2.019 Design of Ocean Systems

Lecture 6

Seakeeping (II)

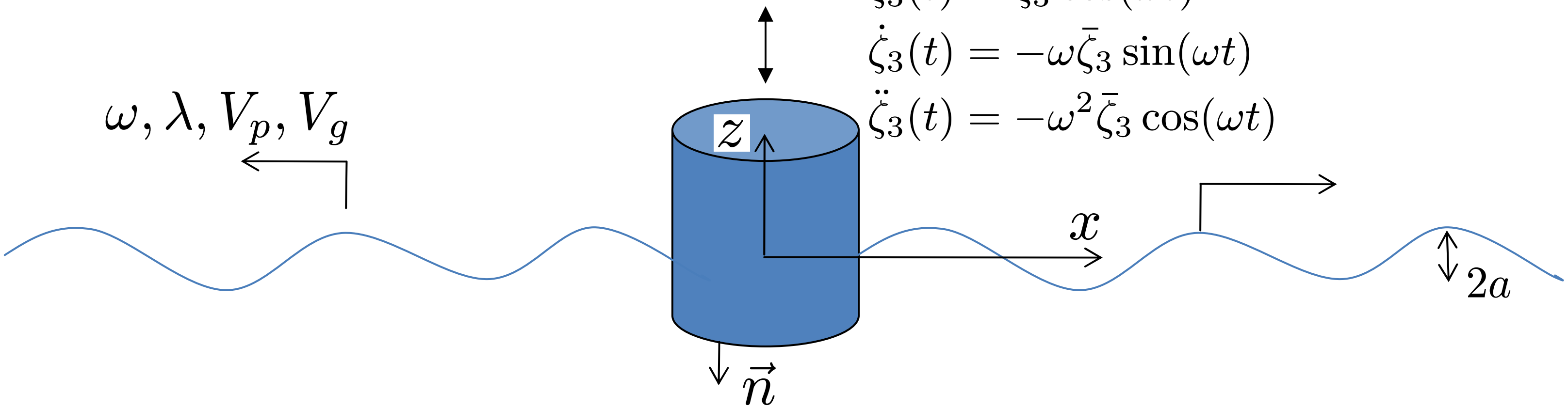
February 21, 2011

Wave Radiation Problem

$$\zeta_3(t) = \bar{\zeta}_3 \cos(\omega t)$$

$$\dot{\zeta}_3(t) = -\omega \bar{\zeta}_3 \sin(\omega t)$$

$$\ddot{\zeta}_3(t) = -\omega^2 \bar{\zeta}_3 \cos(\omega t)$$



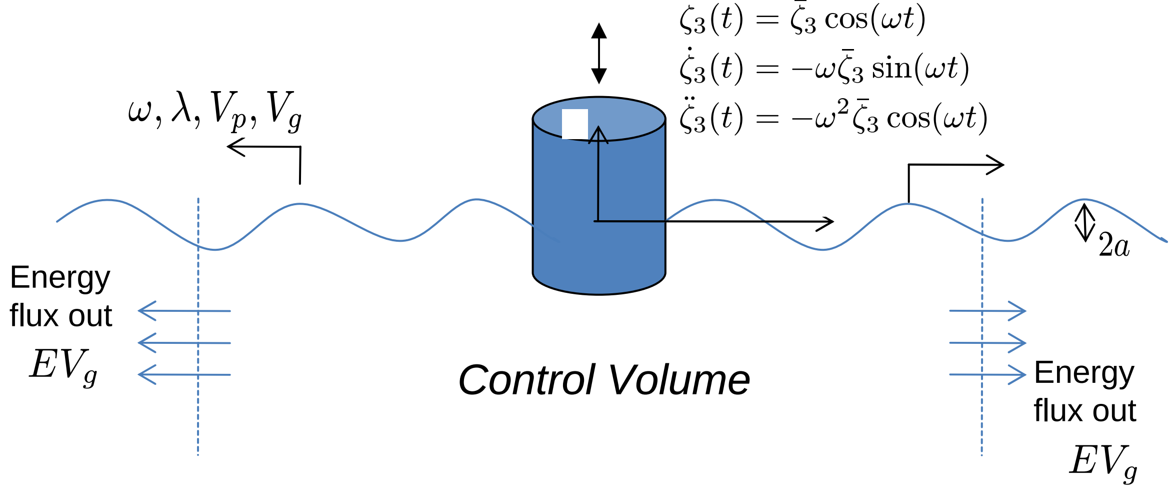
Total: $P(t) = -\rho \frac{\partial \phi}{\partial t} - \rho g z$ Hydrodynamic: $P_d(t) = -\rho \frac{\partial \phi}{\partial t} = \bar{P}_d \cos(\omega t - \psi)$

