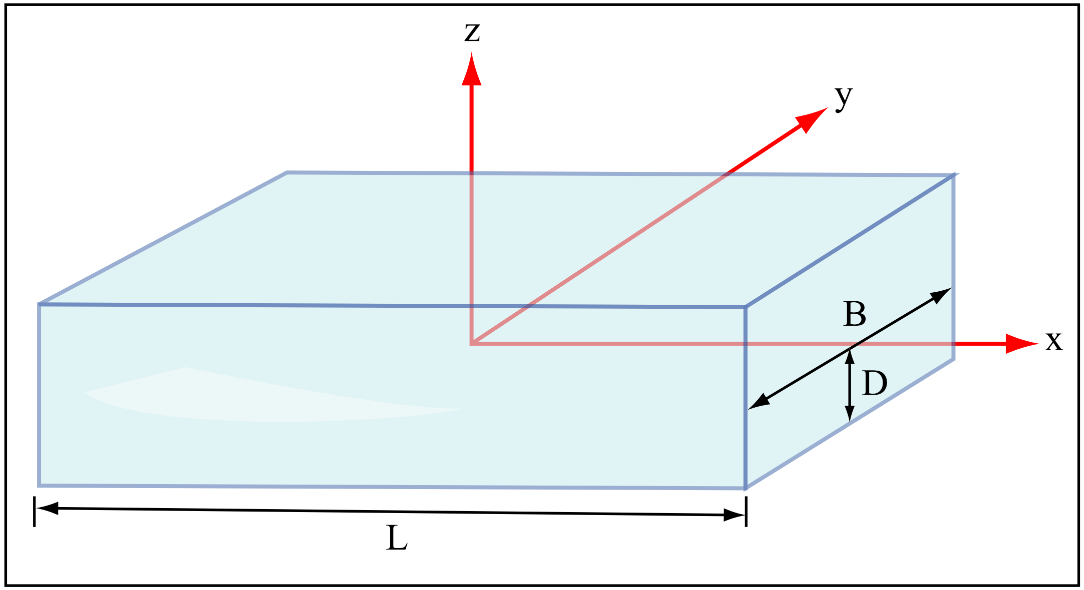
2.019 Design of Ocean Systems

Lecture 7

Seakeeping (III)

February 25, 2011

Motions and Wave Loads on a Barge



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A regular plane progressive incident wave in deep water travels along the x-direction:

$$\eta_I(x,t) = a\cos(\omega t - kx)$$

$$\Phi_I(x,y,z,t) = -\frac{ga}{\omega}\mathrm{e}^{kz}\sin(\omega t - kx)$$

To find the wave force and motion of the barge in the vertical direction using long-wave and strip theory assumptions.

Heave Wave Excitation on a Barge (I)

$$F_{E3} = F_{I3} + F_{D3}$$

Using the strip theory (which is valid for B/L << 1), we have:

$$F_{E3} = \int_{-L/2}^{L/2} f_{E3}(x)\mathrm{d}x, \quad F_{I3} = \int_{-L/2}^{L/2} f_{I3}(x)\mathrm{d}x, \quad F_{D3} = \int_{-L/2}^{L/2} f_{D3}(x)\mathrm{d}x$$

Froude Krylov force component:

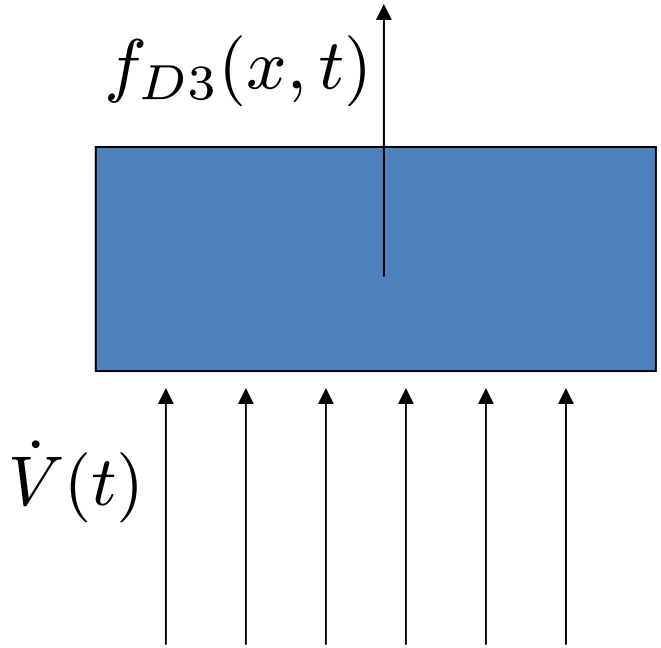
$$\begin{aligned} f_{I3}(x) &= -\int_{-B/2}^{B/2} P_I(x) n_z \mathrm{d}y \\ &= \int_{-B/2}^{B/2} (-\rho\Phi_t(x, y, z = -D, t)\mathrm{d}y \\ &= B\rho g a \mathrm{e}^{-kD}\cos(\omega t - kx) \end{aligned}$$

