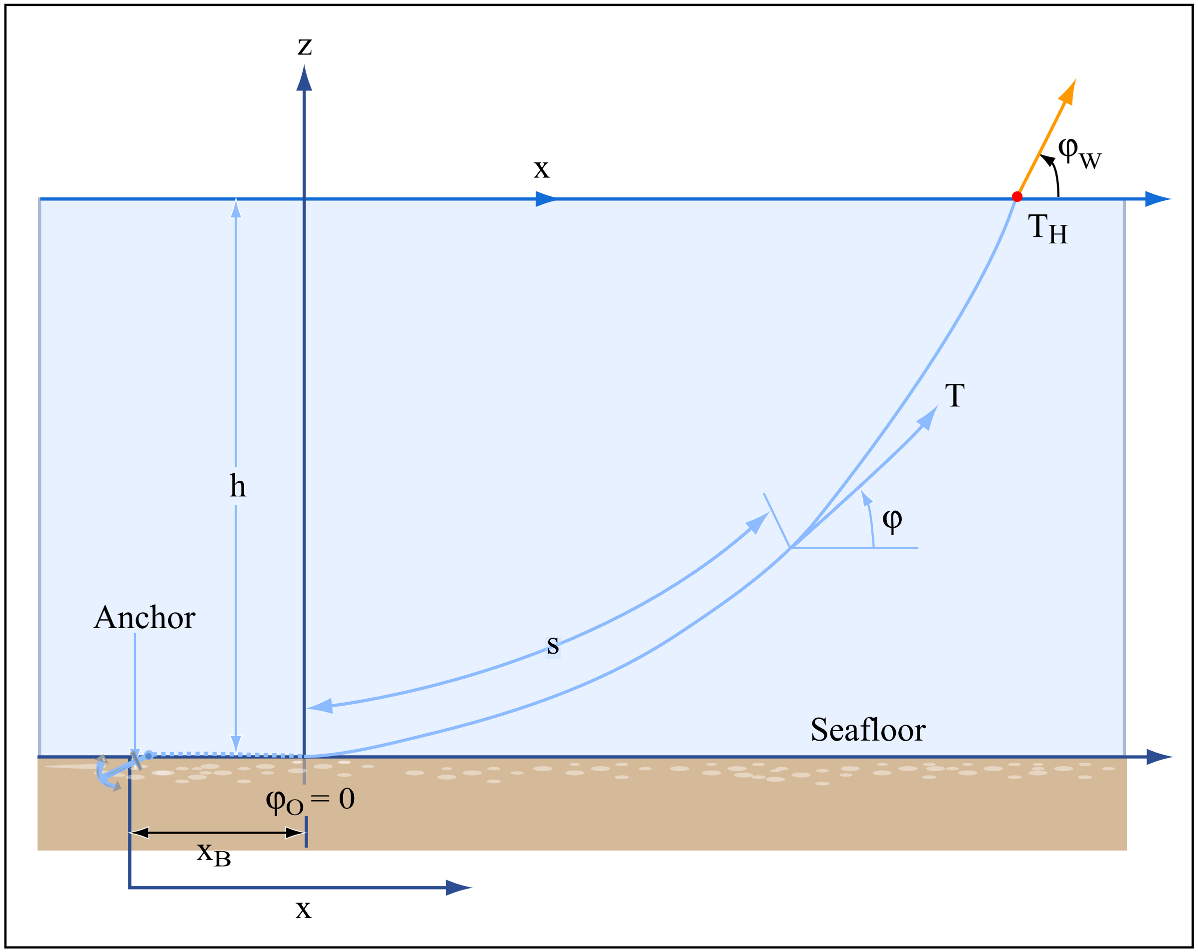
2.019 Design of Ocean Systems

Lecture 13

Mooring Dynamics (II)

March 28, 2011

Static Analysis: Catenary Cable



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Cable configuration:

$$s = \frac{T_H}{\omega}\sinh\left(\frac{\omega}{T_H}x\right)$$

$$z + h = \frac{T_H}{\omega}\left[\cosh\left(\frac{\omega}{T_H}x\right) - 1\right]$$