Hydrodynamic Force:

$$\begin{aligned} F_3(t) &= -\iint_{S_B} P_d n_z \mathrm{d}S = \bar{F}_3 \cos(\omega t - \psi) \\ &= \bar{F}_3 \cos\psi \cos(\omega t) + \bar{F}_3 \sin\psi \sin(\omega t) \\ &= -\frac{\bar{F}_3 \cos\psi}{\bar{\zeta}_3 \omega^2} \ddot{\zeta}_3(t) - \frac{\bar{F}_3 \sin\psi}{\bar{\zeta}_3 \omega} \dot{\zeta}_3(t) \\ &= -A_{33} \ddot{\zeta}_3(t) - B_{33} \dot{\zeta}_3(t) \end{aligned}$$

A_{33}: Added mass; B_{33}: Wave damping

Physical Meaning of Wave Damping



Averaged power into the fluid by the body:

$$\bar{E}_{in} = \frac{1}{T}\int_0^T \{-F_3(t)\}\,\dot{\zeta}_3(t)\mathrm{d}t$$

$$= \frac{1}{T}\int_0^T \left\{A_{33}\ddot{\zeta}_3(t)\dot{\zeta}_3(t) + B_{33}\dot{\zeta}_3(t)\dot{\zeta}_3(t)\right\}\mathrm{d}t = B_{33}(\bar{\zeta}_3\omega)^2/2$$

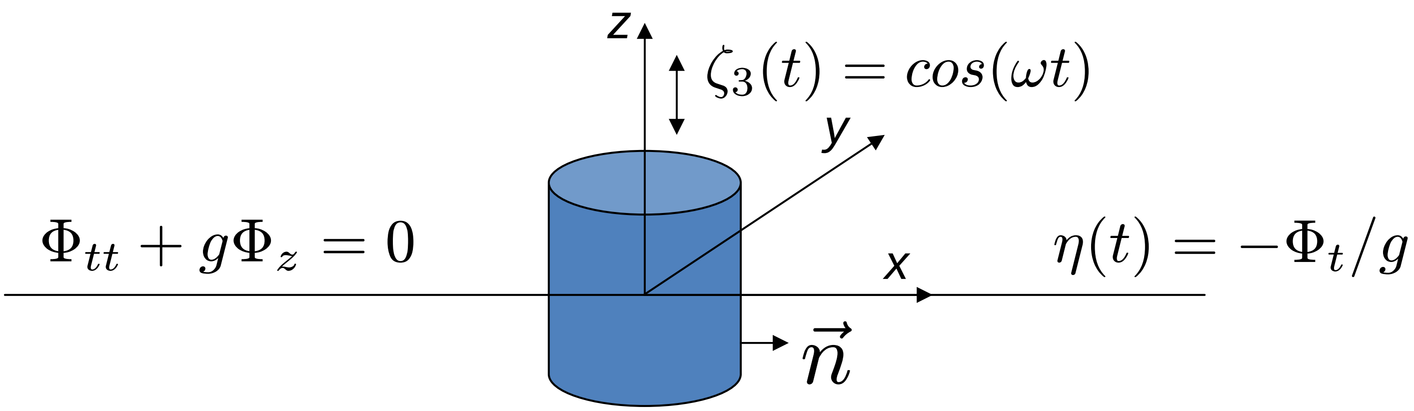
Averaged energy flux out of the control volume: $\bar{E}_{flux} = 2V_gE \sim 2V_ga^2$

Conservation of energy: $\frac{\mathrm{d}\bar{E}}{\mathrm{d}t} \equiv \bar{E}_{in} - \bar{E}_{flux} = 0$

$$\longrightarrow \quad B_{33} \sim (a/\bar{\zeta}_3)^2 > 0$$

- B_{33} =0 if a=0 corresponding to $\omega = \infty$, 0

Mathematical Formulation of Heave Radiation Problem



Radiation condition:
Generated waves must propagate away from the body

$$\nabla^2\Phi(x, y, z, t) = 0$$

Deep water condition:

$$\nabla\Phi \to 0 \quad \text{as} \quad z \to -\infty$$

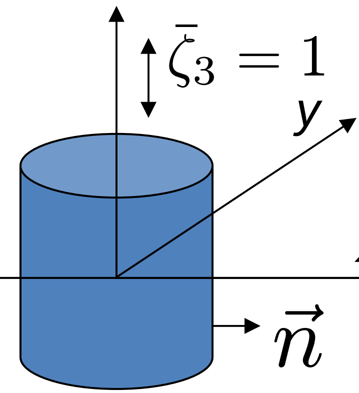
Hydrodynamic Pressure: $$P_d(x, y, z, t) = -\rho\Phi_t$$

Radiation Force: $$\vec{F}_R(t) = -\int_{S_B} P_d\vec{n}\mathrm{d}s$$

Radiation Moment: $$\vec{M}_R(t) = -\int_{S_B} P_d(\vec{x}\times\vec{n})\mathrm{d}s$$

