2.019 Design of Ocean Systems

Lecture 9

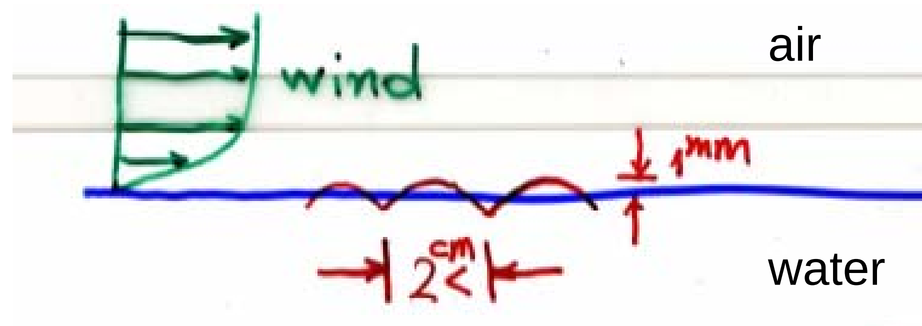
Ocean Wave Environment

March 7, 2011

Ocean Surface Wave Generation

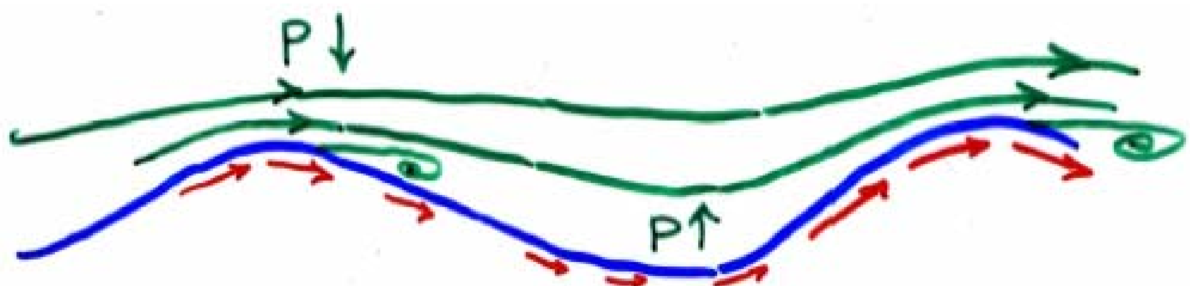
Waves important to offshore structure design and operation: Wind waves or gravity waves with wave period T= 5 ~ 20 seconds, wavelength O(10)m to O(500)m.

Source of forcing: wind
Source of restoring: gravity
Source of damping: wave breaking and viscous effects

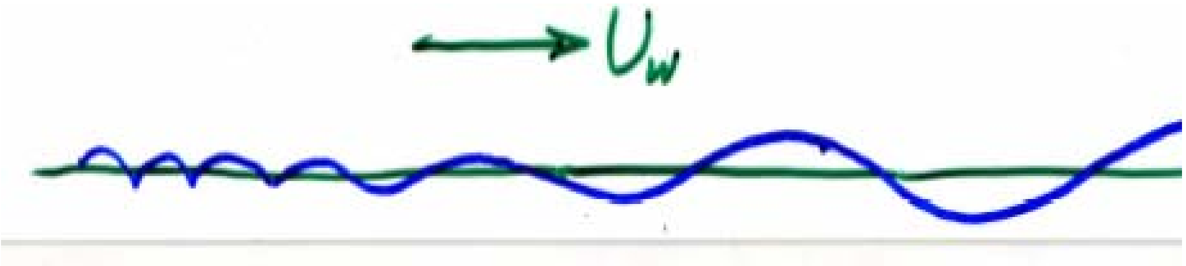


- When wind starts (0.5 ~ 2.0 knots), capillary waves form (e.g Vp =24 cm/s → λ=1.73 cm)
- As wind becomes stronger, waves become longer

Wind energy input into water:



Frictional drag
Separation drag
Bernoulli effect



Wave evolution as wind blows

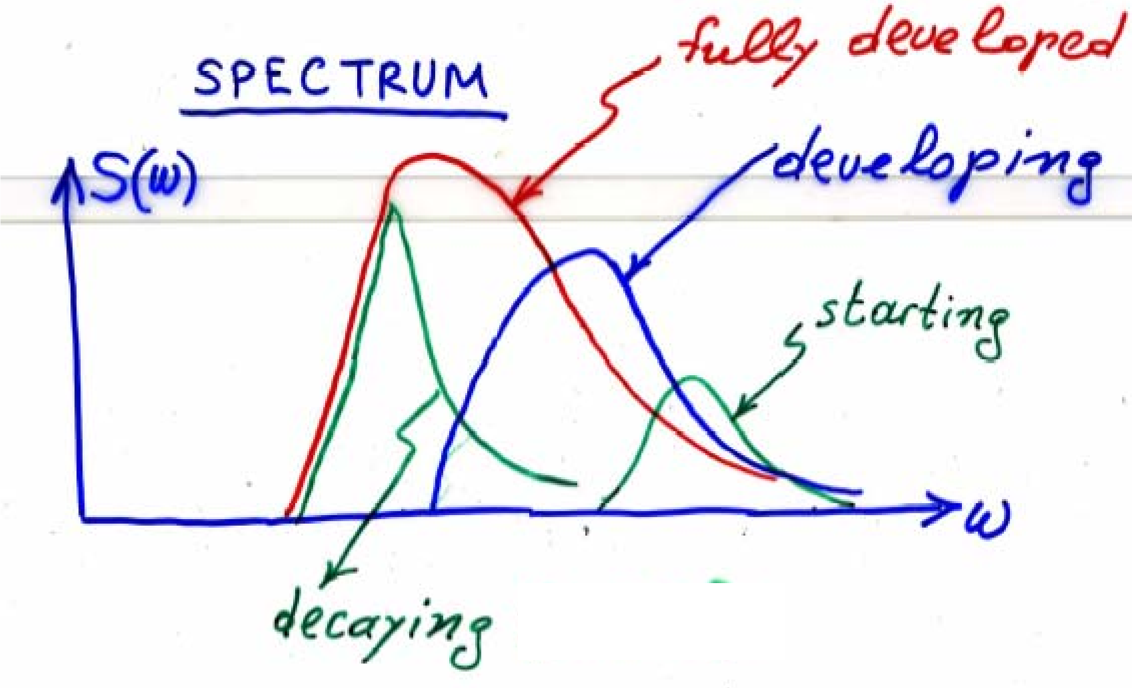
- Nonlinear wave-wave interactions cause energy to be transferred into longer waves
- Certain distance and duration (for wind to blow) are necessary for effective energy transfer
- Equilibrium sea: when energy input from the wind is balanced by dissipation
- When wind input energy is larger than dissipation, waves grow
- When wind input energy is smaller than dissipation, waves decay. Short waves decay faster.

Amplitude decays as $e^{-\gamma t}$

$$\gamma = 2\nu k^2 = 2\nu \omega^4 / g^2$$

(Landau + Lifshitz)

Shorter waves are steeper, and easier to break



- Wind must blow over long periods of time and large distances to reach fully-developed state.
- At fully-developed state, $U_w \sim V_p$ (i.e. $\omega_{limit} \sim g/U_w$)

- <u>Swell</u>: waves are not generated by local wind
- <u>Sea</u>: waves generated by local wind

Required fetch and storm duration:

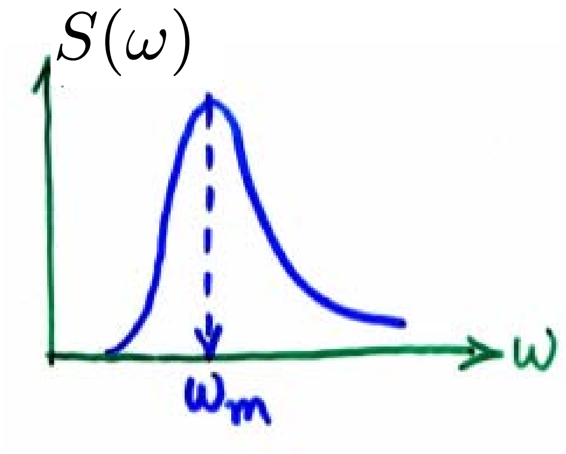
Beaufort scale	wind speed (mph)	fetch (miles)	duration (h)
3-4	12	15	3
5-6	25	100	12
7	35	400	28
9	50	1,050	50

Standard Wave Spectra

Based on measured spectra and theoretical results, standard spectrum forms have been developed:

Bretschneider spectrum:

$$S(\omega) = \frac{1.25}{4}\frac{\omega_m^4}{\omega^5}H_s^2 \exp\{-1.25\left(\frac{\omega_m}{\omega}\right)^4\}$$



