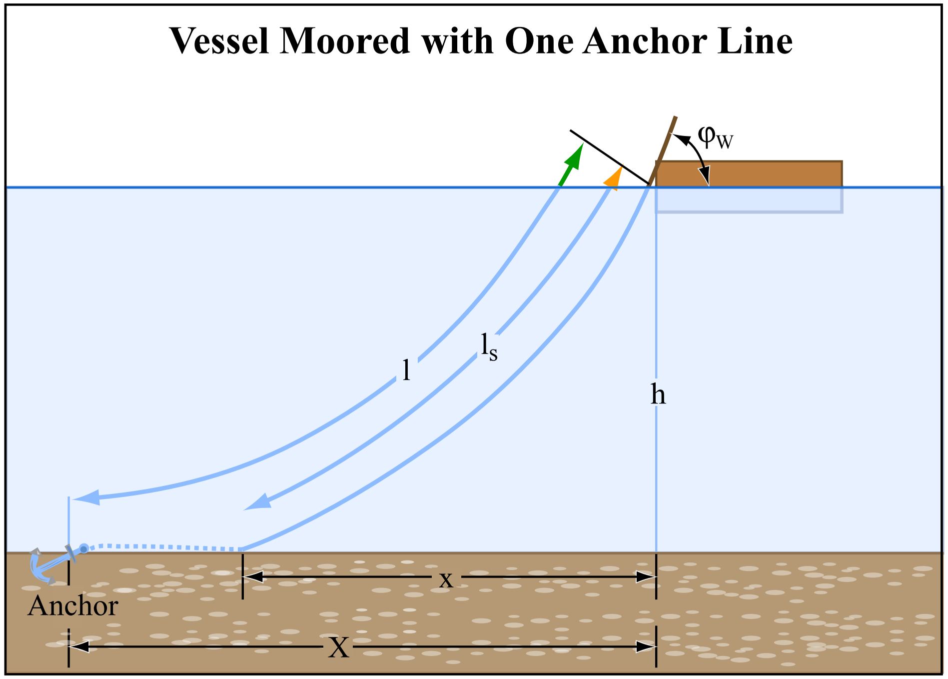
2.019 Design of Ocean Systems

Lecture 14

Mooring Dynamics (III)

April 1, 2011

Cable Load-Excursion Relation



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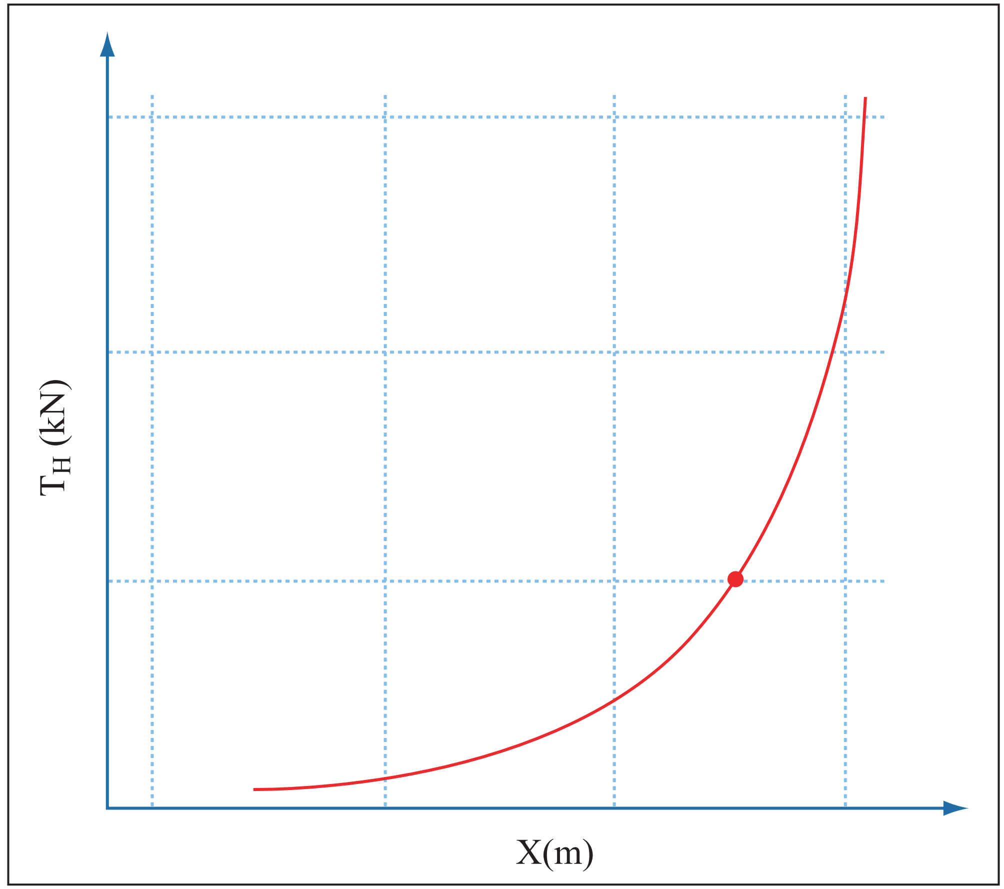

