2.019 Design of Ocean Systems

Lecture 11

Drift and Slowly-Varying Loads and Motions (II)

March 14, 2011

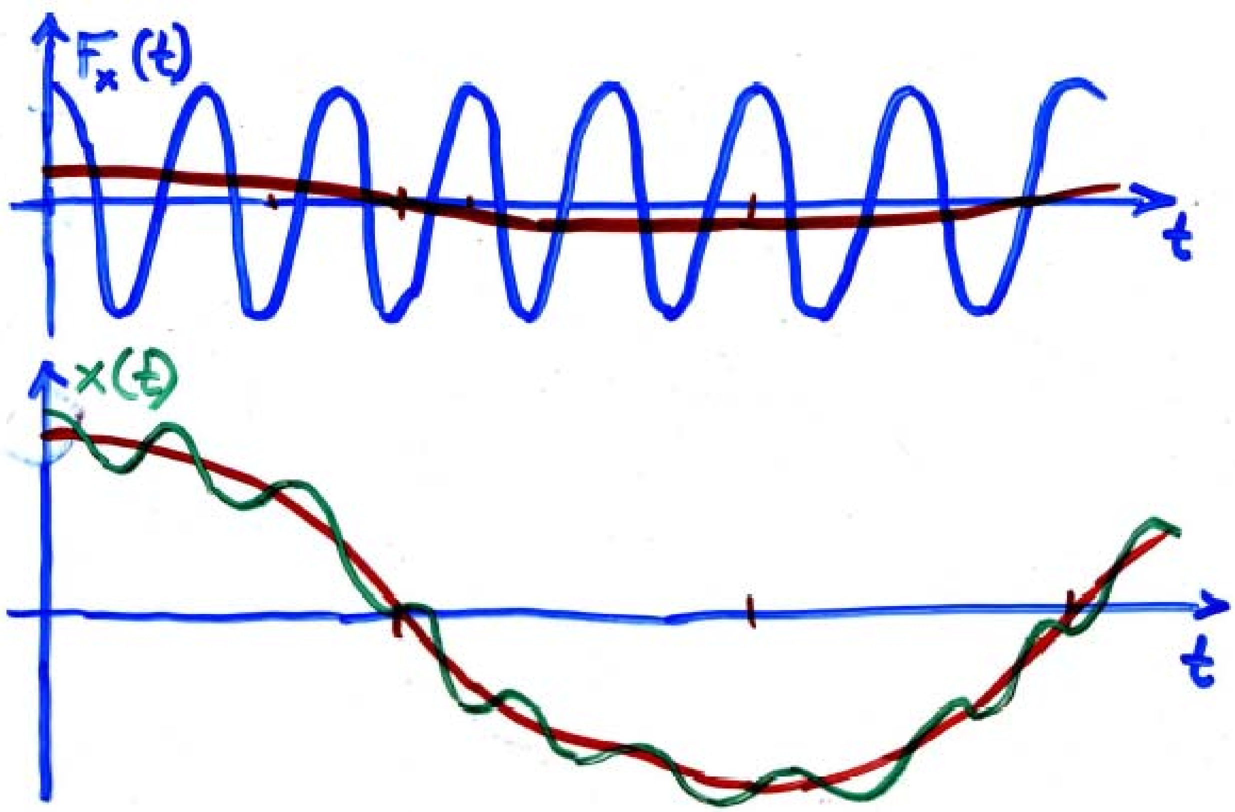
Example of Slowly-Varying Drift Motion

Excitation: $F_x(t) = f_0 \cos(\omega t) + 0.1 f_0 \cos(0.1\omega t)$

Equation of Motion: $M \frac{\mathrm{d}^2 x}{\mathrm{d}t^2} = F_x(t)$

Solution of motion:

$$\begin{aligned} x(t) &= -\frac{f_0}{M\omega^2}\left\{\cos\omega t + \frac{0.1}{0.1^2}\cos(0.1\omega t)\right\} \\ &= -\frac{f_0}{M\omega^2}\left\{\cos\omega t + 10\cos(0.1\omega t)\right\} \end{aligned}$$



Responses of Floating Structures in Ocean

Natural Frequency: $$\omega_n = \sqrt{\frac{C}{(M+M_a)}}$$

- For surge, sway, and yaw: hydrostatic restoring coefficients $C_{11}, C_{22}, C_{66} = 0$

