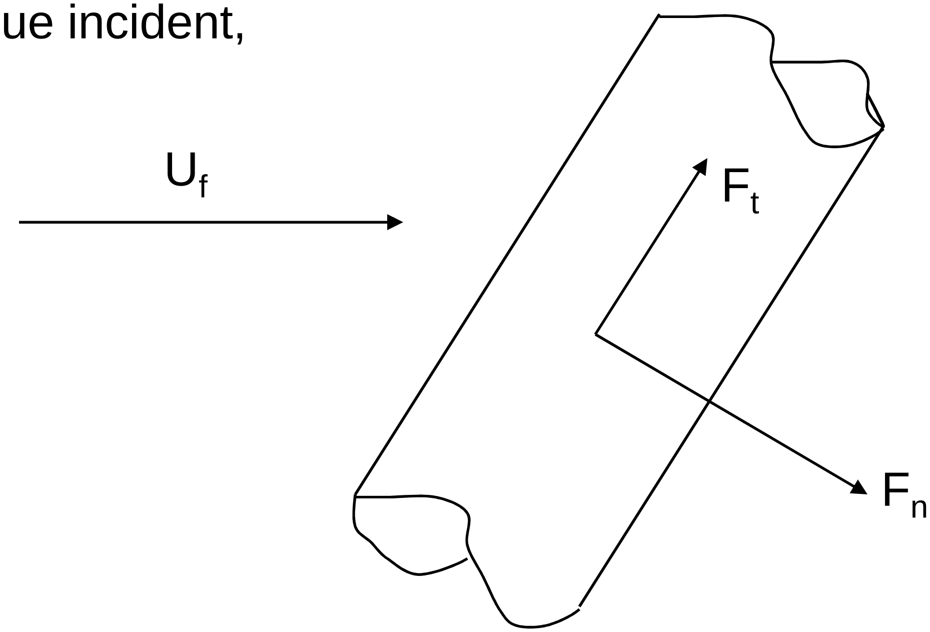
$$F_{current} = \frac{1}{2}\rho_w C_D (U_f - U_b)|U_f - U_b| S$$

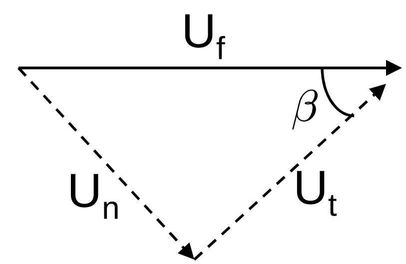
- Frictional drag coefficient on a ship hull: $C_f = \frac{0.075}{(\log_{10}(Rn)-2)^2}$

Rn: Reynolds number in terms of ship length *L*

Current Drag Forces and Moments

In oblique incident,





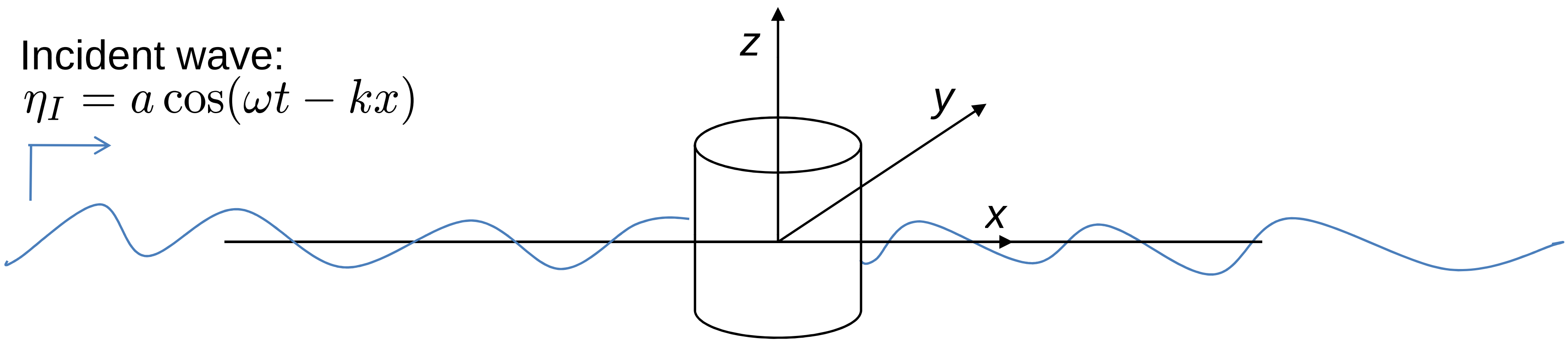
Normal component U_n causes flow separation drag, tangential component U_t causes frictional drag