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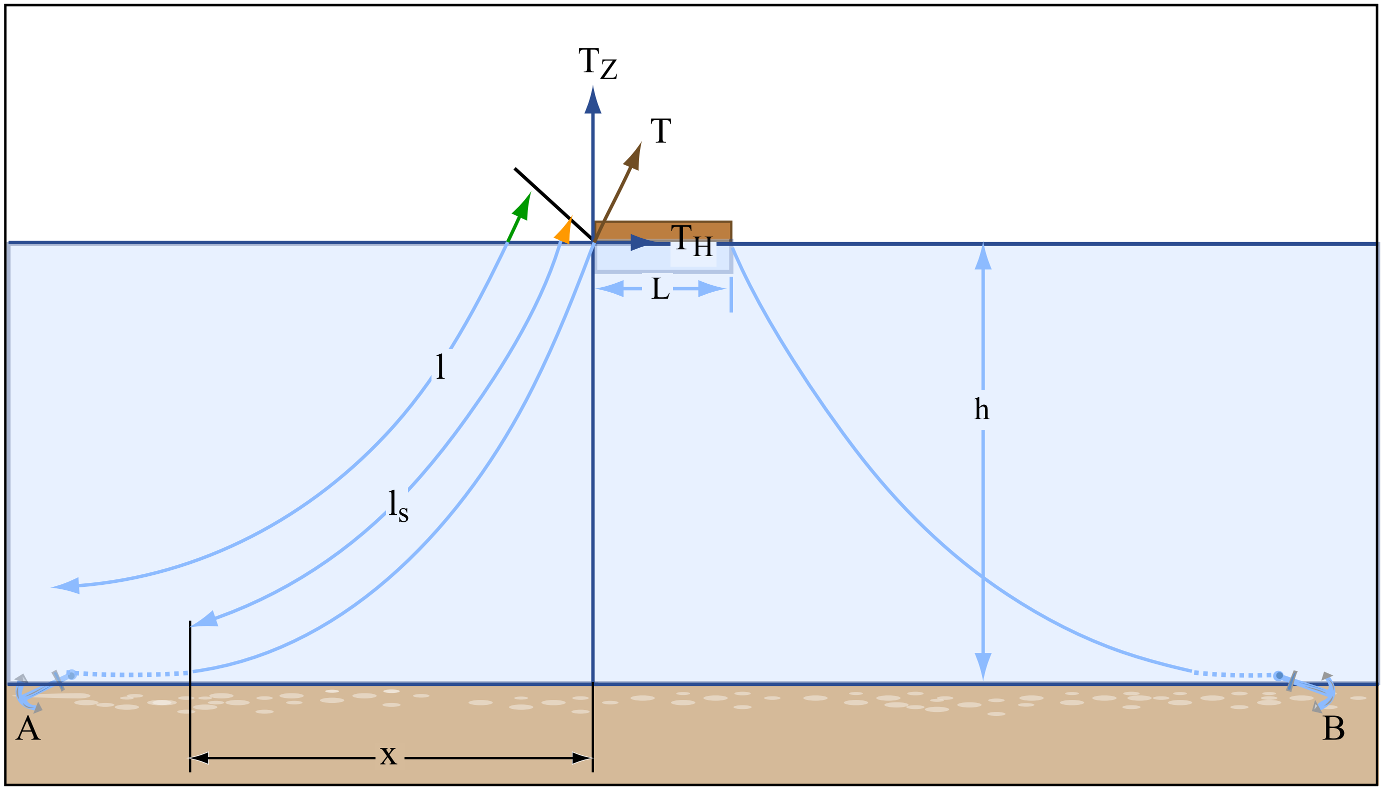
$$X = l - l_s + x$$