$$\begin{aligned} F_{E3} &= \int_{-L/2}^{L/2} f_{E3}(x)\mathrm{d}x \\ &= \int_{-L/2}^{L/2} B\rho g a \mathrm{e}^{-kD}\cos(\omega t - kx)\mathrm{d}x \\ &= \rho g a B \left(\frac{2}{k}\right) \mathrm{e}^{-kD} \sin\frac{kL}{2}\cos\omega t \end{aligned}$$

In the limit $\omega \to 0$: $F_{E3} \to \rho g a B L \cos\omega t = \rho g \eta(t)(BL)$

Heave Wave Excitation on a Barge (II)

Long-wave assumption: wave motion is a flow slowly varying in space and time.
The wave diffraction effect is approximated by the added mass effect.



$$f_{D3}(x,t) = A_{33}^{2D}(x)\dot{V}(x,t)$$

$$\begin{aligned} V(x,t) &= \Phi_{Iz}(x, z=-D/2, t) \\ &= -\frac{gak}{\omega}\mathrm{e}^{-kD/2}\sin(\omega t - kx) \end{aligned}$$

$$\dot{V}(x,t) = -gak\mathrm{e}^{-kD/2}\cos(\omega t - kx)$$

$$f_{D3}(x,t) = -gak\mathrm{e}^{-kD/2}A_{33}^{2D}\cos(\omega t - kx)$$

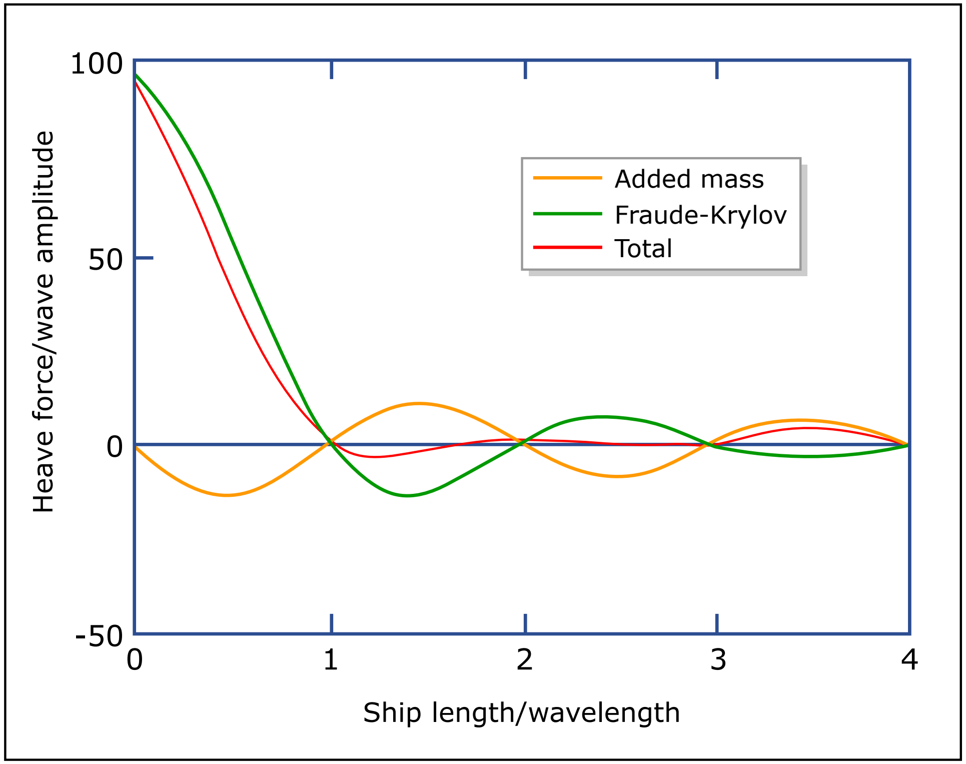
$$F_{D3}(t) = \int_{-L/2}^{L/2} f_{D3}(x,t)\mathrm{d}x = -2ga\mathrm{e}^{-kD/2}A_{33}^{2D}\sin\frac{kL}{2}\cos\omega t$$