Frequency-Domain Formulation of Heave Radiation Problem

$$-\omega^2\phi_3 + g\phi_{3z} = 0$$



$$\bar{\eta} = -i\omega\phi_3/g$$

Radiation condition

Deep water condition:

$$\nabla\phi_3 \to 0 \quad \text{as} \quad z \to -\infty$$

$$\nabla^2\phi_3(\vec{x}) = 0$$

Let:

$$\zeta_3(t) = \cos\omega t = \Re\{e^{i\omega t}\}$$

$$\Phi(\vec{x}, t) = \Re\{\phi_3(\vec{x})e^{i\omega t}\}$$

$$P_d(\vec{x}, t) = \Re\{p_d(\vec{x})e^{i\omega t}\}$$

$$\vec{F}_R(t) = \Re\{\vec{f}e^{i\omega t}\}$$

$$\vec{M}_R(t) = \Re\{\vec{m}e^{i\omega t}\}$$

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$$p_d = -i\rho\omega\phi_3(\vec{x})$$

$$\vec{f} = -\int_{S_B} p_d\vec{n}\,\mathrm{d}S$$

$$\vec{m} = -\int_{S_B} p_d(\vec{x}\times\vec{n})\,\mathrm{d}S$$

$$f_{3R} = i\rho\omega\int_{S_B}\phi_3 n_3\,\mathrm{d}s$$

$$F_{3R}(t) = -A_{33}\ddot{\zeta}_3(t) - B_{33}\dot{\zeta}(t) = \Re\{[\omega^2 A_{33} - i\omega B_{33}]e^{i\omega t}\}$$

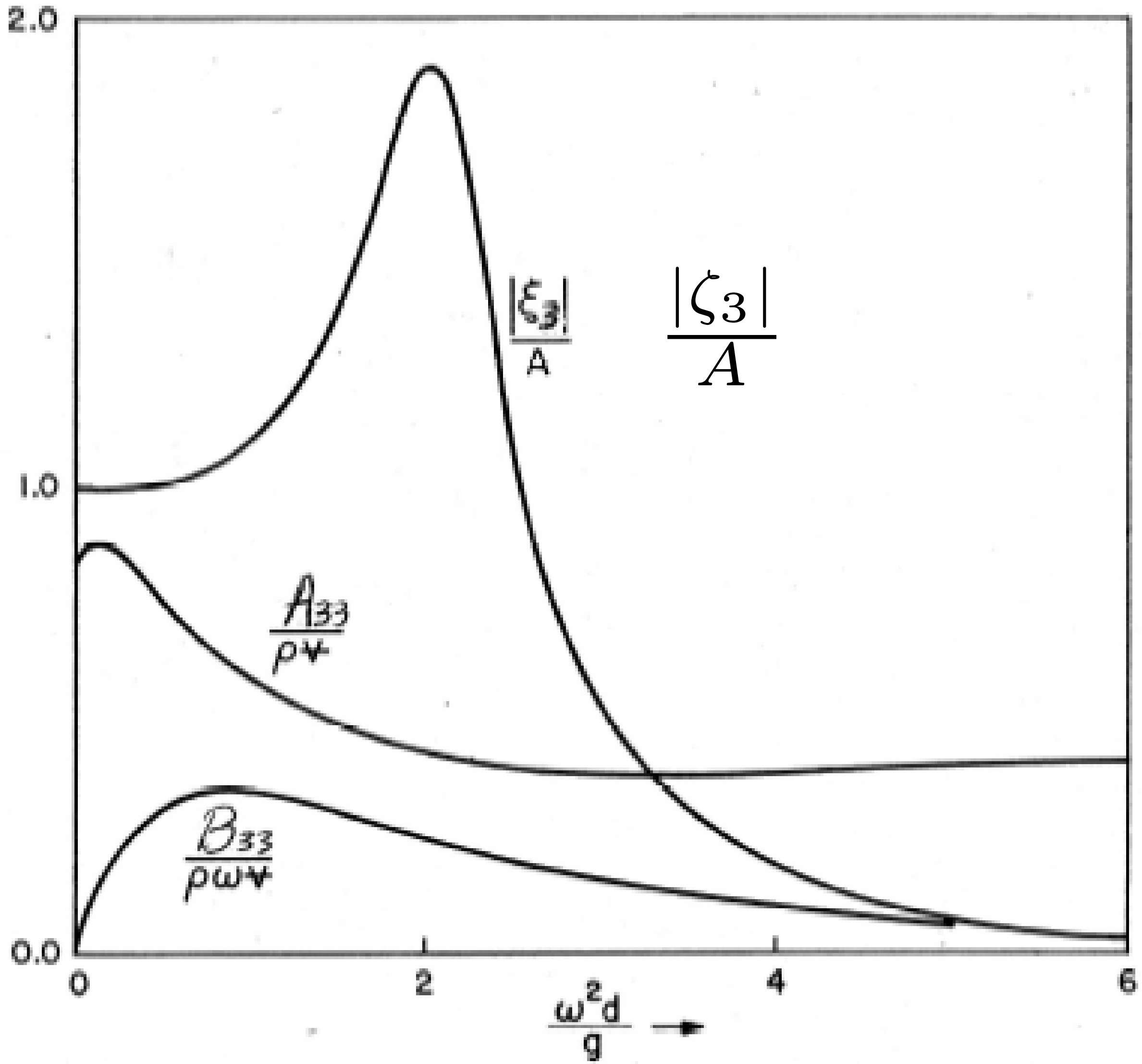
Thus,

$$A_{33} = \Re\left\{\frac{i\rho}{\omega}\int_{S_B}\phi_3 n_3\,\mathrm{d}s\right\}, \quad B_{33} = -\Im\left\{i\rho\int_{S_B}\phi_3 n_3\,\mathrm{d}s\right\}$$