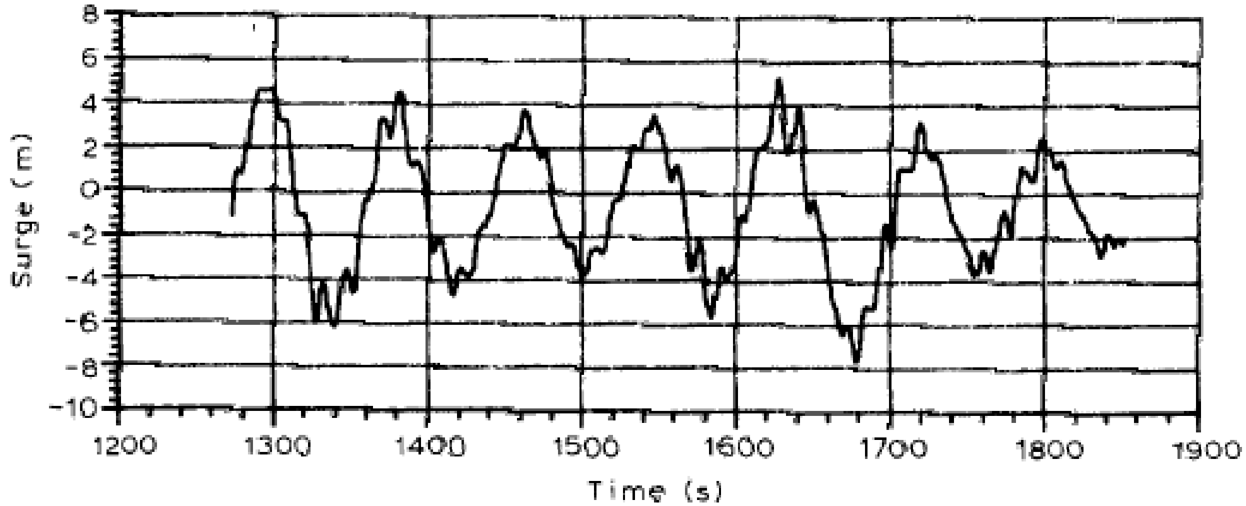
$$\rightarrow \omega_n = 0$$

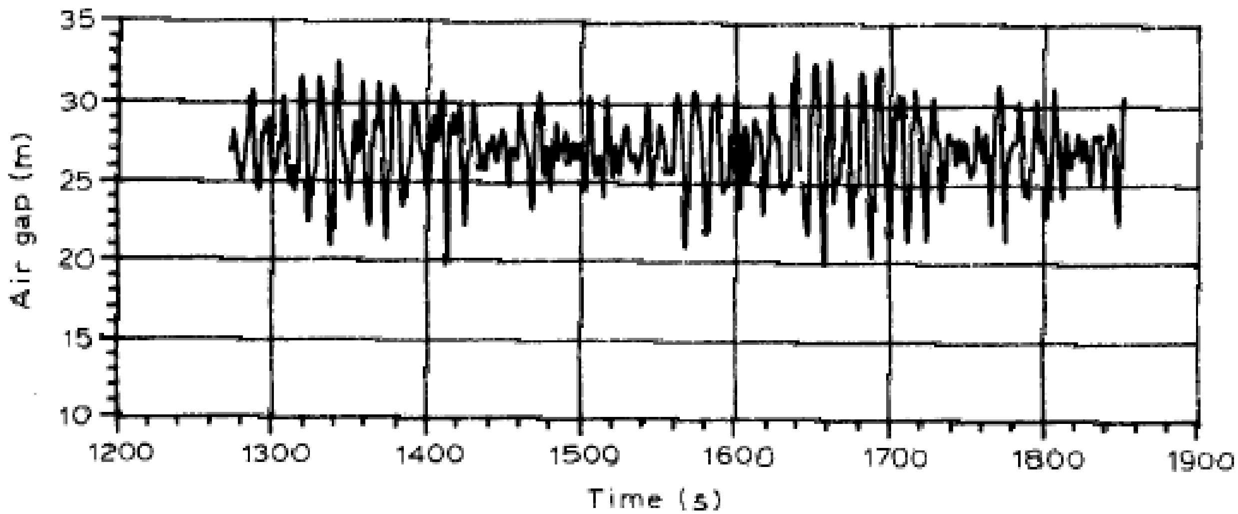
Large-amplitude responses can be excited by slowly-varying excitations.