In the limit $\omega \to 0$: $F_{D3} \to 0$

Heave Wave Excitation on a Barge (III)

$$F_{E3}(t) = F_{I3} + F_{D3} = \Big[\underbrace{\rho g a B \mathrm{e}^{-kD}}_{\text{Froude Krylov}} - \underbrace{g A_{33}^{2D} k a \mathrm{e}^{-kD/2}}_{\text{Added mass effect}}\Big] \left(\frac{2}{k}\right) \sin\frac{kL}{2} \cos\omega t$$

$$A_{33}^{2D} = C_a \left[\rho \frac{\pi}{2} (B/2)^2\right], \qquad C_a \sim 1.0$$



Radiation Force

Added mass coefficient: $A_{33} = \int_{-L/2}^{L/2} A_{33}^{2D}(x)\mathrm{d}x = LA_{33}^{2D}$

Wave damping coefficient: $B_{33} \to 0 \quad \text{with long-wave assumption}$

Radiation force: $F_{R3} = -A_{33}\ddot{\zeta}_3(t) = -LA_{33}^{2D}\ddot{\zeta}_3(t)$

Restoring Force

$$F_{S3} = -C_{33}\zeta_3(t) = -\rho g B L \zeta_3(t)$$

Equation of Motion

$$(M + A_{33})\ddot{\zeta}_3 + B_v\dot{\zeta}_3 + \rho g B L \zeta_3(t) = F_{E3}(t)$$

If B_v =0, $\zeta_3(t) = \bar{\zeta}_3 \cos(\omega t)$

$$\bar{\zeta}_3(\omega) = \frac{\left[\rho g a B \mathrm{e}^{-kD} - g A_{33}^{2D} k a \mathrm{e}^{-kD/2}\right]\left(\frac{2}{k}\right)\sin\frac{kL}{2}}{-\omega^2(M + A_{33}) + \rho g B L}$$

In the limit $\omega \to 0$:

$$\bar{\zeta}_3 = \frac{\rho g B L a}{-\omega^2(M+A_{33})+\rho g B L} = a$$

$$\zeta_3(t) = a\cos\omega t = \eta(x=0,t)$$

Barge responds to move like a fluid particle in the limit of very long wave.

Natural Frequency

$$-\omega_n^2(M + A_{33}) + \rho g B L = 0$$

$$\omega_n = \left(\frac{\rho g B L}{M+A_{33}}\right)^{1/2} = \left(\frac{gB}{BD+A_{33}^{2D}}\right)^{1/2} = \left(\frac{g}{D+C_a(\pi/8)B}\right)^{1/2}$$

Natural period: $T_n = \frac{2\pi}{\omega_n}$

T_n increases with D and B.