$$A_{13} = \Re\left\{\frac{i\rho}{\omega}\int_{S_B}\phi_3 n_1\,\mathrm{d}s\right\}, \quad B_{13} = -\Im\left\{i\rho\int_{S_B}\phi_3 n_1\,\mathrm{d}s\right\}$$

$$A_{23}, B_{23}, \ldots, A_{63}, B_{63}$$

- A_{ij} and B_{ij} are symmetric, i.e. $A_{ij} = A_{ji}$, $B_{ij} = B_{ji}$, i=1, …, 6; j=1, …, 6
- Aij and Bij are functions of frequency ω



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Added-mass and damping coefficients for a sphere of diameter d, half submerged in deep water. ∀ is the displaced volume $\pi d^3/12$. Also shown is the heave-response ratio (190).

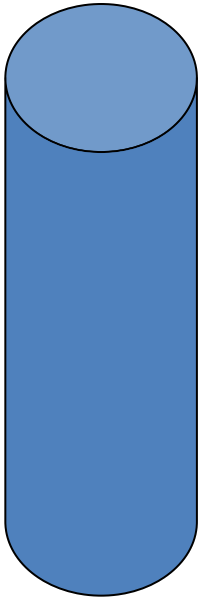
Examples: Added Mass at Low Frequency

At low frequencies, i.e. $\omega \to 0$:

$$\frac{\mathrm{d}^2\Phi}{\mathrm{d}t^2} \sim \omega^2 \to 0 \quad \text{as } \omega \to 0$$

Thus, the free surface boundary conditoin becomes: $\Phi_z = 0$

(1) slender vertical circular cylinder



Surge added mass

$m_{11} = \rho\pi R^2 h$

Wave damping $=0$

(2) slender ship with a semi-circle cross section

Sway added mass

$m_{11} = \rho\frac{\pi R^2}{2}L$

Wave damping $=0$

Examples: Added Mass at High Frequency

At high frequencies, i.e. $\omega \to \infty$:

$$\frac{\mathrm{d}^2\Phi}{\mathrm{d}t^2} \sim \omega^2 \to \infty \quad \text{as } \omega \to \infty$$

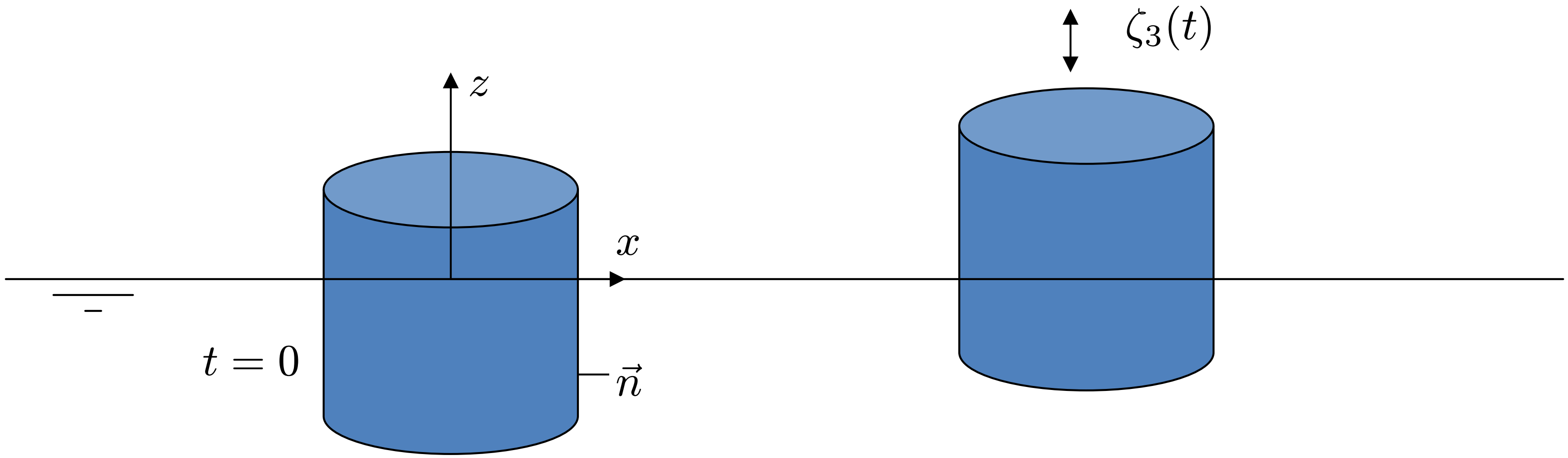
Thus, the free surface boundary conditoin becomes: Φ=0

Slender ship with a semi-circle cross section:

Heave added mass: $m_{33} = \rho \frac{\pi R^2}{2} L$

Wave damping =0

Hydrostatic Restoring Effect in Body Motion



Wetted body surface: S_0 $\qquad$ $S_B(t) = S_0 + \Delta S(t)$

Hydrostatic pressure: $P_s = -\rho g z$

Hydrostatic force: $\vec{F}_s = -\int_{S_B(t)} P_s \vec{n} \mathrm{d}s = -\int_{S_0} P_s \vec{n} \mathrm{d}s - \int_{\Delta S(t)} P_s \vec{n} \mathrm{d}s$

- $\int_{\Delta S(t)} P_s \vec{n} \mathrm{d}s$ → Hydrostatic restoring effect
- $\int_{S_0} P_s \vec{n} \mathrm{d}s$ → Balanced by other forces at equilibrium

$$F_{s3}(t) = \rho g \int_{Vol} \mathrm{d}Vol = \bar{F}_{s3} - \rho g S_{wl} \zeta_3(t)$$

S_{wl} : Water plane surface area of the body

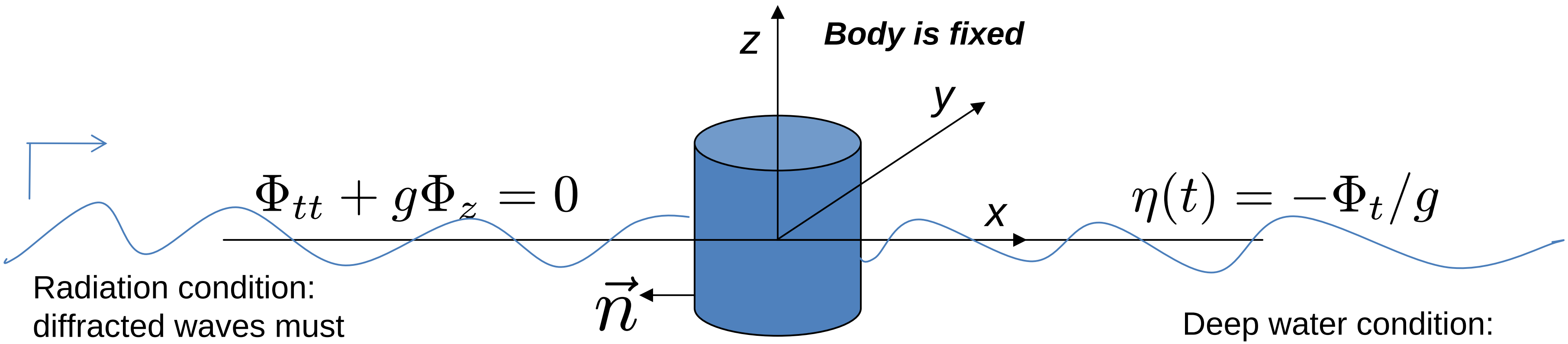
$C_{33} = \rho g S_{wl}$: Hydrostatic restoring coefficient (i.e. spring constant)

Hydrostatic moment: $\vec{M}_s = -\int_{S_B(t)} P_s (\vec{x} \times \vec{n}) \mathrm{d}s = -\int_{S_0} P_s (\vec{x} \times \vec{n}) \mathrm{d}s - \int_{\Delta S(t)} P_s (\vec{x} \times \vec{n}) \mathrm{d}s$

Hydrostatic restoring force/moment: $F_{si3}(t) = -C_{i3} \zeta_3(t), \quad i = 1, \ldots, 6$

In general $F_{sij}(t) = -\sum_{j=1}^{6} C_{ij} \zeta_j \; , i = 1, \ldots, 6$ where C_{ij} is 6×6 restoring coef. matrix

Wave Diffraction Problem



Radiation condition: diffracted waves must propagate away from the body

Deep water condition:

$$\nabla\Phi \to 0 \quad \text{as} \quad z \to -\infty$$

$$\nabla^2\Phi(x, y, z, t) = 0$$

$$\vec{F}_E, \vec{M}_E = ???$$

Total potential: $\Phi(\vec{x}, t) = \Phi_I(\vec{,}xt) + \Phi_D(\vec{x}, t)$

Φ_I : Incident wave potential (of a plane progressive wave)

Φ_D : Diffracted (or scattered) wave potential

Total dynamic pressure: $P_d = -\rho\Phi_{It} - \rho\Phi_{Dt}$

Diffraction effect

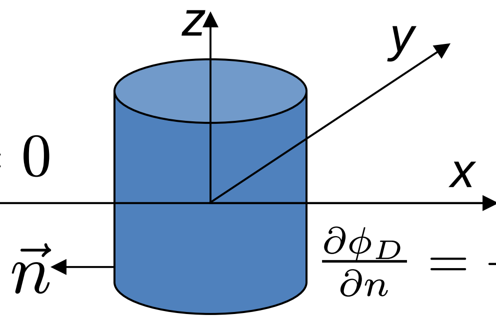
Total wave excitations (force/moment): $\vec{F}_E(t) = \int_{S_B} \rho\Phi_{It}\vec{n}\mathrm{d}s + \int_{S_B} \rho\Phi_{It}\vec{n}\mathrm{d}s = \vec{F}_I + \vec{F}_D$