$$F_n = \frac{1}{2}\rho C_D U_n |U_n| S$$

$$F_t = \frac{1}{2}\rho C_f U_t |U_t| S_t$$

Valid for β in the range of 30° to 150°.

Wave Drift (Mean) Force/Moment



Wave drift force/moment comes from:

(1) 2nd-order hydrodynamic pressure due to the first order wave

(2) Interaction between the first-order motion and the first-order wave

2^{nd}-order Hydrodynamic Mean Pressure

Consider a simple plane progressive wave in deep water:

$$\Phi(x,z,t) - \frac{gA}{\omega}\mathrm{e}^{-kz}\sin(\omega t \quad kx)$$
$$\eta(x,t) = a\cos(\omega t - kx)$$

We look at the pressure field of the wavefield:

$$\frac{P(x,z,t)}{\rho} = -\frac{\partial\Phi}{\partial t} - \frac{1}{2}\nabla\Phi\cdot\nabla\Phi - gz$$

$$\nabla\Phi\cdot\nabla\Phi = \Phi_x^2 + \Phi_y^2 + \Phi_z^2$$

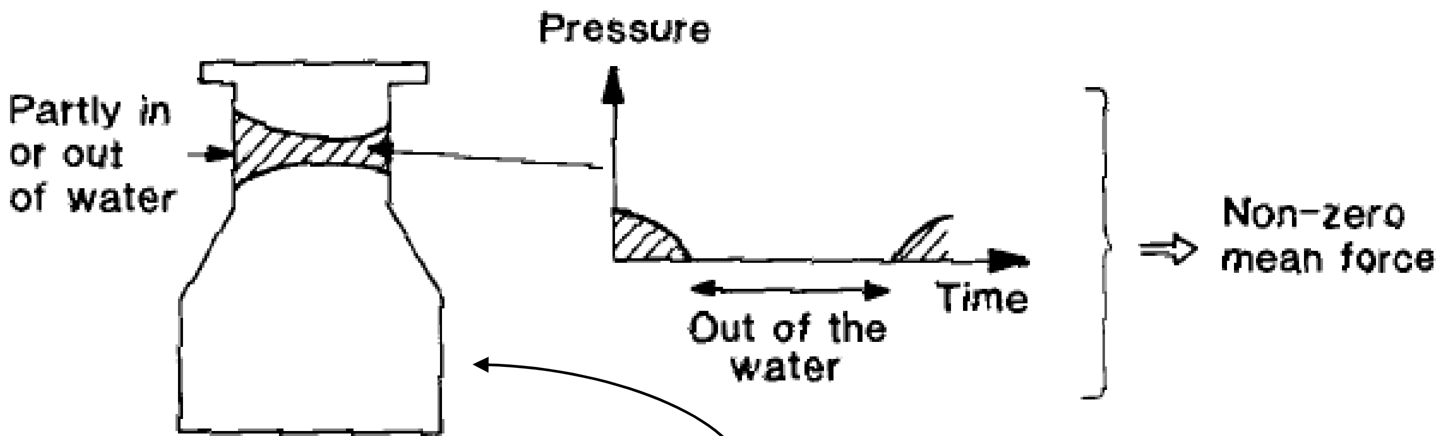
$$\Phi_x^2 = \left\{-\frac{gAk}{\omega}\mathrm{e}^{kz}\cos(\omega t - kx)\right\}^2 = (\omega A\mathrm{e}^{kz})^2\cos^2(\omega t - kx)$$
$$\Phi_z^2 - \left\{\frac{gAk}{\omega}\mathrm{e}^{kz}\sin(\omega t - kx)\right\}^2 - (\omega A\mathrm{e}^{kz})^2\sin^2(\omega t - kx)$$