For example, for D=20m, B=60m, we have T_n = *13s.*

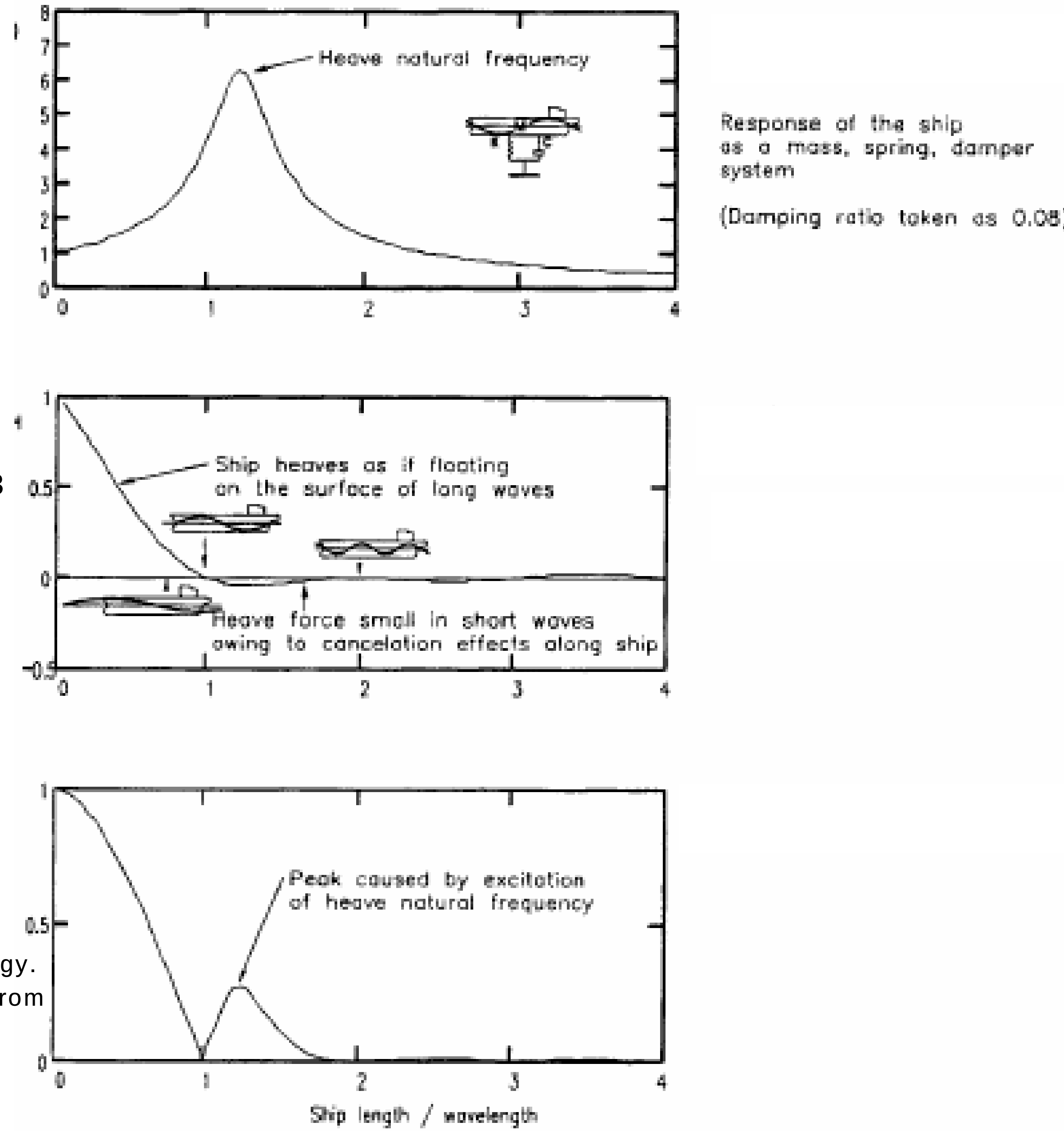
Sample Results for Heave Motion

Draft: D=12 m
Width: B=40 m

Natural period: T_n= 11.4 s
B_v = 8% critical damping

F_{E3}

RAO



Pitch Motion and Wave Loads on a Barge

Wave Excitation:
$$F_{E5}(t) = \int_{-L/2}^{L/2} -x f_{E3} \mathrm{d}x$$
$$= \left[\rho g B a \mathrm{e}^{-kD} - gka\mathrm{e}^{-kD/2} A_{33}^{2D}\right] \int_{-L/2}^{L/2} [-x\cos(\omega t - kx)]\mathrm{d}x$$

Added mass and wave damping: $A_{55} = \int_{-L/2}^{L/2} x^2 A_{33}^{2D} \mathrm{d}x$, $\quad B_{55} = 0 \quad \text{as } \omega \to 0$

Radiation moment: $F_{R5} = -A_{55}\ddot{\zeta}_5(t) - B_{55}\dot{\zeta}_5(t)$

Hydrostatic restoring moment: $F_{S5} = -C_{55}\zeta_5(t) = -[\rho g \nabla(Z_B - B_G) + \rho g \int_{A_{wp}} x^2 \mathrm{d}s]\zeta_5(t)$

Moment of inertia: $I_{55} = \rho D B \int_{-L/2}^{L/2} x^2 \mathrm{d}x$

From the equation of motion for pitch, we can get pitch motion: $\zeta_5(t) = \bar{\zeta}_5 \sin \omega t$