ω_m is peak or modal frequency

H_s is significant wave height

$$\int_0^\infty S(\omega)\mathrm{d}\omega = M_0 = \left(\frac{H_s}{4}\right)^2$$

For fully developed sea (Pierson-Moskowitz spectrum):

$$\omega_m = 0.4\sqrt{\frac{g}{H_s}}$$

JONSWAP Spectrum:

$$S(\omega) = \frac{\alpha g^2}{\omega^5}\exp\{-1.25\left(\frac{\omega_p}{\omega}\right)^4\}\cdot\gamma^{\exp\{-0.5\left(\frac{\omega-\omega_p}{\sigma\omega_p}\right)^2\}}$$

$$\alpha = 5.061\left(\frac{\omega_p}{2\pi}\right)^4 H_s^2[1-0.287\log\gamma]$$

$$\sigma = 0.07 \text{ for } \sigma < \omega_p\text{, and } \sigma = 0.09 \text{ for } \omega \geq \omega_p$$

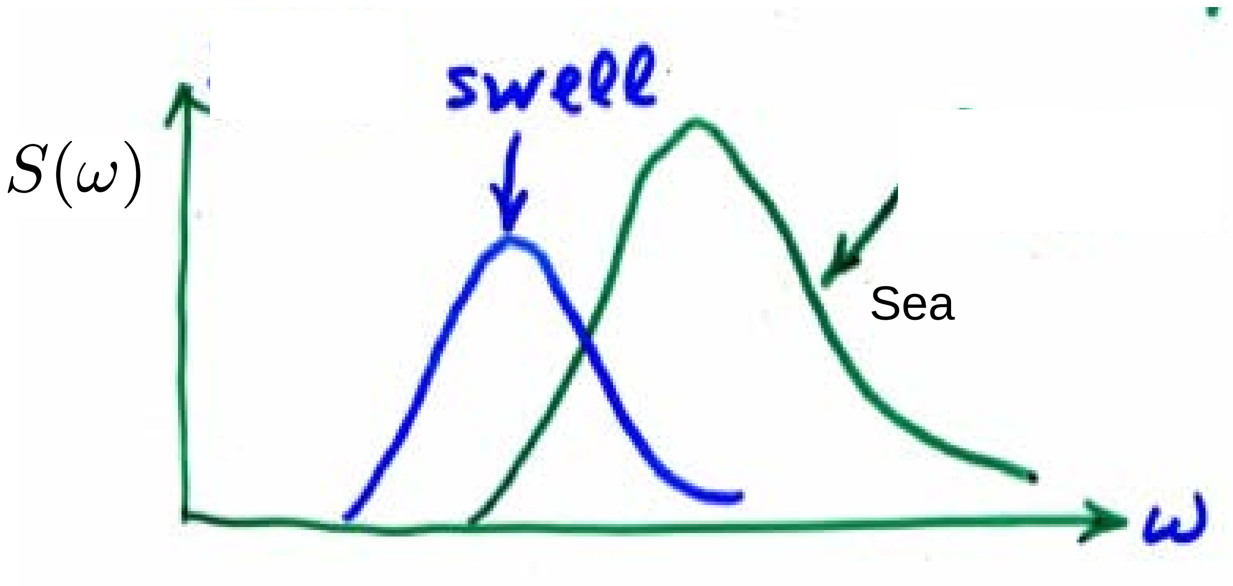
H_s: significant wave frequency

ω_p: peak frequency

γ: peak enhance coefficient

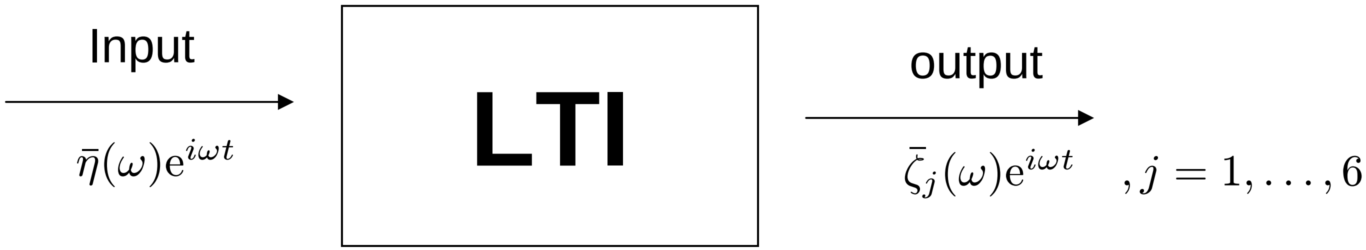
Combined Sea and Swell:

$$S(\omega) = S_{swell}(\omega) + S_{sea}(\omega)$$



Response Spectra of a Floating Structure in Irregular Waves

For a linear time-invariant (LTI) system:



$$H_j(\omega) = \frac{\bar{\zeta}_j(\omega)}{\bar{\eta}(\omega)} \quad j = 1, \ldots, 6$$