Froude-Krylov force

$$\vec{M}_E(t) = \int_{S_B} \rho\Phi_{It}(\vec{x} \times \vec{n})\mathrm{d}s + \int_{S_B} \rho\Phi_{Dt}(\vec{x} \times \vec{n})\mathrm{d}s = \vec{M}_I + \vec{M}_D$$

Frequency-Domain Formulation of Wave Diffraction Problem

$$-\omega^2\phi_D + g\phi_{Dz} = 0 \qquad \bar{\eta}_D = -i\omega\phi_D/g$$



$$\frac{\partial\phi_D}{\partial n} = -\frac{\partial\phi_I}{\partial n}$$

Radiation condition: diffracted waves must propagate away from the body

Deep water condition:

$$\nabla\phi_D \rightarrow 0 \quad \text{as} \quad z \rightarrow -\infty$$

$$\nabla^2\phi_D(x, y, z) = 0$$

Incident $\eta_I(x, y, t) = a\cos(\omega t - kx) = \Re\{a\mathrm{e}^{-ikx}\mathrm{e}^{i\omega t}\} = \Re\{\bar{\eta}_I\mathrm{e}^{i\omega t}\}$

Diffraction potential: $\Phi_D(\vec{x}, t) = \Re\{\phi_D(\vec{x})\mathrm{e}^{i\omega t}\}$

Total dynamic pressure: $P_d(\vec{x}, t) = \Re\{p_d(\vec{x})\mathrm{e}^{i\omega t}\}, \qquad p_d(\vec{x}) = p_I + p_D$

$$p_I = -i\rho\omega\phi_I, \qquad p_D = -i\rho\omega\phi_D$$

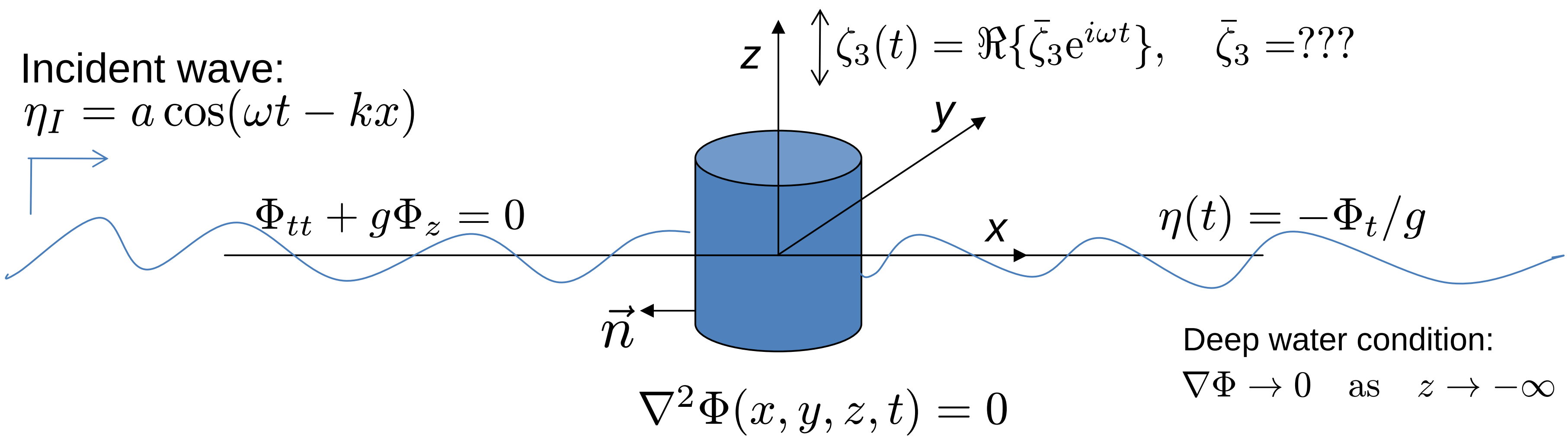
Total wave excitations: $\vec{F}_E(t) = \Re\{\vec{f}_E\mathrm{e}^{i\omega t}\}, \qquad \vec{f}_E = \vec{f}_{EI} + \vec{f}_{ED}$

Froude-Krylov force: $\vec{f}_{EI} = -\int_{S_B} p_I\vec{n}\mathrm{d}s = i\rho\omega\int_{S_B}\phi_I\vec{n}\mathrm{d}s$

Diffraction force: $\vec{f}_{ED} = -\int_{S_B} p_D\vec{n}\mathrm{d}s = i\rho\omega\int_{S_B}\phi_D\vec{n}\mathrm{d}s$

$$\vec{M}_E(t) = \Re\{\vec{m}_E\mathrm{e}^{i\omega t}\}, \qquad \vec{m}_E = \vec{m}_{EI} + \vec{m}_{ED}$$

Heave Response of A Floating Body to Ambient Waves



- Decompose the total problem into a sum of diffraction problem and radiation problem:

$$\Phi(\vec{x},t) = \Phi_I(\vec{x},t) + \underbrace{\Phi_D(\vec{x},t)}_{\text{Diffraction problem}} + \underbrace{\Phi_R(\vec{x},t)}_{\text{Radiation problem}}$$

- From the diffraction problem:

Wave excitation force: $F_{E3}(t) = \Re\{f_{E3}e^{i\omega t}\}, \qquad f_{E3} = f_{3I} + f_{3D}$

- From the radiation problem:

Wave radiation force: $F_{R3}(t) = -A_{33}\ddot{\zeta}_3(t) - B_{33}\dot{\zeta}_3(t) = \Re\{(-\omega^2 A_{33} - i\omega B_{33})\bar{\zeta}_3 e^{i\omega t}\}$

Hydrostatic restoring force: $F_{s3} = -C_{33}\zeta_3(t) = \Re\{(-C_{33}\bar{\zeta}_3)e^{i\omega t}\}$

• Total hydrodynamic and hydrostatic forces:

$$F_{E3} + F_{R3} + F_{s3} = \Re\{[f_{E3} - (\omega^2 A_{33} + i\omega B_{33} + C_{33})\bar{\zeta}_3]\mathrm{e}^{i\omega t}\}$$

• Applying Newton's second law:

$$F_{E3} + F_{R3} + F_{s3} = m\ddot{\zeta}_3(t)$$

$$\Re\{(-\omega^2 m)\mathrm{e}^{i\omega t}\} = \Re\{[f_{E3} - (-\omega^2 A_{33} + i\omega B_{33} + C_{33})\bar{\zeta}_3]\mathrm{e}^{i\omega t}\}$$

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Equation of Motion: $[-\omega^2(m + A_{33}) + i\omega B_{33} + C_{33}]\bar{\zeta}_3 = f_{3I} + f_{3D}$

• Heave motion amplitude: $\bar{\zeta}_3 = \frac{f_{3I}+f_{3D}}{-\omega^2(m+A_{33})+i\omega B_{33}+C_{33}}$

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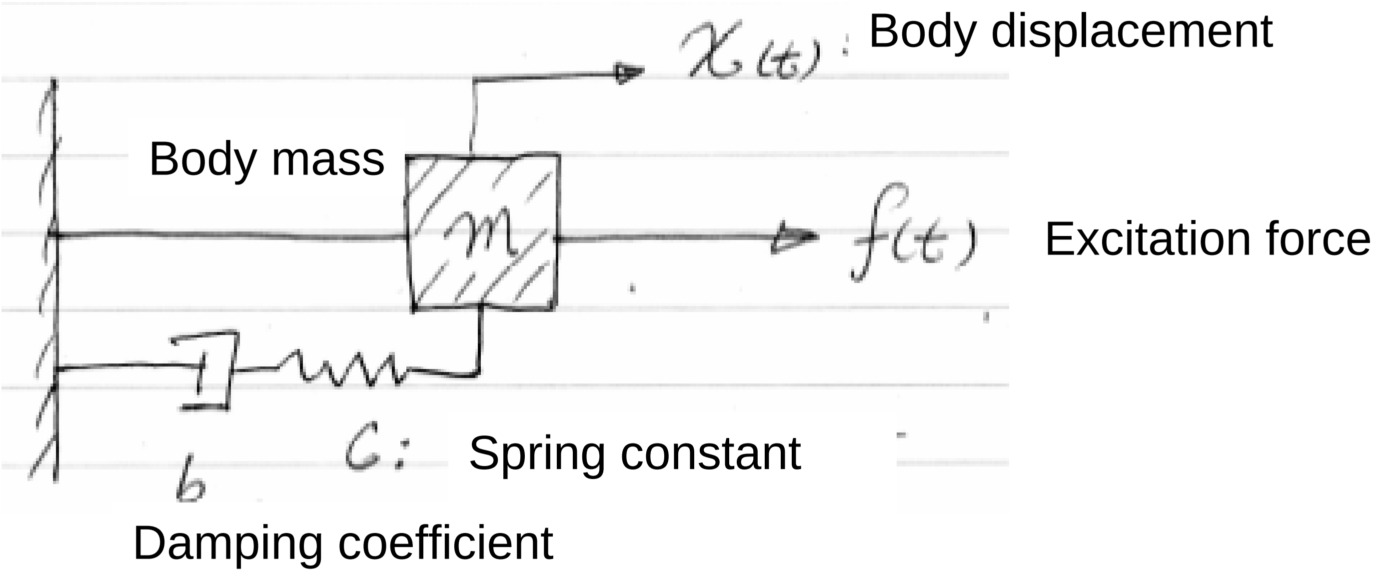
Response Amplitude Operator (**RAO**):

$$\frac{\bar{\zeta}_3(\omega)}{a} = \frac{(f_{3I} + f_{3D})/a}{-\omega^2(m + A_{33}) + i\omega B_{33} + C_{33}}$$

$$= \left|\frac{(f_{3I} + f_{3D})/a}{-\omega^2(m + A_{33}) + i\omega B_{33} + C_{33}}\right|\mathrm{e}^{i\alpha}$$

Heave natural frequency: $-\omega_{n3}(m + A_{33}) + C_{33} = 0 \quad \rightarrow \quad \omega_{3n} = \left(\frac{C_{33}}{m+A_{33}}\right)^{\frac{1}{2}}$

