2.019 Design of Ocean Systems

Lecture 12

Mooring Dynamics (I)

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Position keeping

Chain, wire, rope, …

Steel, natural fibre, synthetic fibre

Good for tension, ineffective for compression, bending moment,

Tension are provided by weight and elasticity of cables

Vertical mooring: TLP

Spreading mooring: FPSO, SPAR

Reference:

O. M. Faltinsen, 1990 Sea Loads on Ships and Offshore Structures, Cambridge University Press

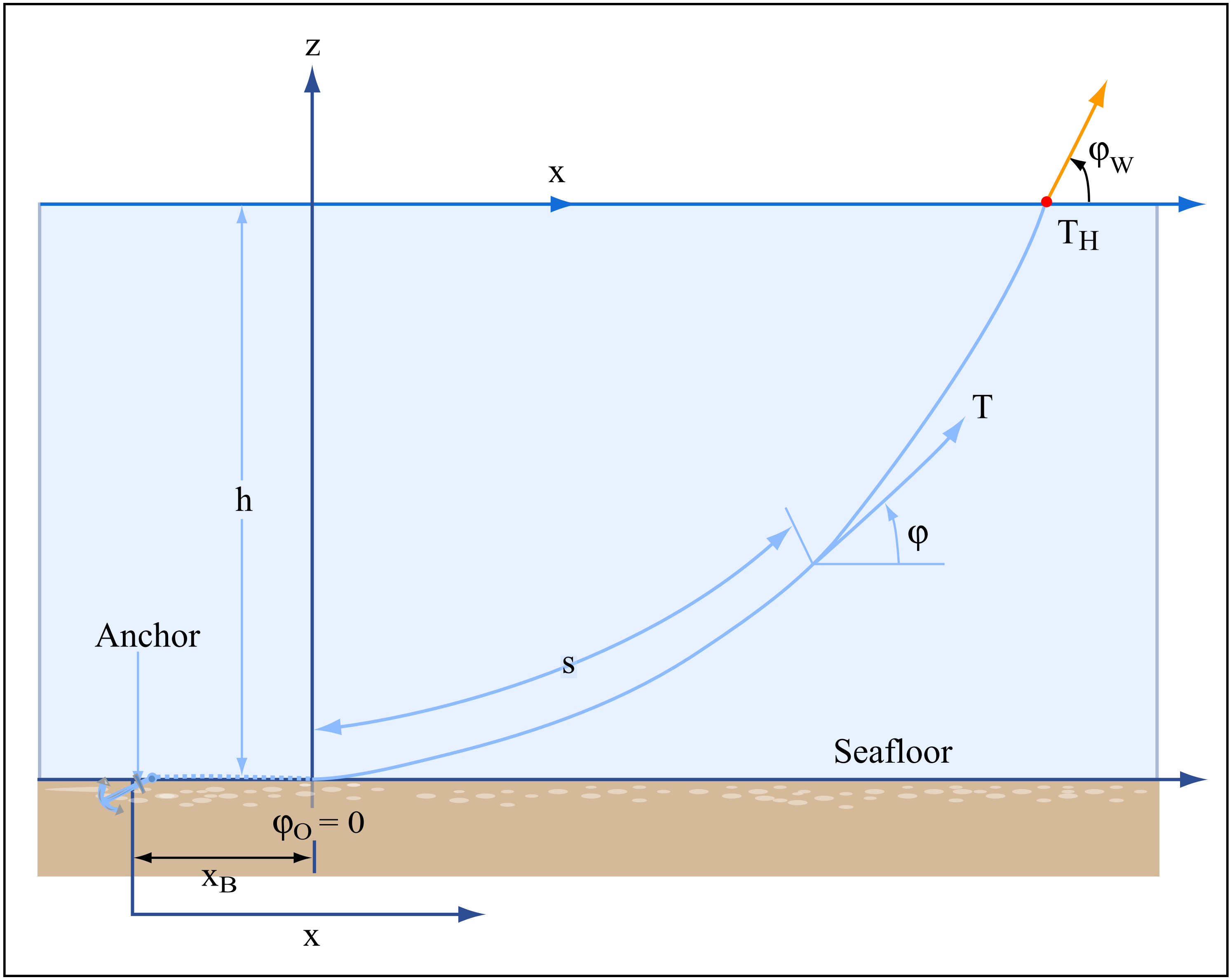
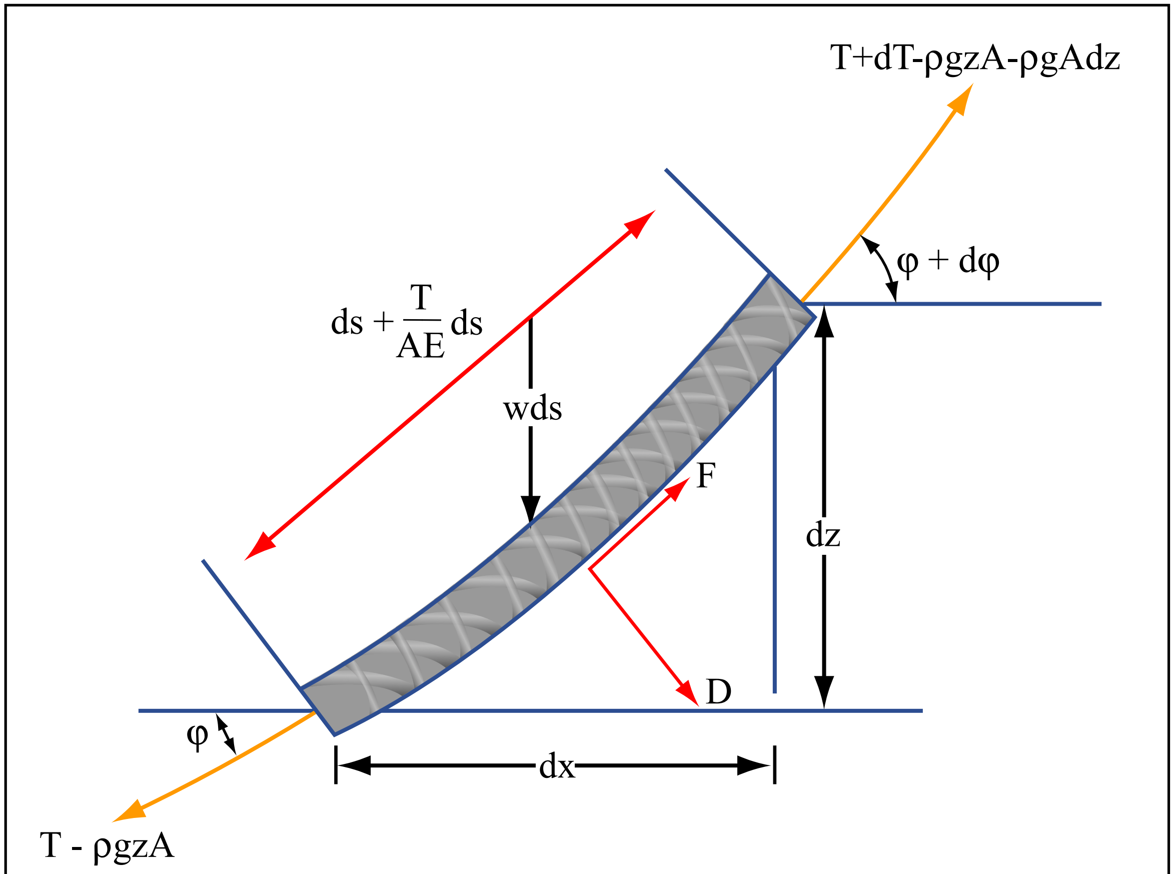


Image by MIT OpenCourseWare.