- For structures with small water-plane area such as semi-submersibles, the hydrostatic restoring for heave, pitch, and roll are small, the natural frequencies are small. In this case, large-amplitude responses can also be excited by slowly-varying excitations.

- In general, very little wave energy at low frequency is present in the ocean. Thus low frequency wave excitation (based on linear wave theory) is small. Thus, from linear theory, no large-amplitude slowly-varying responses can be caused by the action of waves!!

- Source of slowly-varying excitations:
  - –Nonlinear wave structure interaction
  - –Wind loads

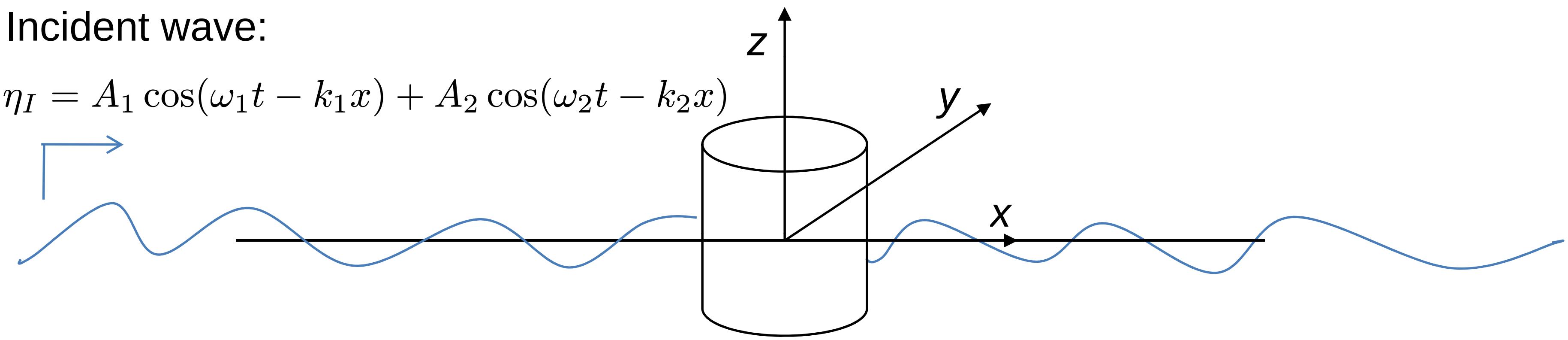

Surge (m)
8
6
4
2
0
-2
-4
-6
-8
-10
1200
1300
1400
1500
1600
1700
1800
1900
Time (s)


Air gap (m)
35
30
25
20
15
10
1200
1300
1400
1500
1600
1700
1800
1900
Time (s)

Slowly-Varying Wave Force/Moment

Incident wave:

$$\eta_I = A_1 \cos(\omega_1 t - k_1 x) + A_2 \cos(\omega_2 t - k_2 x)$$



Slowly-varying wave force/moment comes from:

(1) 2^{nd}-order hydrodynamic pressure due to the first order wave

(2) Interaction between the first-order motion and the first-order wave

(3) 2^{nd}-order potential due to slowly-varying forcing on body surface and free-surface

2^{nd}-order Slowly-Varying Hydrodynamic Pressure

Consider two simple plane progressive waves in deep water:

$$\Phi(x,z,t) = -\frac{gA_1}{\omega_1}\mathrm{e}^{k_1 z}\sin(\omega_1 t - k_1 x) - \frac{gA_2}{\omega_2}\mathrm{e}^{k_2 z}\sin(\omega_2 t - k_2 x)$$

$$\eta(x,t) = A_1\cos(\omega_1 t - k_1 x) + A_2\cos(\omega_2 t - k_2 x)$$

We look at the pressure field of the wavefield: $\frac{P(x,z,t)}{\rho} = -\frac{\partial\Phi}{\partial t} - \frac{1}{2}\nabla\Phi\cdot\nabla\Phi - gz$

$$\nabla\Phi \quad \nabla\Phi = \Phi_x^2 + \Phi_y^2 + \Phi_z^2$$

$$\Phi_x = \frac{gA_1k_1}{\omega_1}\mathrm{e}^{k_1 z}\cos(\omega_1 t - k_1 x) + \frac{gA_2k_2}{\omega_2}\mathrm{e}^{k_2 z}\cos(\omega_2 t - k_2 x)$$

$$\begin{aligned}\Phi_x^2 &= \cdots + 2\omega_1\omega_2 A_1 A_2 \mathrm{e}^{(k_1+k_2)z}\cos(\omega_1 t - k_1 x)\cos(\omega_2 t - k_2 x) + \cdots \\ &= \cdots + \omega_1\omega_2 A_1 A_2 \mathrm{e}^{(k_1+k_2)z}\{\cos[(\omega_1-\omega_2)t - (k_1-k_2)x) + \cos[(\omega_1+\omega_2)t - (k_1+k_2)x)\} + \cdots\end{aligned}$$