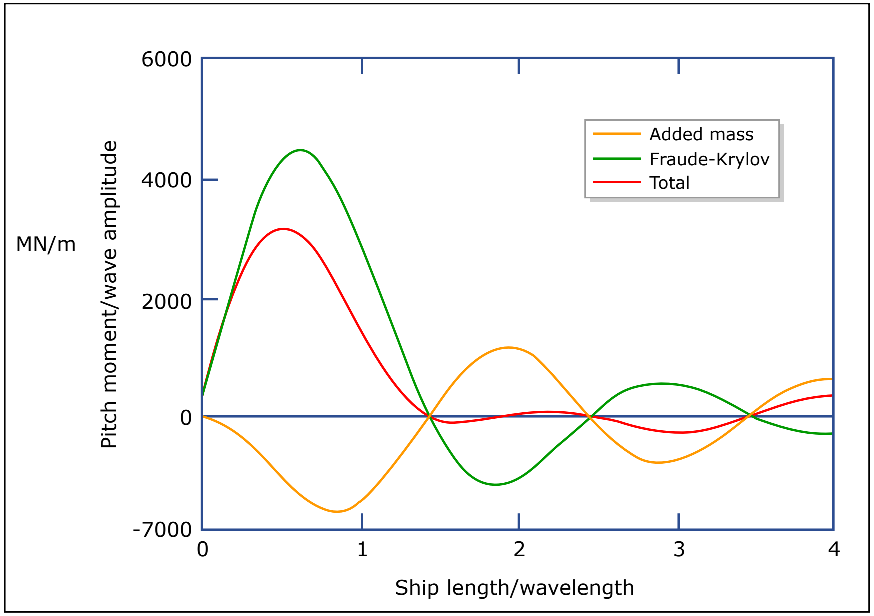
$$\frac{\bar{\zeta}_5}{a} = \ldots..$$

Sample Results for Pitch Motion

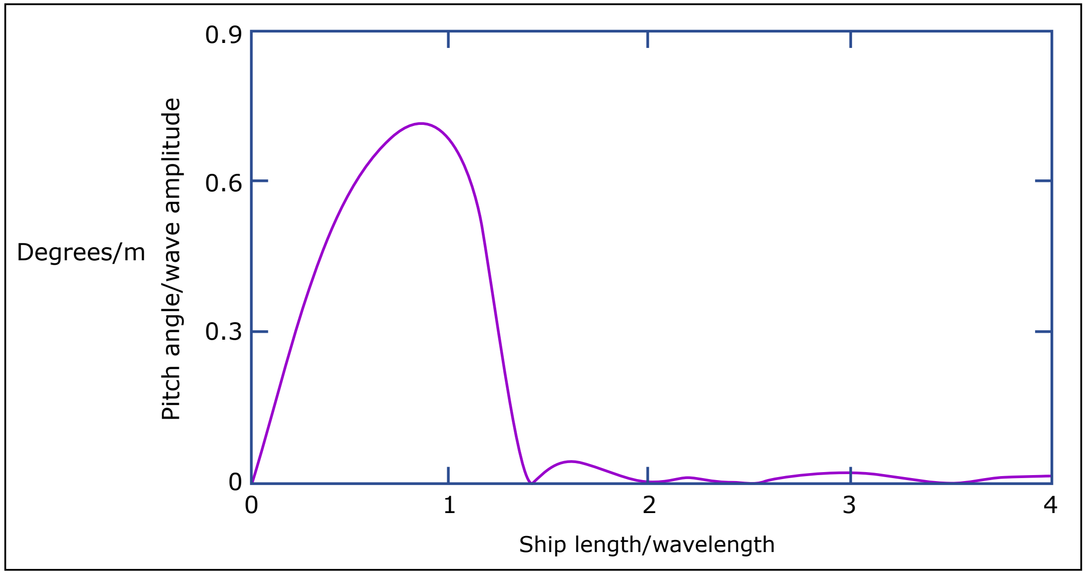
Draft: D=12 m
Width: B=40 m

B_v = 8% critical damping

F_{E5}



RAO



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Deck elevation at bow:

$$Z_D = \zeta_3(t) - (L/2)\zeta_5(t) + H \quad \text{where } H \text{ is deck height}$$

Bottom elevation at bow:

$$Z_B = \zeta_3(t) - (L/2)\zeta_5(t) - D \quad \text{where } D \text{ is draft}$$

Wave elevation at bow:

$$Z_w = \eta(x = L/2, t) = a\cos(\omega t - kL/2)$$

If $Z_D < Z_w$, wave overtoping occurs;
If $Z_B > Z_w$, ship bottom is out of water

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