$$X = l - h\left(1 + 2\frac{T_H}{wh}\right)^{\frac{1}{2}} + \frac{T_H}{w}\cosh^{-1}\left(1 + \frac{wh}{T_H}\right)$$

Restoring Coefficient:

$$C_{11} = \frac{\mathrm{d}T_H}{\mathrm{d}X} = w\left[\frac{-2}{\left(1+2\frac{T_H}{wh}\right)^{1/2}} + \cosh^{-1}\left(1 + \frac{wh}{T_H}\right)\right]^{-1}$$



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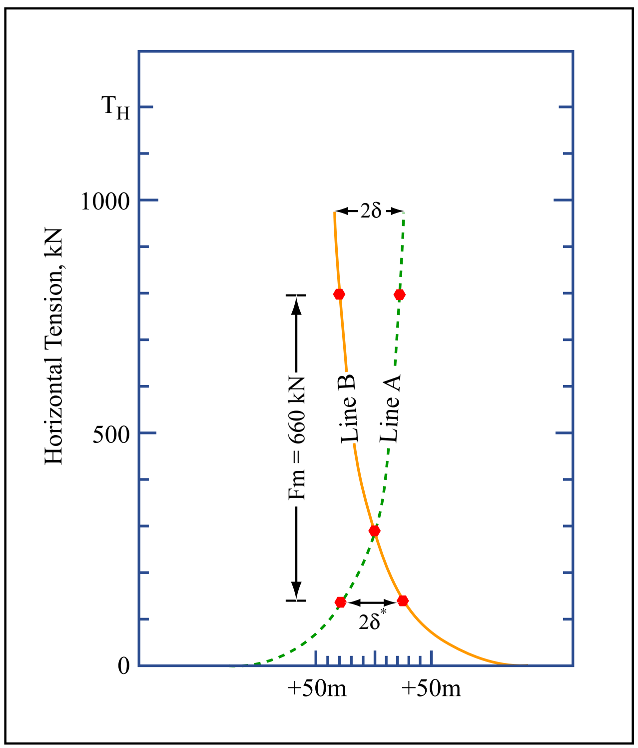

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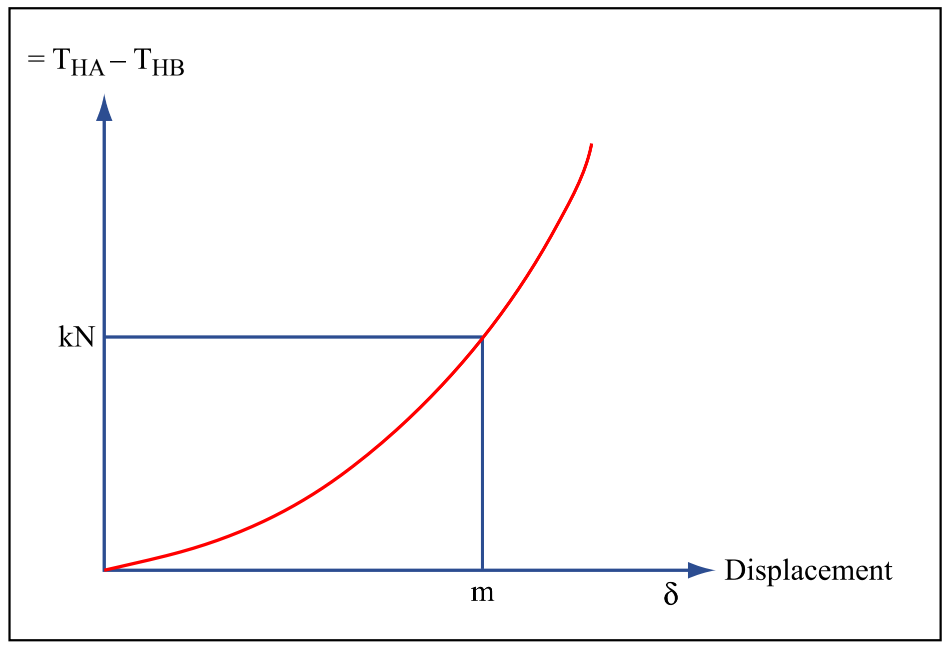