→ 2^{nd}-order In wave amplitude

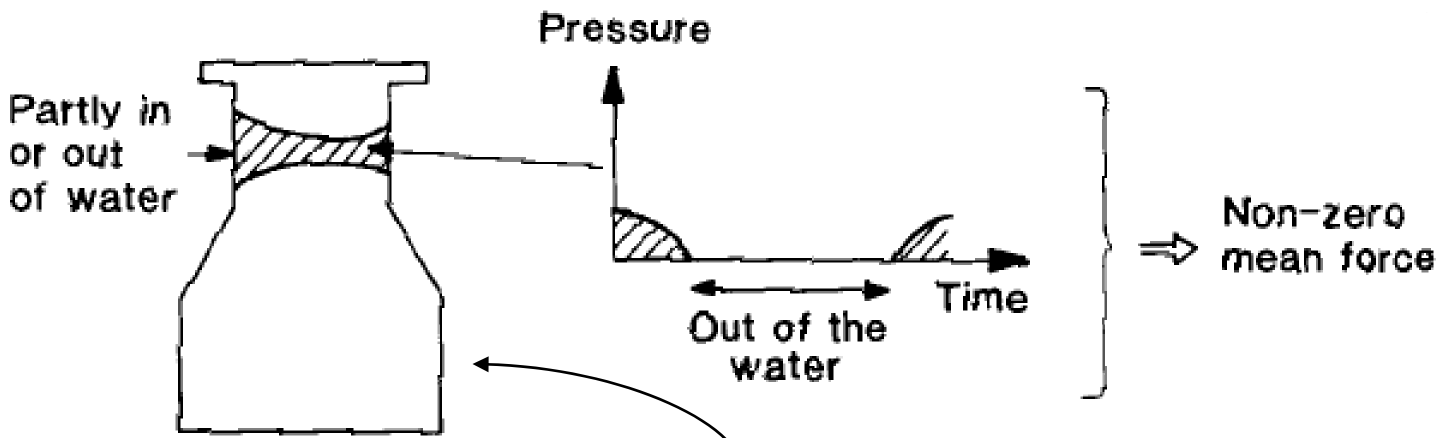
→ Difference –frequency (slowly-varying)

→ Sum-frequency

2^{nd}-order slowly-varying pressure component: $\sim A_1A_2\mathrm{e}^{(k_1+k_2)z}\cos[(\omega_1-\omega_2)t - (k_1-k_2)x]$

Integration this slowly-varying pressure component over body surface to give slowly-varying force/moment

Interaction Between Body Motion and First-Order Wave



$$\vec{F}(t) = \int_{S(t)} P(t)\vec{n}\mathrm{d}s = \int_{S_0} P^{(2)}(t)\vec{n}\mathrm{d}s + \int_{\Delta S(t)} P^{(1)}(t)\vec{n}\mathrm{d}s$$

2nd-order slowly varying pressure effect

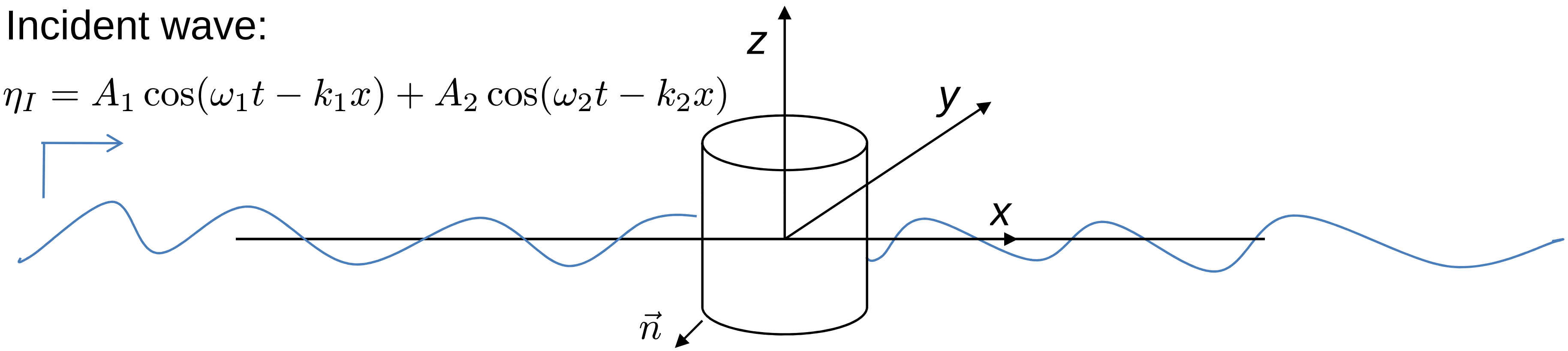
$$\Delta S(t) \sim A_1 \cos(\omega_1 t + A_2 \cos(\omega_2 t)$$