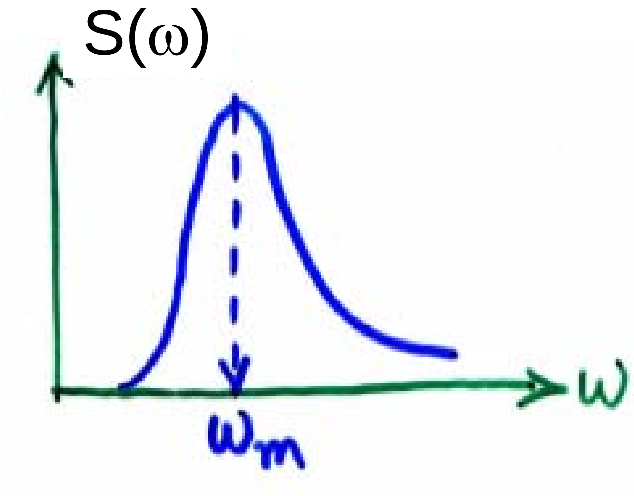
$H_j(\omega)$:Transfer function (or RAO) of the linear system, determined by the system itself

Spectra of the response: $S_{\zeta_j}(\omega) = |H_j(\omega)|^2 \, S_\eta(\omega)$, $\quad$ where $S_\eta(\omega)$ is wave spectrum

Design strategy:

- To avoid large response, make the peak of $H_j(\omega)$ away from the peak of $S\eta(\omega)$
- For wave energy extraction, make the peak of $Hj(\omega)$ close to the peak of $S\eta(\omega)$

Short-Term Statistics



- Once the spectrum of a random process is given, the statistics of the random process can be obtained in terms of moments and bandwidth of the spectrum.

Moments:

$$m_0 = \int_0^\infty S(\omega)\mathrm{d}\omega$$

$$m_2 = \int_0^\infty \omega^2 S(\omega)\mathrm{d}\omega$$

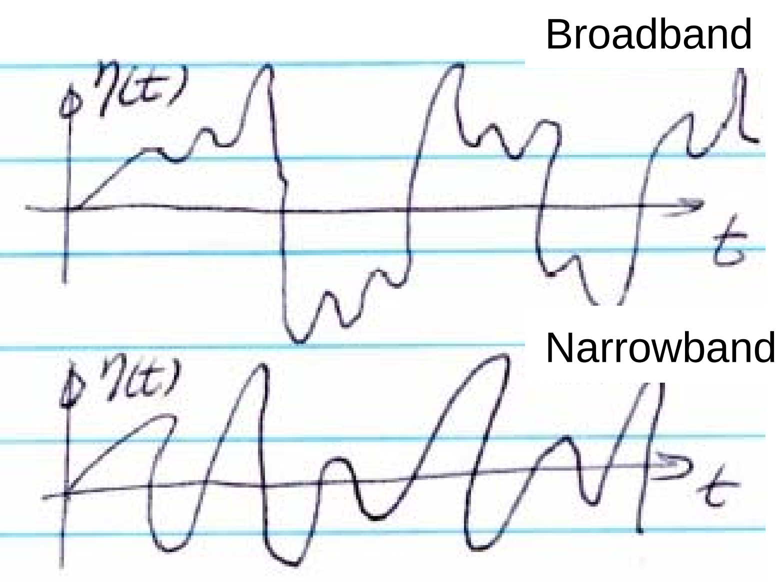
$$m_4 = \int_0^\infty \omega^4 S(\omega)\mathrm{d}\omega$$

$$\ldots\ldots$$

Bandwidth coefficient:

$$\epsilon = \sqrt{1 - \frac{m_2^2}{m_0 m_4}}\,, \quad 0 \le \epsilon \le 1$$

If $\varepsilon \geq$ ~0.5, called broadband
If $\varepsilon <$ ~0.5, called narrowband
In typical ocean, ε=0.5 ~ 0.6



Issue in Evaluating m_n (with $n \geq 4$)

Typically for ocean waves, we have $S(\omega) \sim \omega^{-5}$ as $\omega \to \infty$

$$m_4 = \int_0^{\infty} \omega^4 S(\omega) \mathrm{d}\omega = \int_0^{\omega_1} \omega^4 S(\omega) \mathrm{d}\omega + \int_{\omega_1}^{\infty} \omega^4 \times \omega^{-5} \mathrm{d}\omega$$

$$\rightarrow (\log \omega)\big|_{\omega_1}^{\infty} \to \infty$$ Diverges !!

There are two solutions for this issue:

- Truncate the integration up to $3\omega_m$. This has advantage that scaling can be applied between model and full-scale tests.
- Choose a constant upper limit, typically ω_{max} = 2.0 radian/sec.. When scaling, this may be necessary to avoid very large frequency tests.