$$\nabla\Phi\cdot\nabla\Phi = \Phi_x^2 + \Phi_z^2 = (\omega A\mathrm{e}^{kz})^2$$

Steady, independent of time, and ~*A^2*.

2^{nd}-order mean pressure component: $\rho A^2(\omega\mathrm{e}^{kz})^2$
Integration this mean pressure component over body surface to give mean force/moment

Interaction Between Body Motion and First-Order Wave



$$\vec{F}(t) = \int_{S(t)} P(t)\vec{n}\mathrm{d}s = \int_{S_0} P(t)\vec{n}\mathrm{d}s + \int_{\Delta S(t)} P(t)\vec{n}\mathrm{d}s$$

$$\int_{S_0} \overline{P}\vec{n}\mathrm{d}s$$

→ 2^{nd}-order mean pressure effect

Direct Pressure Integration

Drift force can be obtained by direct integration of the pressure over the body surface and then taking time average:

$$\overline{\vec{F}(t)} - \overline{\int_{S(t)} P(t)\vec{n}\mathrm{d}s} - \overline{\int_{S(t)} \left\{ \quad \rho g z \quad\quad \rho\frac{\partial\Phi}{\partial t} \quad\quad \frac{\rho}{2}(\Phi_x^2 + \Phi_y^2 + \Phi_z^2)^2 \right\} \vec{n}\mathrm{d}s} - \cdots\cdots$$

For example,

$$\Phi(x,z,t) = \frac{2gA}{\omega}\mathrm{e}^{kz}\cos\omega t\cos kx \quad \text{standing wave}$$

$$\eta(x,t) = 2A\sin\omega t\cos kx$$

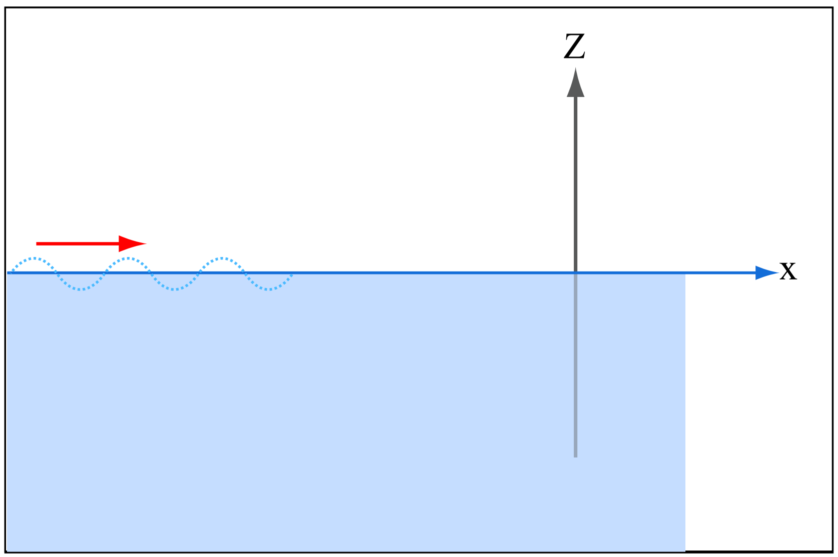


Diagram showing drift forces and incident waves on a vertical wall, Z. Image by MIT OpenCourseWare.