Tension along the cable:

$$T = T_H + \omega h + (\omega + \rho g A) z$$

$$T_z = \omega s$$

Catenary Solution —— Key Results (without Elasticity)

- Minimum line length required (or suspended length for a given fairlead tension) for gravity anchor:

$$l_{\min} = h\left(\frac{2T_{\max}}{wh} - 1\right)^{\frac{1}{2}}$$

- Horizontal force for a given fairlead tension:

$$T_H = T - wh$$

- Horizontal scope (length in plan view from fairlead to touchdown point):

$$x = \frac{T_H}{w}\sinh^{-1}\left(\frac{wl_{\min}}{T_H}\right)$$

- Vertical force at the fairlead:

$$T_z = wl_{\min}$$

Simple Examples

- Given T_{max}= T_{br} =1510 KN, w=828 N/m, h=25m, then

$$l_{\min} = h\left(\frac{2T_{\max}}{wh} - 1\right)^{\frac{1}{2}} = 25 * \sqrt{\frac{2*1510*10^3}{828*25} - 1} = 300.93m$$

$$T_H = T - wh = 1510 * 10^3 - 828 * 25 = 1489KN$$

$$T_z = wl_{\min} = 828 * 301 = 249KN$$

$$x = \frac{T_H}{w}\sinh^{-1}\left(\frac{wl_{\min}}{T_H}\right) = \frac{1489*10^3}{828}\sinh^{-1}\frac{828*301}{1489*10^3} = 300m$$

- Given x=270m, w=828 N/m, h=25m, then

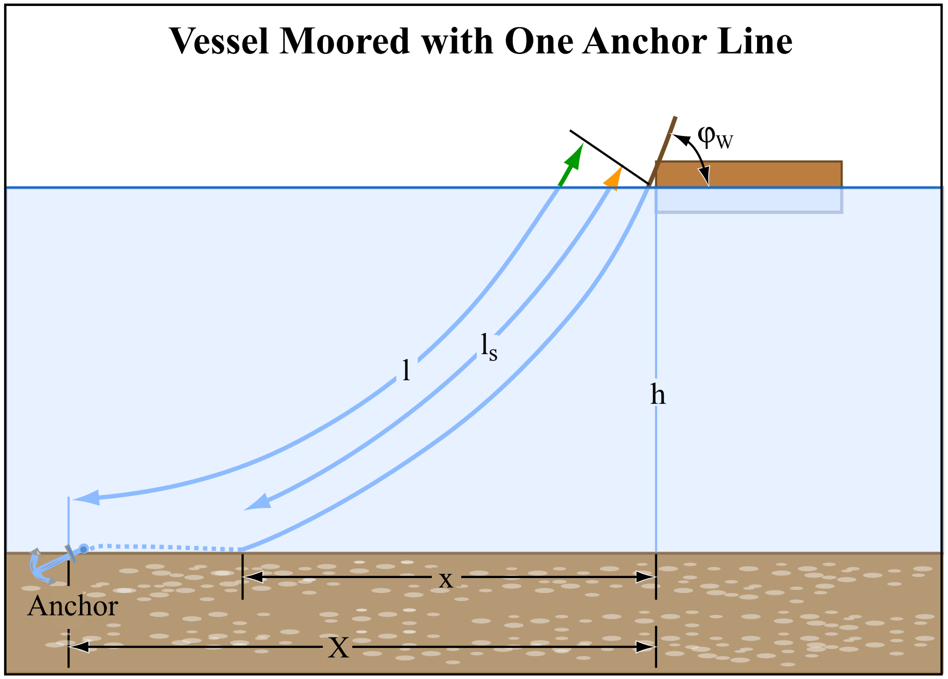
$$h = \frac{T_H}{w}\left[\cosh \quad \frac{wx}{T_H} \quad - 1\right] \longrightarrow T_H = 1200kN$$