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Catenary Solution —— Key Results (with Elasticity)

- Horizontal force for a given fairlead tension T:

$$T_H - AE\sqrt{\left(\frac{T}{AE} + 1\right)^2 - \frac{2wh}{AE}} - AE$$

- Minimum line length required (or suspended length for a given fairlead tension) for gravity anchor:

$$l_{\min} = \frac{1}{w}\sqrt{T^2 - T_H^2}$$

- Vertical force at the fairlead:

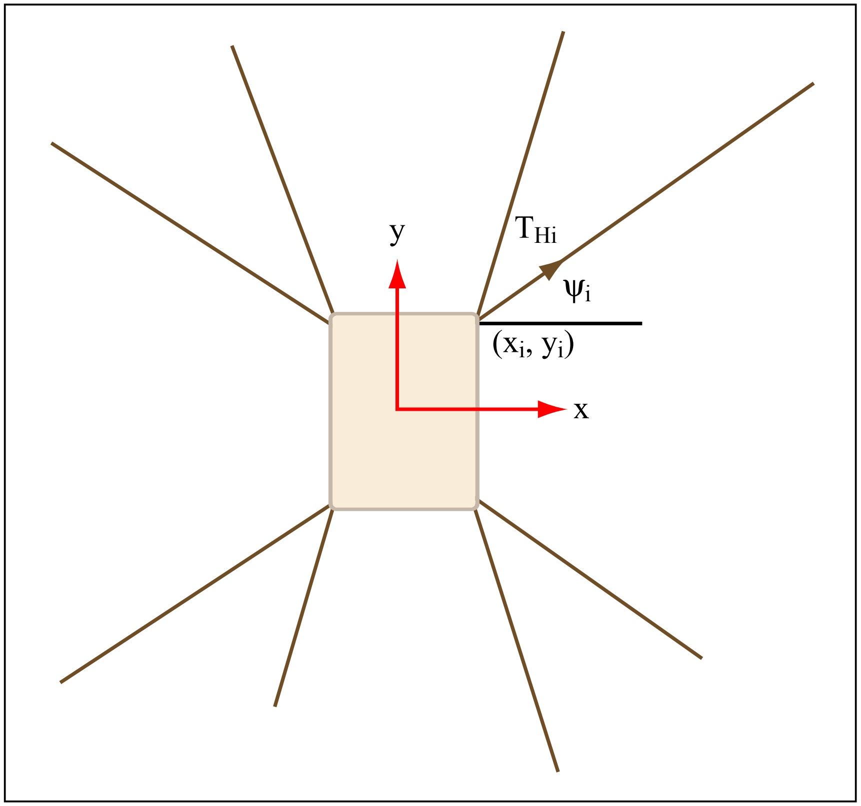
$$T_z = wl_{\min}$$

- Horizontal scope (length in plan view from fairlead to touchdown point):

$$x = \frac{T_H}{w}\sinh^{-1} \quad \frac{wl_{\min}}{T_H} \quad + \frac{T_H l_{\min}}{AE}$$

AE: stiffness of the cable

Analysis of Spread Mooring System



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- Mean position of the body is determined by balancing force/moment between those due to environments and mooring lines