Interaction gives 2nd-order slowly-varying force/moment

2^{nd}-Order Slowly-Varying Potential Due to Forcing on Body Surface and Free-Surface

Incident wave:

$$\eta_I = A_1 \cos(\omega_1 t - k_1 x) + A_2 \cos(\omega_2 t - k_2 x)$$



Body has a first-order motion resulting from the action of the incident wave, for example, the heave motion:

$$\zeta_3(t) = a_1 \cos(\omega_1 t) + a_2 \cos(\omega_2 t)$$

General body boundary condition imposed on instantaneous body position S_B(t):

$$\frac{\partial \Phi}{\partial n} = \frac{\mathrm{d}\zeta_3(t)}{\mathrm{d}t} \cdot n_z = -n_z[a_1\omega_1 \cos(\omega_1 t) - \omega_2 a_2 \sin(\omega_2 t)]$$

Applying Taylor series expansion of the body boundary condition about the mean body position $\bar{S}_B$

$$\frac{\partial \Phi}{\partial n}\Big|_{S_B(t)} = \frac{\partial \Phi}{\partial n}\Big|_{\bar{S}_B} + \zeta_3(t) \cdot \frac{\partial^2 \Phi}{\partial z \partial n}\Big|_{\bar{S}_B} + \cdots$$

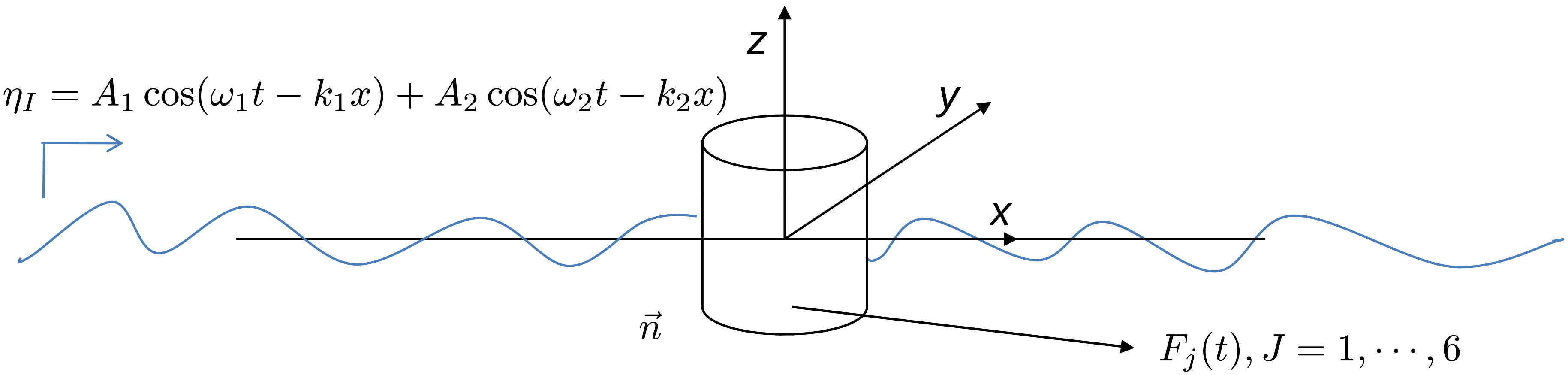
This terms gives slow-varying terms: $\sim \cos[(\omega_1 - \omega_2)t]$ $\sim \sin[(\omega_1 - \omega_2)t]$

Similar forcing terms are also obtained on the free-surface boundary condition.