Analogy to a Simple Mass-Spring-Dashpot System



Equation of motion: $m\ddot{x} + b\dot{x} + cx = f(t)$

For harmonic excitation, $f(t) = f_0 \cos \omega t$, we have harmonic response: $x(t) = x_0 \cos(\omega + \alpha), \quad x_0 = ??$

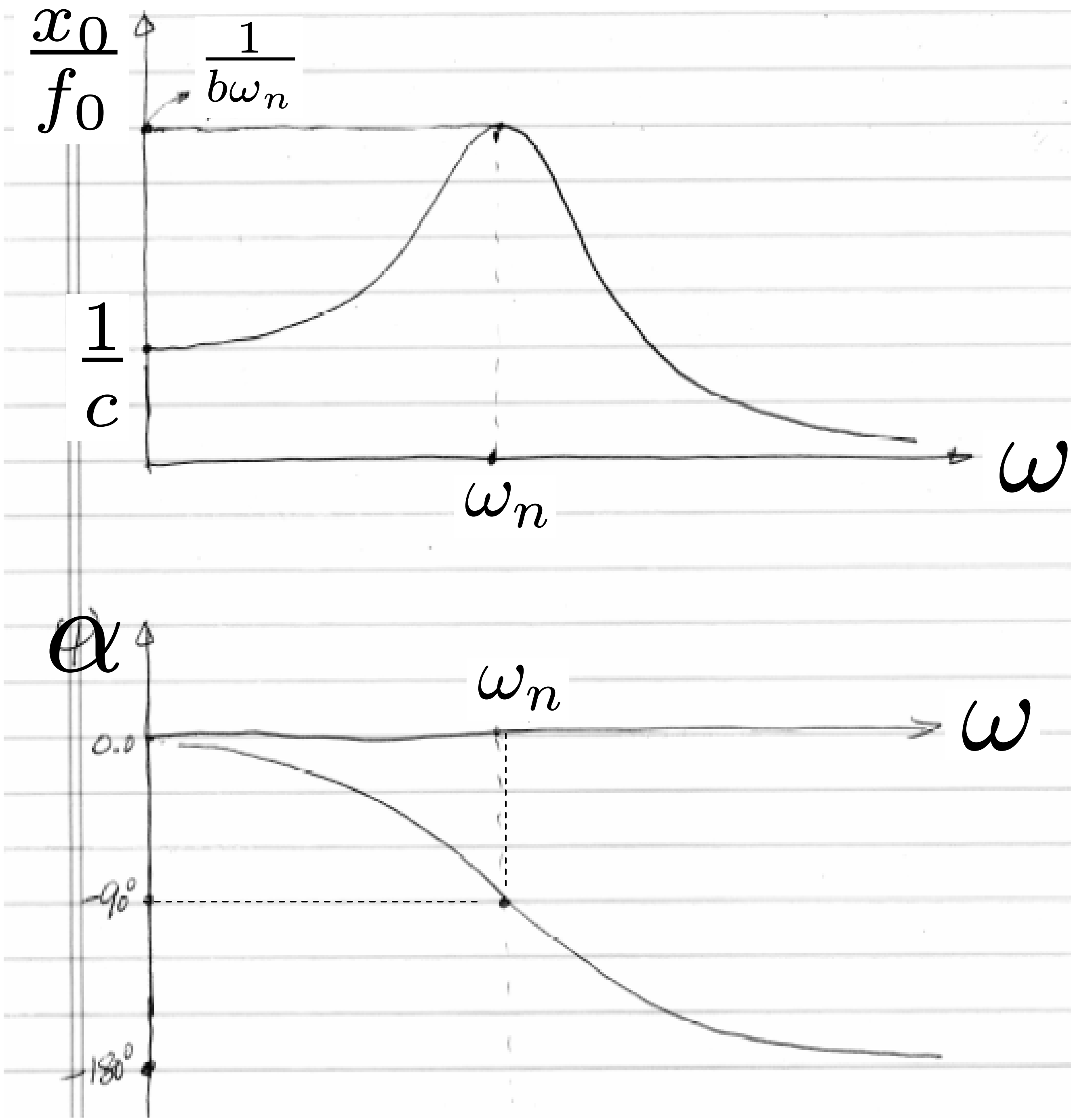
From equation of motion, we obtain: $x_0 = \frac{f_0}{[(c - m\omega^2)^2 + b^2\omega^2]^{1/2}}$ and $\alpha = \tan^{-1}\left(\frac{-b\omega}{c - m\omega^2}\right)$

Natural frequency: $\omega_n = (c/m)^{1/2}$

$$\frac{x_0}{f_0} = \frac{1}{c}, \quad \text{at } \omega{=}0$$

$$\frac{x_0}{f_0} = \frac{1}{b\omega_n}, \quad \text{at } \omega{=}\omega_n$$

$$\frac{x_0}{f_0} \to 0, \quad \text{as } \omega \to \infty$$


$\frac{x_0}{f_0}$
$\frac{1}{b\omega_n}$
$\frac{1}{c}$
ω_n
ω
α
ω_n
ω
0.0
-90°
-180°

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