$$T = T_H + wh = 1221kN$$

$$l_{\min} = h\left(\frac{2T}{wh} - 1\right)^{\frac{1}{2}} = 271m$$

$$T_z = wl_{\min} = 828 * 271 = 224KN$$

Cable Load-Excursion Relation



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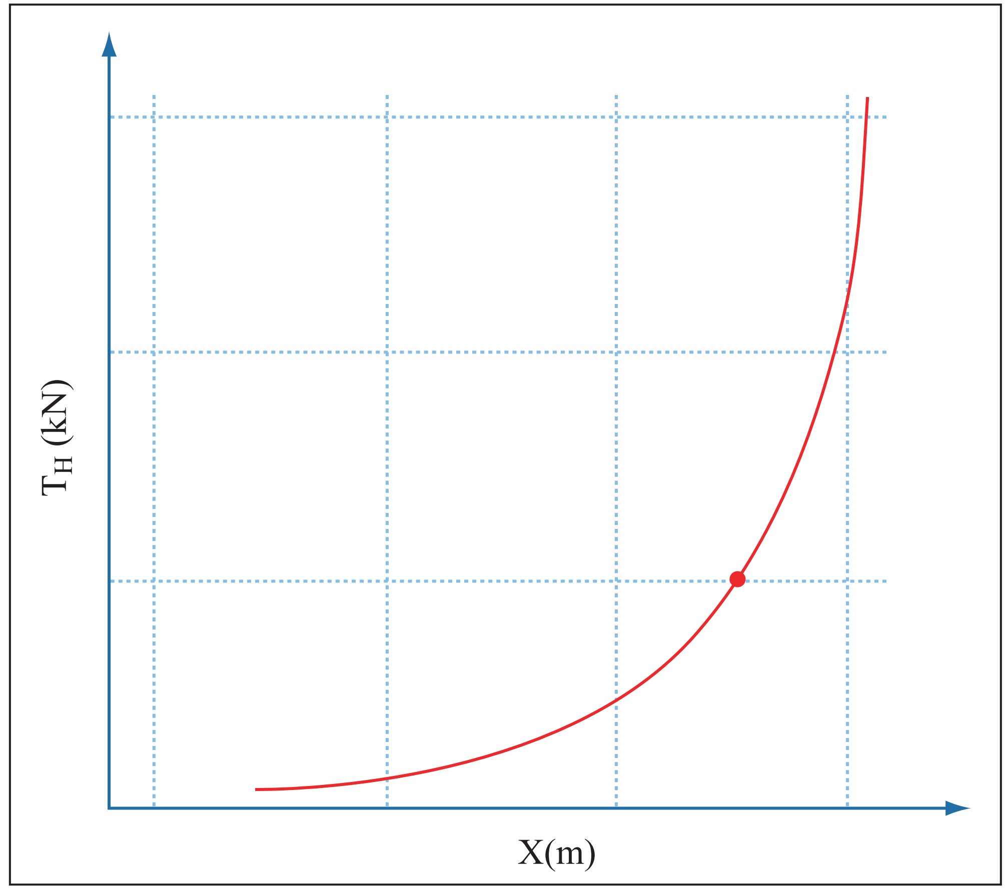

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$$X = l - l_s + x$$

$$X = l - h\left(1 + 2\frac{T_H}{wh}\right)^{\frac{1}{2}} + \frac{T_H}{w}\cosh^{-1}\left(1 + \frac{wh}{T_H}\right)$$

Restoring Coefficient:

$$C_{11} = \frac{\mathrm{d}T_H}{\mathrm{d}X} = w\left[\frac{-2}{\left(1+2\frac{T_H}{wh}\right)^{1/2}} + \cosh^{-1}\left(1 + \frac{wh}{T_H}\right)\right]^{-1}$$

Simple Example

Given: A ship experiences a total mean drift force (in surge) of 50KN, wave frequency oscillation of amplitude ζ_1 = 3 m and frequency 2π/10 rad/s, what is the total tension in the cable?