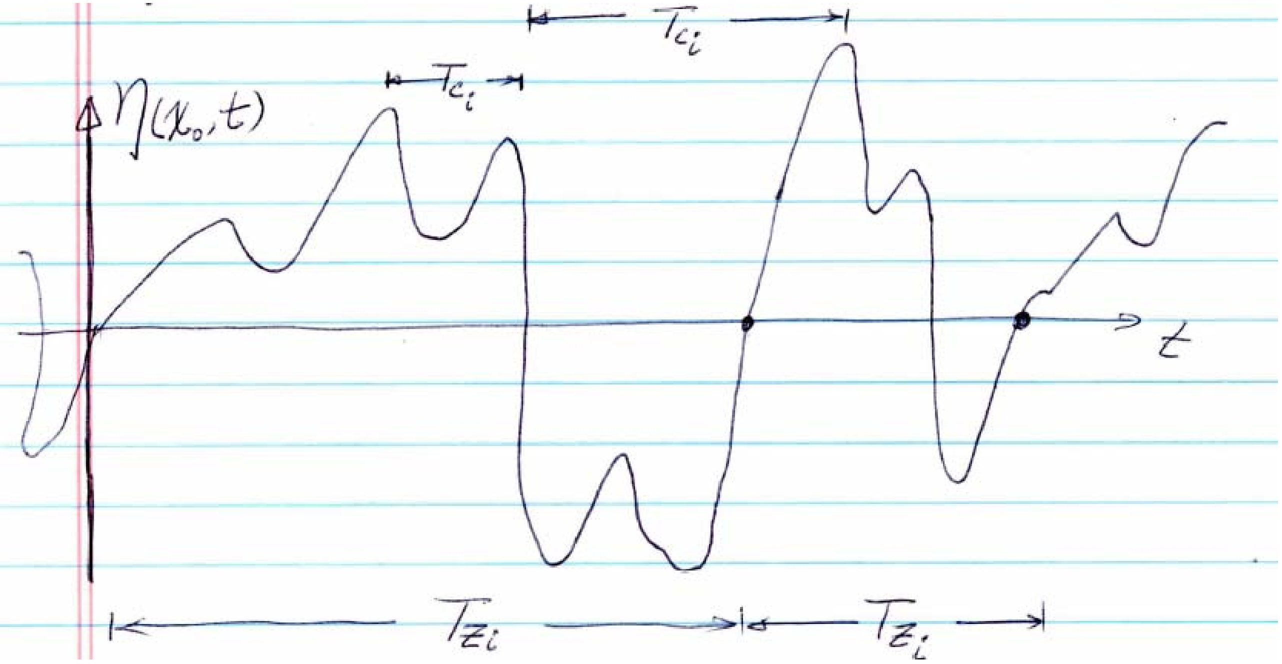
Zero Up-crossing Period and Peak to Peak Period

Simulation of a random process from a given spectrum S(ω):

$$\eta(x,t) = \sum_j^N A_j \cos(\omega_j t - k_j x + \psi_j)$$

$$\omega_j^2 = g k_j \tanh k_j H \; , \qquad A_j^2 = \tfrac{1}{2} S(\omega_j) \Delta\omega_j$$

ψ_j: random phase uniformly distributed in $[0, 2\pi]$.