- These lead to a 2^{nd}-order potential: $\Phi^{(2)} \sim \cos[(\omega_1 - \omega_2)t] \text{and} \sin[(\omega_1 - \omega_2)t]$

From Bernoulli equation, this potential gives a slowly-varying pressure $P^{(2)} = -\rho \frac{\partial \Phi^{(2)}}{\partial t}$

Determination of Slowly-Varying Wave Force/Moment



$$F_j^{sv}(t) = A_1 A_2 \{ Q_{12}^{jc} \cos(\omega_1 - \omega_2)t + Q_{12}^{js} \sin(\omega_1 - \omega_2)t \} \qquad j = 1, \cdots, 6$$

$Q_{12}^{jc}(\omega_1, \omega_2)$ and $Q_{12}^{js}(\omega_1, \omega_2)$ are the slowly-varying force/moment transfer functions.

How to find the slowly-varying force/moment transfer functions??

- By experiments — accurate measurement of 2nd-order slowly-varying force/moment is challenge in laboratory

- By numerical computation — using WAMIT or other nonlinear computational tools (state-of-the-are research in this area is still going on….)

Determination of Slowly Varying Force/Moment in Irregular Seas

- Incident wave travels in x direction in deep water:

$$\eta^I(x,t) = \sum_{\ell=1}^{N} A_\ell \cos(\omega_\ell t - k_\ell x + \epsilon_\ell)$$

$$A_\ell(\omega_\ell) = \sqrt{2S(\omega_\ell)\Delta\omega} \quad \text{and} \quad \Delta\omega = (\omega_{max} - \omega_{min})/N$$

$S(\omega)$ is the spectrum of the irregular waves

- Slowly-varying force/moment on a floating body is given by:

$$F_j^{sv} = \frac{1}{2}\sum_{\ell=1}^{N}\sum_{k=1}^{N} A_\ell A_k \left\{ Q_{\ell k}^{jc}\cos[(\omega_k - \omega_\ell)t + (\epsilon_k - \epsilon_\ell)] + Q_{\ell k}^{js}\sin[(\omega_k - \omega_\ell)t + (\epsilon_k - \epsilon_\ell)] \right\}$$

$$j = 1, 2, \ldots, 6 \qquad Q_{\ell k}^{jc} = Q_{k\ell}^{jc} \quad \text{and} \quad Q_{\ell k}^{js} = -Q_{k\ell}^{js}$$

- Applying ***<u>Newman's Approximation</u>***:

$$Q_{\ell k}^{jc} = Q_{k\ell}^{jc} = Q_{\ell\ell}^{jc} + Q_{kk}^{jc} \quad \text{and} \quad Q_{\ell k}^{js} = Q_{k\ell}^{js} = 0$$

where $Q_{\ell\ell}^{jc}$ and Q_{kk}^{jc} are the transfer function for drift force/moment, i.e.

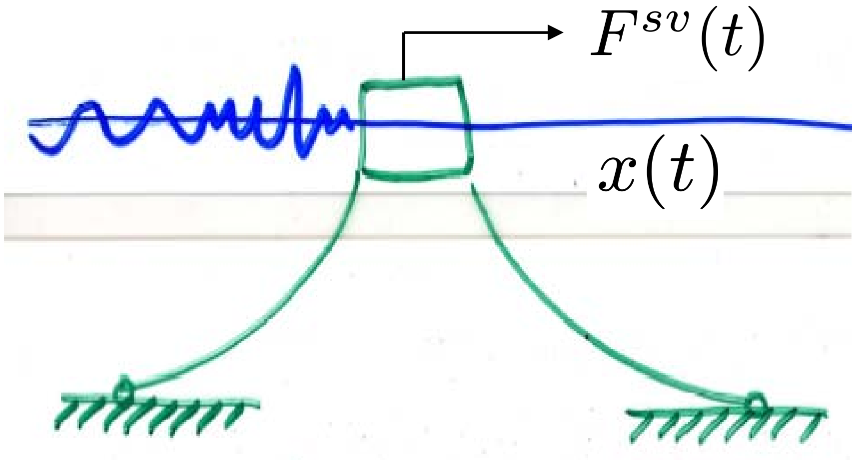
$$\bar{F}_j(\omega_\ell) = A_\ell^2 Q_{\ell\ell}^{jc} \quad \text{and} \quad \bar{F}_j(\omega_k) = A_k^2 Q_{kk}^{jc}$$