The first two terms' contribution:

$$\overline{-\rho g\int_0^\eta z\mathrm{d}z} - \overline{\rho\left.\frac{\partial\Phi}{\partial t}\right|_{z=0}\eta} = \rho g A^2$$

The third term's contribution:

$$\overline{-\frac{\rho}{2}\int_{-\infty}^{0}(\Phi_x^2 + \Phi_y^2 + \Phi_z^2)^2\mathrm{d}s}$$

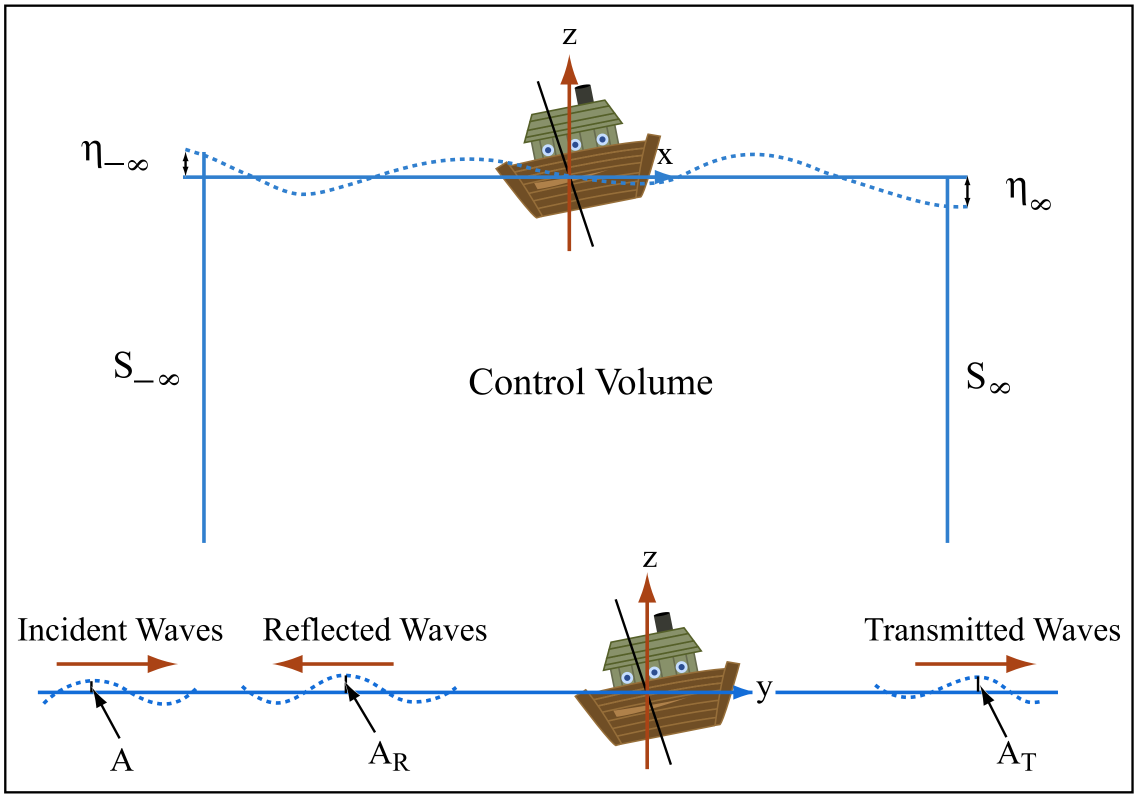
$$= -\frac{\rho}{2}\int_{-\infty}^{0}\frac{1}{2}\frac{4g^2A^2}{\omega^2}k^2\mathrm{e}^{2kz}\mathrm{d}z$$

$$= -\frac{1}{2}\rho g A^2$$

The total horizontal drift force on the wall is: $\frac{1}{2}\rho g A^2$

Far-Field Formula

Mean force or moment on a floating body can also be obtained using the so-called far-field formula developed from the momentum theorem.