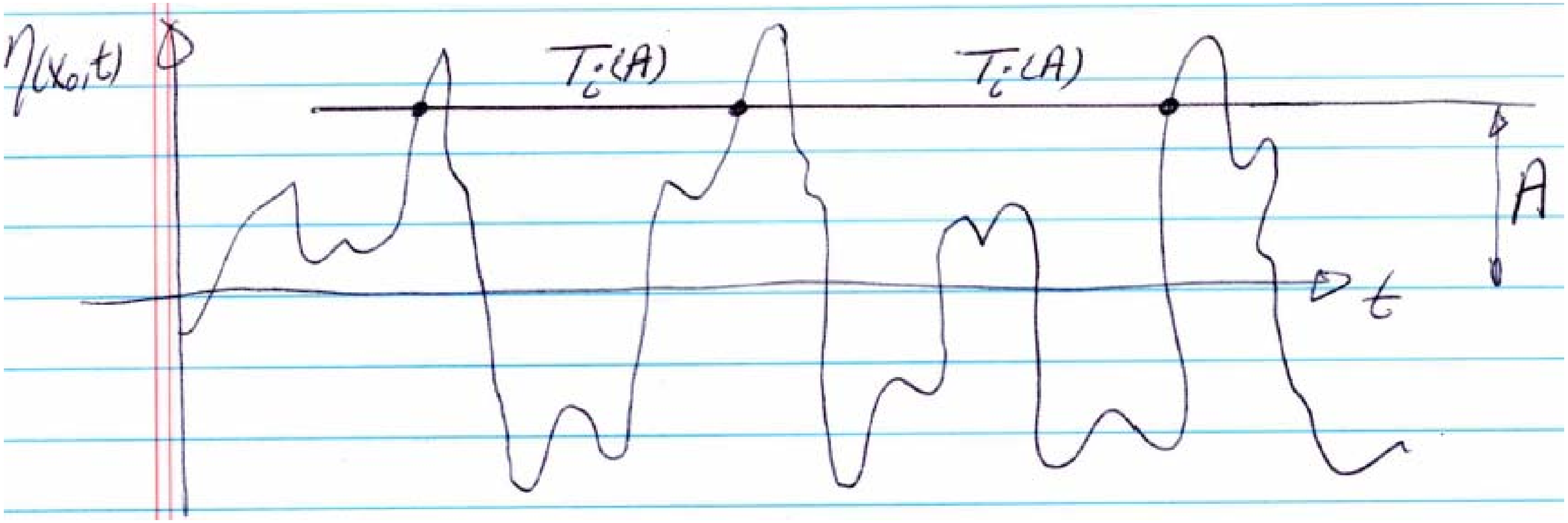
T_{zi}: Zero up-crossing period

T_{ci}: Peak to Peak period

Average zero up-crossing period: $\bar{T}_z = 2\pi \sqrt{\frac{m_0}{m_2}}$

Average peak-to-peak period: $\bar{T}_c = 2\pi \sqrt{\frac{m_2}{m_4}}$

How Often is "A" Level Exceeded??



$\bar{T}(A)$: Average period for "A" level exceeded

If $A = 0$, then $T_i(A) = T_i(0) = T_{zi} \quad \rightarrow \bar{T}(0) = \bar{T}_z$

$\bar{n}(A) = \frac{1}{\bar{T}(A)}$: Average frequency of up-crossings if level "A" (i.e. number of up-crossings of level "A" per second)

$\bar{n}(0) = \frac{1}{\bar{T}_z}$: Average frequency of zero up-crossings

$$\bar{n}(A) = \frac{1}{2\pi}\sqrt{\frac{m_2}{m_0}}\mathrm{e}^{-A^2/2m_0} = \frac{1}{\bar{T}_z}\mathrm{e}^{-A^2/2m_0} = \bar{n}(0)\mathrm{e}^{-A^2/2m_0}$$

Example

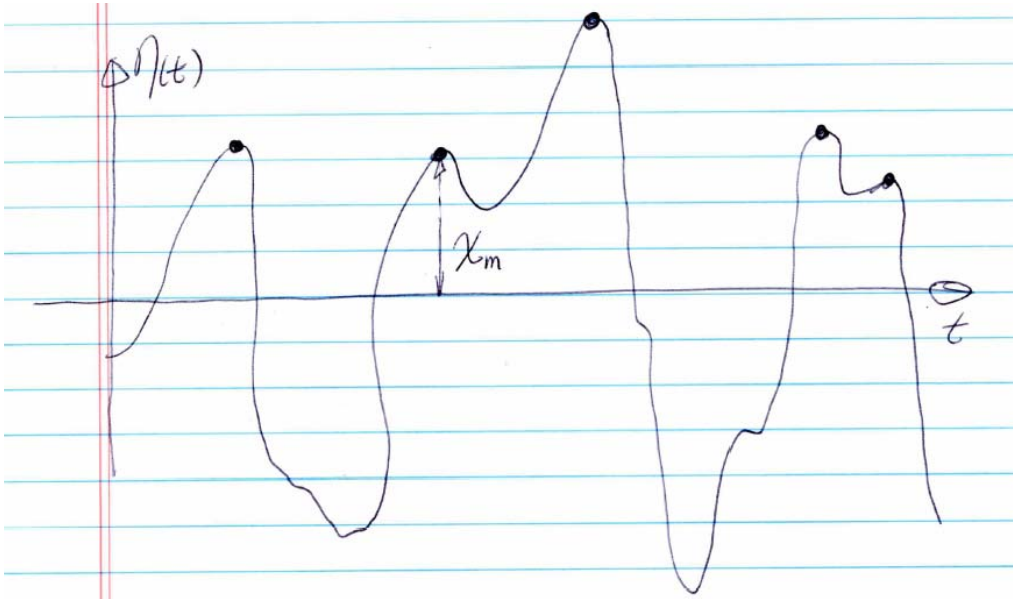
A FPSO is exposed to a storm with waves of m_0 = 4 meter^2 and average period of *T*=8 seconds. Design the free deck height *h* so that the deck is flooded by green water only once every 10 minutes. (Neglect body motion and diffracted wave effects).