- Spectrum of the slowly-varying force/moment:

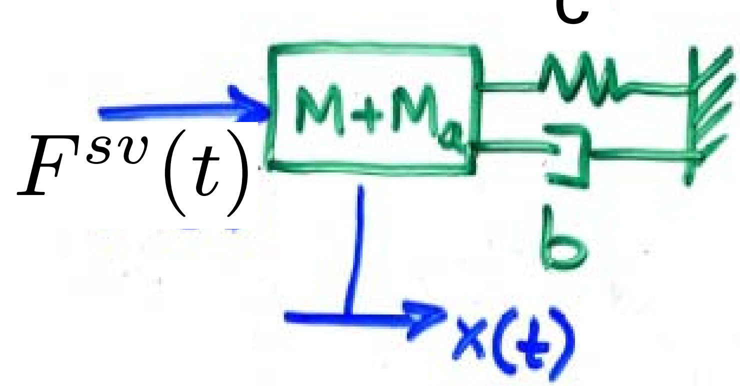
$$S_{F_j}(\mu) = 8 \int_0^\infty S(\omega)S(\omega+\mu)[Q_{\ell\ell}^{jc}(\omega+\mu/2)]^2 \mathrm{d}\omega$$

Slowly-Varying Motion

Moored system in waves:



Mass-Spring-Dashpot system:



c: mooring line spring equivalent
b: total damping in the system

Equation of motion:

$$(M + M_a)\frac{\mathrm{d}^2}{\mathrm{d}t^2}x(t) + b\frac{\mathrm{d}}{\mathrm{d}t}x(t) + cx(t) = F^{sv}(t)$$

In the frequency domain:

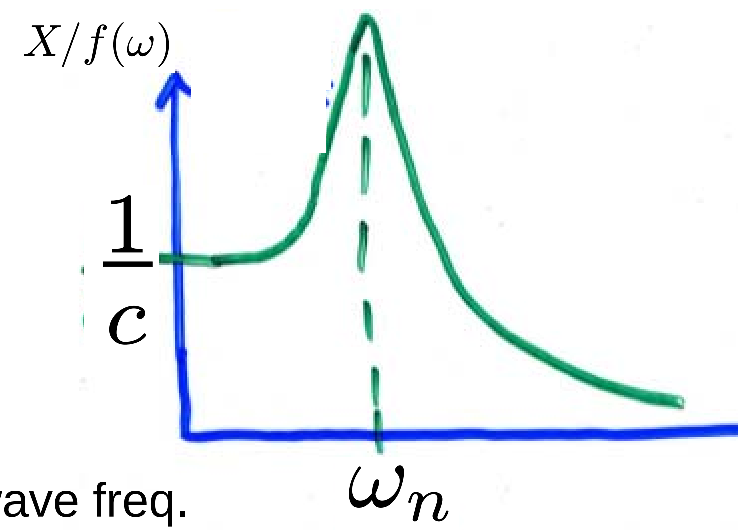
$$-\omega^2(M + M_a)X(\omega) + i\omega bX(\omega) + cX(\omega) = f(\omega)$$

$$X(\omega) = \frac{f(\omega)}{[c-\omega^2(M+M_a)]+i\omega b}$$

$$\omega_n = \sqrt{\frac{c}{M+M_a}}$$

$\omega_n \sim 0.1\ \text{rad/s}$

Much lower than wave freq.



Spectrum of slowly-varying motion:

$$S_x(\omega) = \frac{S_{F^{sv}}(\omega)}{[c-\omega^2(m+M_a)]^2+b^2\omega^2}$$

Variance:

$$\begin{aligned}
\sigma_x &= \int_0^\infty S_x(\omega)\mathrm{d}\omega \\
&= \int_0^\infty \frac{S_{F^{sv}}(\omega)}{[c-\omega^2(m+M_a)]^2+b^2\omega^2}\mathrm{d}\omega \\
&\approx S_{F^{sv}}(\omega_n)\int_0^\infty \frac{\mathrm{d}\omega}{[c-\omega^2(m+M_a)]^2+b^2\omega^2} \\
&= \frac{\pi}{2cb}S_{F^{sv}}(\omega_n)
\end{aligned}$$

Source of c: mooring lines

Source of b: (i) related to hull from — friction, flow separation, current/wind, **wave drift damping**

(ii) mooring lines