Given top end position or tension at the top end, to find:

- Configuration of the cable: $s(\phi)$ or $x(\phi)$, $z(\phi)$ or $z(x)$
- Tension along the cable: $T(s)$ or $T(x)$ or $T(z)$.

Static Analysis of a Cable Line



T	Cable tension
A	Area of the cross section of the cable
W	Water weight per unit length
F, D	Current induced hydrodynamic loads

Image by MIT OpenCourseWare.

Tangential direction: $$dT - \rho g A \, dz = \left[\omega \sin \phi - F\left(1 + T/(AE)\right)\right] ds$$

Normal direction: $$T \, d\phi - \rho g A z \, d\phi = \left[\omega \cos \phi + D\left(1 + T/(AE)\right)\right] ds$$

Solution of Inelastic Cables

By introducing

Effective tension:

$$T' = T - \rho g z A$$

we can write

Governing equations:

$$dT' = w \sin \phi \, \mathrm{d}s$$

$$T' \, \mathrm{d}\phi = w \cos \phi \, \mathrm{d}s$$

By dividing these two equations we see that

$$\frac{dT'}{T'} = \frac{\sin \phi}{\cos \phi} \mathrm{d}\phi \longrightarrow \frac{\mathrm{d}T'}{T'} = -\frac{\mathrm{d}\cos\phi}{\cos\phi}$$

i.e.

Tension along the cable:

$$T' = T_0{}' \frac{\cos \phi_0}{\cos \phi}$$

By integrating equation (8.3) we find that

Solution of s(ϕ):
$$s - s_0 = \frac{1}{w}\int_{\phi_0}^{\phi} \frac{T_0'}{\cos\theta}\frac{\cos\phi_0}{\cos\theta}\,\mathrm{d}\theta = \frac{T_0'\cos\phi_0}{w}[\tan\phi - \tan\phi_0] \tag{8.5}$$

Since $\mathrm{d}x = \cos\phi\,\mathrm{d}s$ we can write

Solution of x(ϕ):
$$\begin{aligned} x - x_0 &= \frac{1}{w}\int_{\phi_0}^{\phi} \frac{T_0'\cos\phi_0}{\cos\theta}\,\mathrm{d}\theta \\ &= \frac{T_0'\cos\phi_0}{w}\left(\log\left(\frac{1}{\cos\phi} + \tan\phi\right)\right. \\ &\qquad \left. - \log\left(\frac{1}{\cos\phi_0} + \tan\phi_0\right)\right) \end{aligned} \tag{8.6}$$

Since $\mathrm{d}z = \sin\phi\,\mathrm{d}s$ we find that

Solution of z(ϕ):
$$\begin{aligned} z - z_0 &= \frac{1}{w}\int_{\phi_0}^{\phi} \frac{T_0'\cos\phi_0\sin\theta}{\cos^2\theta}\,\mathrm{d}\theta \\ &= \frac{T_0'\cos\phi_0}{w}\left[\frac{1}{\cos\phi} - \frac{1}{\cos\phi_0}\right] \end{aligned} \tag{8.7}$$

Choose ϕ_0 to be the point of contact between the cable line and the sea bed, i.e. ϕ_0 =0. What is T_0' ??

$$T_0' = T'(\phi)\cos\phi = T'(\phi_w)\cos\phi_w = T_H$$

Given the horizontal component of the tension at the waterline T_H, we then have:

Cable configuration:

$$s = \frac{T_H}{\omega}\sinh\left(\frac{\omega}{T_H}x\right)$$

$$z + h = \frac{T_H}{\omega}\left[\cosh\left(\frac{\omega}{T_H}x\right) - 1\right]$$

Tension along the cable:

$$T - \rho g z A = \frac{T_H}{\cos\ \phi} = T_H + \omega(z + h)$$

$$T = T_H + \omega h + (\omega + \rho g A)z$$

Vertical component of the tension T_z at the waterplae:

$$T_z = ws$$