Wave spectrum S(ω): $m_0 = 4, \quad \bar{T}_z = 8$

$$\bar{n}(h) = \frac{1}{\bar{T}_z} \mathrm{e}^{-h^2/(2m_0)} = \frac{1}{10 \times 60}$$

$$h = \sqrt{-2m_0 \ln\left(\frac{\bar{T}_z}{600}\right)} = 5.8 meters$$

Maxima

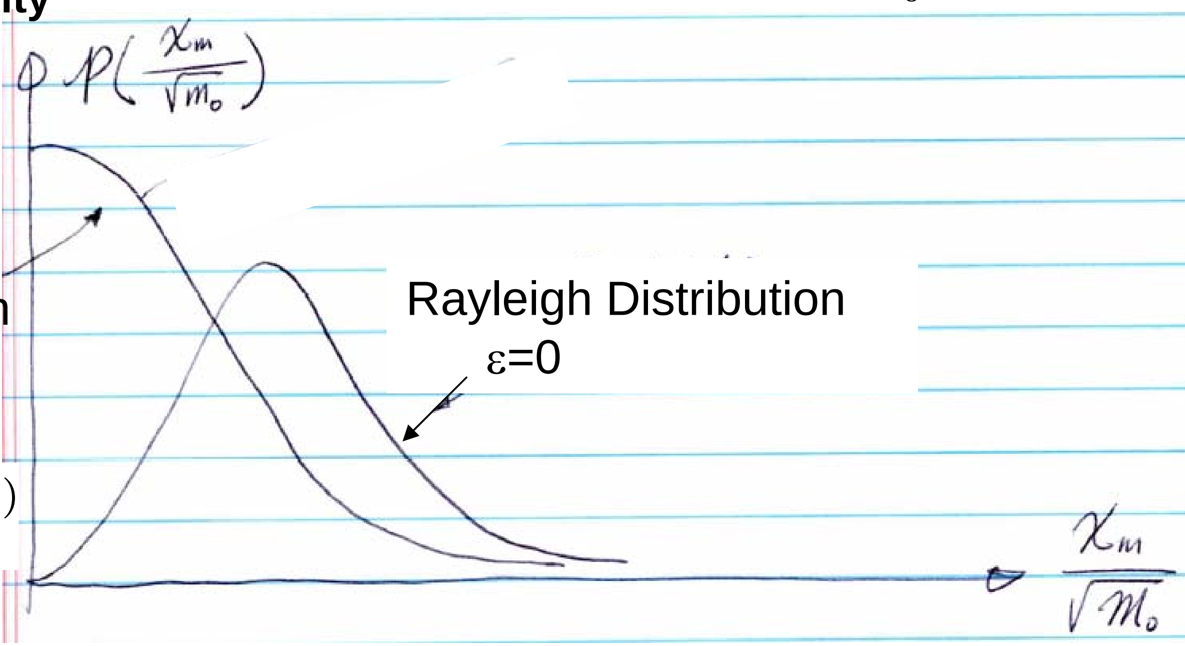


X_m: Local maxima of wave elevation

Rayleigh Distribution:

$$p(x_m) = \frac{x_m}{m_0} e^{-x_m^2/(2m_0)}$$

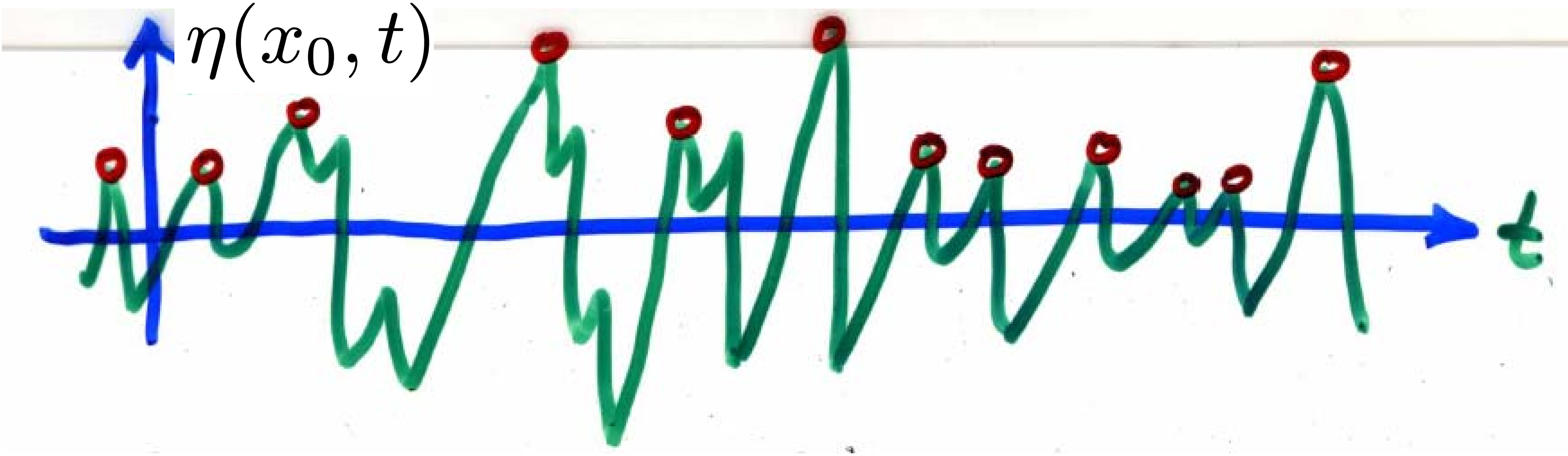
Probability density function



Gaussian distribution
ε=1

$$p(x_m) = \frac{1}{\sqrt{2\pi m_0}} \mathrm{e}^{-x_m^2/(2m_0)}$$

1/N th Highest Maxima



$a_1, a_2, \ldots, a_n, \ldots,$ are the maxima of $\eta(x_0, t)$

$a^{1/N}$ is the value that is exceeded by $1/N$ th maxima.

Example: $N = 10$, $a^{1/10}$ is the value that is exceeded (on the average) by 10% of the maxima.

$$a^{1/N} = \sqrt{m_0}\sqrt{2\ln\left(\frac{2\sqrt{1-\epsilon^2}}{1+\sqrt{1-\epsilon^2}}N\right)}$$

1/N th Highest Average Maxima

$\overline{a^{1/N}}$: The average value of all the maxima above $a^{1/N}$.

$$\overline{a^{1/N}} = E\{a_m|_{a_m > a^{1/N}}\}$$

This is called the $1/N$ highest average amplitude.

$$\overline{a^{1/N}} = 2N\sqrt{m_0}\frac{\sqrt{1-\epsilon^2}}{1+\sqrt{1-\epsilon^2}}\int_{\eta^{1/N}}^{\infty}\zeta^2 \mathrm{e}^{-\zeta^2/2}\mathrm{d}\zeta$$

$$\text{with } \eta^{1/N} = a^{1/N}/\sqrt{m_0}$$

$\overline{a^{1/3}}$ Significant amplitude: the 1/3 highest average amplitudes

$H^{1/3}$ Significant height: twice of significant amplitude. $H^{1/3} = 2\overline{a^{1/3}}$

For narrow baneded spectrum (ε<0-.5) , the following holds:

$$\overline{a^{1/3}} = 2\sqrt{m_0} \qquad H_{1/3} = 4\sqrt{m_0}$$

Example

Given spectrum S(ω) with m0=4, m2=1.75, m4=1.0, find

Q1: what is the probability that the amplitude is large than A = 10m?

$$\epsilon = \sqrt{1 - \frac{m_2^2}{m_0 m_4}} = \sqrt{1 - \frac{1.75^2}{4*1.0}} = 0.484$$

$$P(A > 10m) = \frac{2\sqrt{1-\epsilon^2}}{1+\sqrt{1-\epsilon^2}} \mathrm{e}^{-10^2/(2m_0)} = 3.5 \times 10^{-6}$$

Q2: obtain $A^{1/10}, \overline{A^{1/10}}, \overline{A^{1/3}}, H^{1/3}$

$$A^{1/10} = \sqrt{2m_0 \ln\left(\frac{2\sqrt{1-\epsilon^2}}{1+\sqrt{1-\epsilon^2}} N\right)} = 4.23 meters$$

$$\eta^{1/10} = \frac{A^{1/10}}{\sqrt{m_0}} = 2.12$$

$$\overline{A^{1/10}} = \sqrt{m_0} 2N \frac{\sqrt{1-\epsilon^2}}{1+\sqrt{1-\epsilon^2}} \int_{\eta^{1/10}}^{\infty} \zeta^2 \mathrm{e}^{-\zeta^2/2} \mathrm{d}\zeta = \ldots$$

$$\overline{A^{1/3}} = 2\sqrt{m_0} = 4 \text{ meter (assuming narrow band)}$$

$$H_{1/3} = 2\overline{A_{1/3}} = 8 \text{ meters}$$