- Iterative solver is usually applied

Total mooring line force/moment:

$$F_1^M = \sum_{i=1}^{n} T_{Hi} \cos \psi_i$$

$$F_2^M = \sum_{i=1}^{n} T_{Hi} \sin \psi_i$$

$$F_6^M = \sum_{i=1}^{n} T_{Hi} \left[x_i \sin \psi_i - y_i \cos \psi_i \right]$$

Total mooring line restoring coefficients:

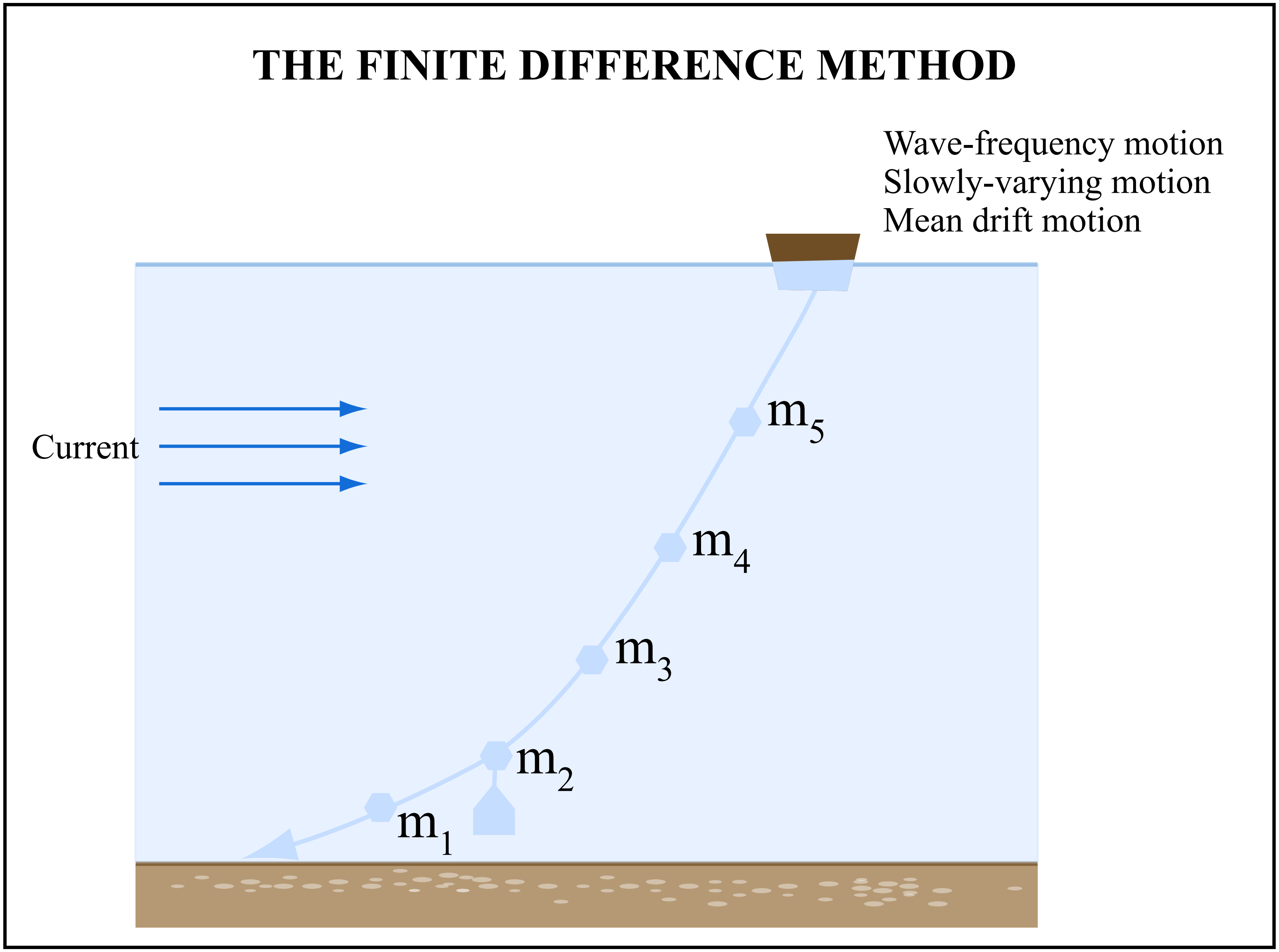
$$C_{11} = \sum_{i=1}^{n} k_i \cos^2 \psi_i$$