Steady tension: $T_{0H} = 50kN$

Mean position: $X_0 = 92.5m$

Restoring coefficient: $C_{11}(T_{0H}) \approx 10kN/m$

$$T_H = T_{0H} + T_H(\omega) = T_{0H} + [-C_{11}\zeta_1 \cos(\omega t + \alpha)] = 50 - 30\cos(\omega t + \alpha)$$

$$|T_H| = 80kN$$

Plus effect due to slowly varying motion

$$T = T_H + wh$$

Catenary Solution —— Key Results (with Elasticity)

- Horizontal force for a given fairlead tension T:

$$T_H - AE\sqrt{\left(\frac{T}{AE} + 1\right)^2 - \frac{2wh}{AE}} - AE$$

- Minimum line length required (or suspended length for a given fairlead tension) for gravity anchor:

$$l_{\min} = \frac{1}{w}\sqrt{T^2 - T_H^2}$$

- Vertical force at the fairlead:

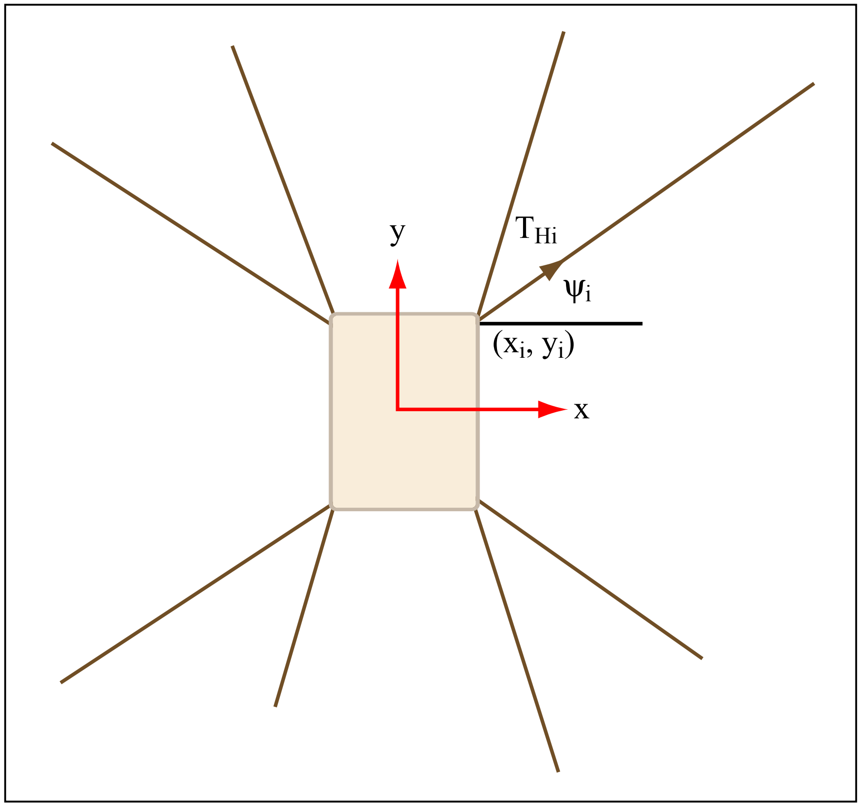
$$T_z = wl_{\min}$$

- Horizontal scope (length in plan view from fairlead to touchdown point):

$$x = \frac{T_H}{w}\sinh^{-1} \quad \frac{wl_{\min}}{T_H} \quad + \frac{T_H l_{\min}}{AE}$$

AE: stiffness of the cable

Analysis of Spread Mooring System



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- Mean position of the body is determined by balancing force/moment between those due to environments and mooring lines

- Iterative solver is usually applied

Total mooring line force/moment:

$$F_1^M = \sum_{i=1}^{n} T_{Hi} \cos \psi_i$$

$$F_2^M = \sum_{i=1}^{n} T_{Hi} \sin \psi_i$$

$$F_6^M = \sum_{i=1}^{n} T_{Hi} \left[x_i \sin \psi_i - y_i \cos \psi_i \right]$$

Total mooring line restoring coefficients:

$$C_{11} = \sum_{i=1}^{n} k_i \cos^2 \psi_i$$

$$C_{22} = \sum_{i=1}^{n} k_i \sin^2 \psi_i$$

$$C_{66} = \sum_{i=1}^{n} k_i \left(x_i \sin \psi_i - y_i \cos \psi_i \right)^2$$

$$C_{26} = C_{62} = \sum_{i=1}^{n} k_i \left(x_i \sin \psi_i - y_i \cos \psi_i \right) \sin \psi_i$$

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Spring 2011