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Momentum theorem:

$$\sum F_x(t) + \sum FM = \frac{\mathrm{d}M_x}{\mathrm{d}t}$$

$$\overline{\sum F_x(t) + \sum FM} = \overline{\frac{\mathrm{d}M_x}{\mathrm{d}t}}$$

F_x is the force acting on control volume, FM is total momentum flux into the control volume, M_x is linear momentum in control volume

$$\overline{\frac{\mathrm{d}M_x}{\mathrm{d}t}} = 0$$

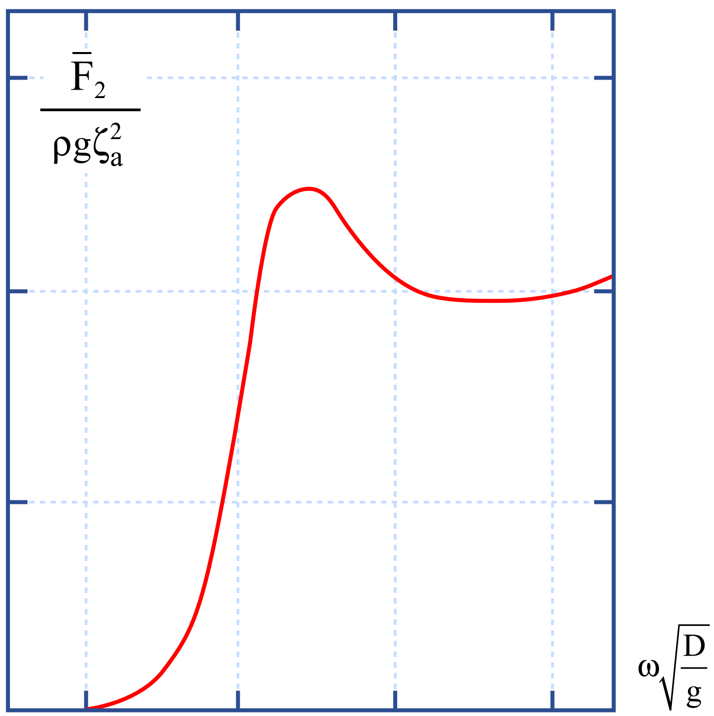
F_x contains force on body, force on $S_{-\infty}$ and $S_{+\infty}$

FM contains momentum flux into control volume from boundaries S_{-} and $S_{+\infty}$

$$\overline{F}_x = \frac{\rho g}{4}(A^2 + A_R^2 - A_T^2)$$

Since $A^2 = A_R^2 + A_T^2$, we finally have $\overline{F_x} = \frac{\rho g}{2} A_R^2$

•Mean force/moment is 2nd-order in wave amplitude



$\bar{F}_2$ is the horizontal force in wave direction on the 2D body;
D = draught, ζ_a = wave amplitude of the incident waves,
ω = circular frequency of oscillations.

F2 is the horizontal force in wave direction on the 2D body

Mean Force/Moment in Irregular Sea

$$\bar{F}_j = H_{\bar{F}_j}(\omega)A^2 \ , j = 1, \ldots, 6$$

where $H_{\bar{F}_j}$ is the transfer function for mean force $\bar{F}_j$

In irregular sea,

$$\bar{F}_j = \sum_{i=1}^{N} H_{\bar{F}_j}(\omega_i)A_i^2 \quad j = 1, \ldots, 6$$

A_i is the wave amplitude of the i-th wave component.

$$A_i - \sqrt{2S(\omega_i)\Delta\omega}$$

$$\begin{aligned}
\bar{F}_j &= \sum_{i=1}^{N} H_{\bar{F}_j}(\omega_i)2S(\omega_i)\Delta\omega \quad j = 1, \ldots, 6 \\
&= 2\int_0^{\infty} S(\omega)H_{\bar{F}_j}(\omega)\mathrm{d}\omega \quad j = 1, \ldots, 6
\end{aligned}$$