$$C_{22} = \sum_{i=1}^{n} k_i \sin^2 \psi_i$$

$$C_{66} = \sum_{i=1}^{n} k_i \left(x_i \sin \psi_i - y_i \cos \psi_i \right)^2$$

$$C_{26} = C_{62} = \sum_{i=1}^{n} k_i \left(x_i \sin \psi_i - y_i \cos \psi_i \right) \sin \psi_i$$

Mooring Line Dynamics



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Mooring Forces and Displacement vs. Mooring Stiffness

$$\begin{aligned} \textit{Mooring force} \; &= \; \textit{steady force} \qquad\qquad (\textit{independent of stiffness}) \\ &+ \; \textit{slow drift mooring force} \; \left(\propto \sqrt{\textit{stiffness}}\right) \\ &+ \; \textit{wave frequency motion} \; \left(\propto \textit{stiffness}\right) \end{aligned}$$

$$\begin{aligned} \textit{Displacement} \; &= \; \frac{\textit{steady force}}{\textit{stiffness}} \; \left(\frac{1}{\propto} \; \textit{stiffness}\right) \\ & \textit{slow drift displacement} \; \left(\frac{1}{\propto} \; \sqrt{\textit{stiffness}}\right) \\ & \textit{wave frequency motion} \; (\textit{independent of stiffness}) \end{aligned}$$

Thus as a general rule, as a system is made less stiff, the mooring forces will be smaller and the displacements will be larger.

Load/Displacement Combinations and Extreme Values

Tensions and excursions in a mooring system have three components:

(1) a static component known as T_{static} which arises from wind, wave drift, and current

(2) a wave frequency component, which occurs in the range of 0.03 to 0.3 Hz and is caused by first order wave loads

(3) a low frequency component, which occurs in the range of 0 to 0.02 Hz and is caused by second order waves and wind dynamics

Significant wave frequency motion: $x_{wfsig} = 2\sigma_{wf}$

Maximum wave frequency motion: $x_{wfmax} = \sqrt{\left[2\ln\left(T_{exp} / T_{zwf}\right)\right]}\sigma_{wf}$

Significant low-frequency motion: $x_{lfsig} = 2\sigma_{lf}$

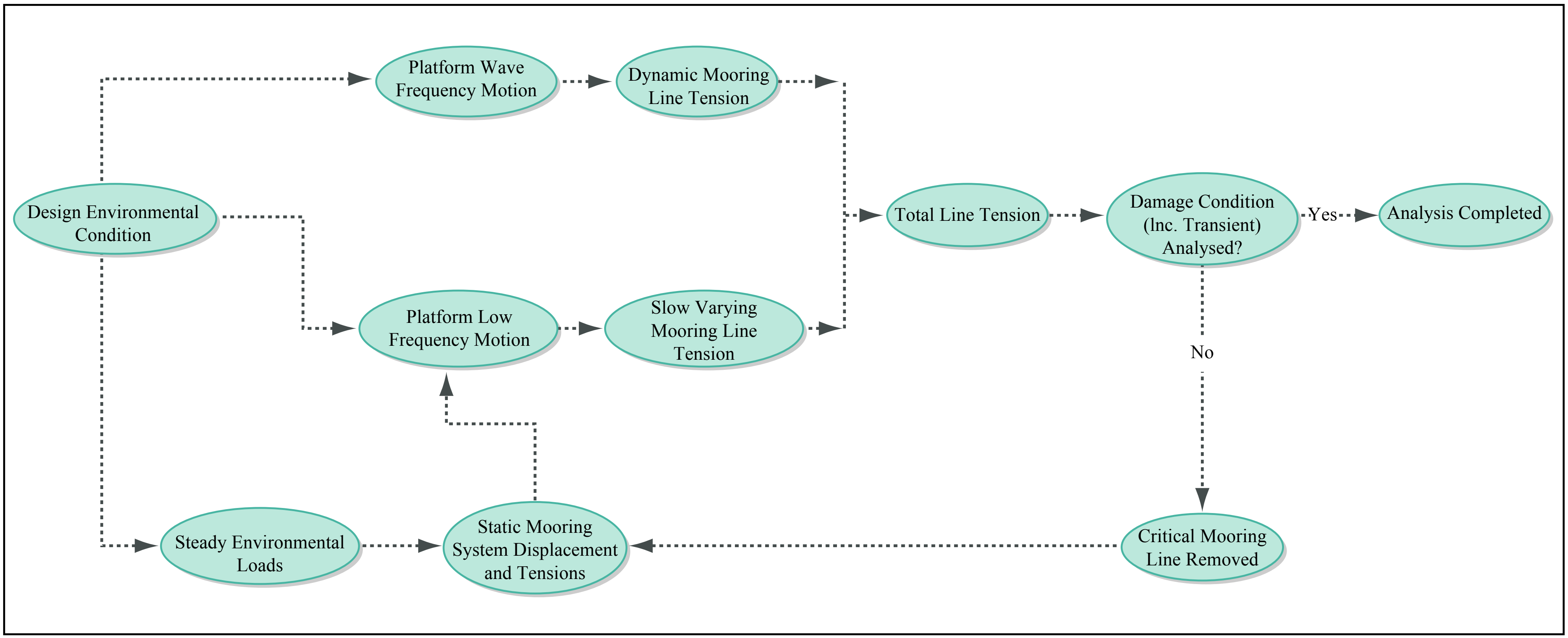
Maximum low-frequency motion: $x_{lfmax} = \sqrt{\left[2\ln\left(T_{exp} / T_{zlf}\right)\right]}\sigma_{lf}$

T_{exp} ~ 3 to 6 hours; T_z: peak period

Maximum combined dynamic tension/excursion:

$$x_{dyn} = \max[(x_{wfmax} + x_{lfsig}),(x_{wfsig} + x_{lfmax})]$$

Mooring Analysis Flowchart



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Mooring Line Materials

Chain:

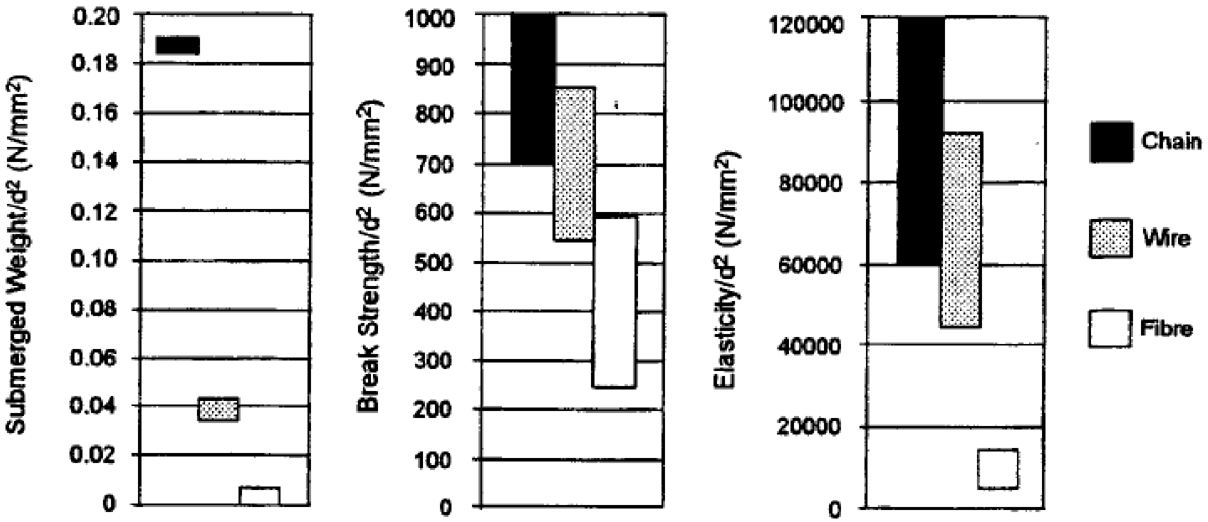
- Studless or studded chain links
- Heavy, highbreaking strength, high elastisity
- No bending effect
- Most popular, all chain in shallow water (< 100M)
- Chain segments are used near fairlead and bottom (in deepwater)

Wire:

- Lighter than chain
- Slight bending effect
- Used as main mooring line segments in deep water (to reduce vertical loads)

High-Tech Fibre:

- Light weight (almost neutrally buoyant)
- Highly extensible
- Potentially useful for very deep water



Chain

Weight and Stiffness:

Submerged weight per unit length, $w = 0.1875D^2$ N/M (D in mm)
Axial stiffness per unit length, $AE = 90000D^2$ N (D in mm)

Breaking Strength:

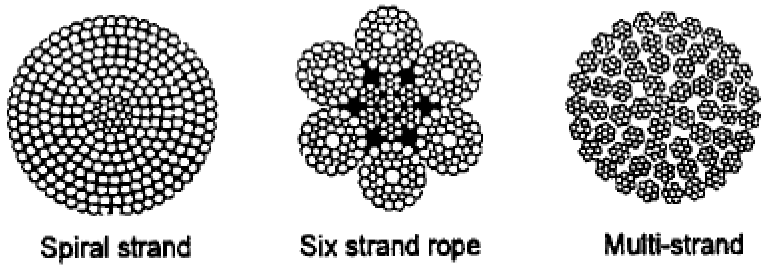
$$\text{CBS or proof load} = c(44 - 0.08D)D^2 \text{ N } (D \text{ in mm})$$

Catalogue breaking strength

Values of c:

Grade (specification)	Catalogue Break Strength	Proof Load
ORQ	21.1	14.0
3	22.3	14.8
3S	24.9	18.0
4	27.4	21.6

Wire Rope



Weight and Stiffness:

Construction	Submerged weight/length, *w*	Stiffness/length, AE
Six strand (IWRC)	$0.034d^2$ N (*d* in mm)	$45000d^2$ N (*d* in mm)
Spiral strand	$0.043d^2$ N (*d* in mm)	$90000d^2$ N (*d* in mm)

Breaking Strength:

Construction	Ultimate Tensile Stress (N/mm^2)	Breaking Strength (N)
Six strand (IWRC)	1770	$525d^2$ (*d* in mm)
Six strand (IWRC)	1860	$600d^2$ (*d* in mm)
Spiral strand	1570	$900d^2$ (*d* in mm)

High Technology Fibre Rope

Weight and Stiffness:

Fibre Rope Type	Weight Per Unit Length (N/m)
Polyester	$0.0067d^2$ (d in mm)
Aramid	$0.00565d^2$ (d in mm)
HMPE	$0.0062d^2$ (d in mm)

Breaking Strength:

Fibre Rope Type	Breaking Strength (N)
Polyester	$250d^2$ (d in mm)
Aramid	$450d^2$ (d in mm)
HMPE	$575d^2$ (d in mm)

Properties of Typical Systems

Extreme Excursions as a Percentage of Water Depth

Water Depth (m)	Mooring Type	Semi-submersible	Ship
30	Chain/wire	30-45%	40-55%
150	Chain	15-25%	30-40%
500	Chain/wire	25-30%	20-30%
1000	Fibre ropes	5-10%	5-15%

Typical Natural Periods of Mooring Systems

Water Depth (m)	Mooring Type	Semi-submersible (s)	Ship (s)
30	Chain/wire	30	45
150	Chain	60-120	60-150
500	Chain/wire	120-180	150-250
1000	Fibre ropes	90-110	120-150

Guidance, Rules, and Regulations

IACS (International Association of Classification Societies) safety factors:
for survival conditions

Condition	Safety factor (= Break strength/Max.tension)
Intact	1.8
One line removed	1.25
Transient	1.1

IACS *(International Association of Classification Societies) safety factors: for survival conditions for operating conditions, these safety factors are increased by about 50%.*

API RP 2SK Safety Factors:

Condition	Safety factor (= Break strength/Max.tension)
Intact	1.67
One line removed	1.25
Transient	1.05

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