2.032 DYNAMICS Fall 2004

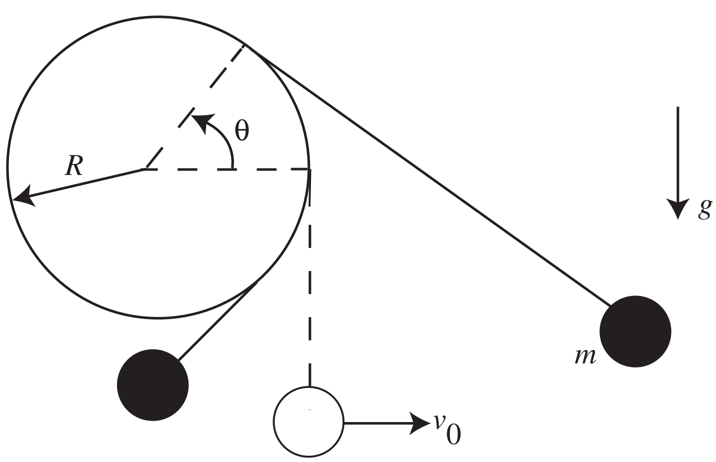
Problem Set No. 1

Out: Wednesday, September 15, 2004

Due: Wednesday, September 22, 2004 *at the beginning of class*

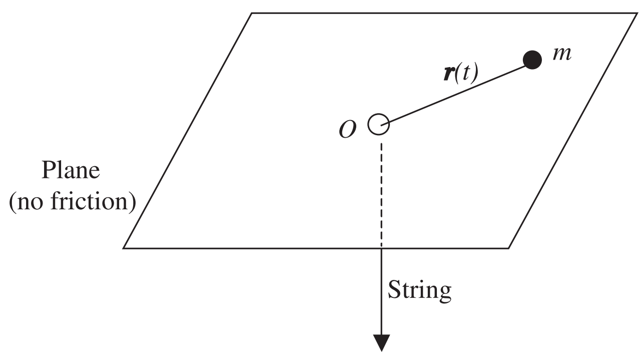
Problem 1 (Doctoral Exam, 1999)

A pendulum is constructed by attaching a mass m to an extensionless string of fixed length l. The upper end of the string is connected to the uppermost point of a vertical fixed disk of radius R ($R < l/\pi$), as shown below. At $t = 0$ the mass hangs at rest at the equilibrium position $\theta = 0$, when it is given an initial velocity v_0 along the horizontal. Derive expressions for the two extreme deflections (in terms of θ) of the pendulum resulting from this initial perturbation. Do *not* make a small-angle approximation.



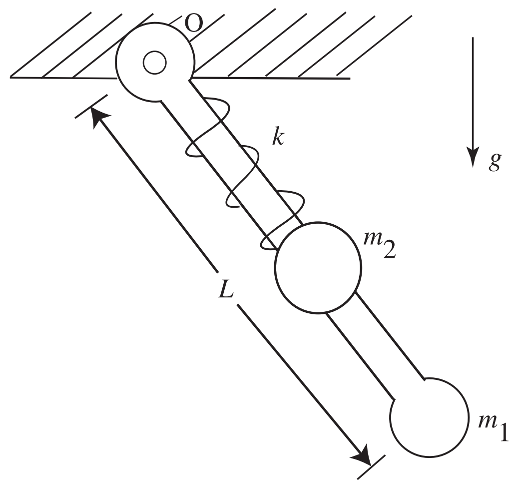
Problem 2

A point mass moves without friction on a horizontal plane. A massless inextensible string is attached to the point mass and led through a hole (see figure below). At time $= t_0$ the mass moves along a circle with constant velocity v_0. We gradually pull the free end of the string downwards until, at time t_1, we have $|\boldsymbol{r}(t_1)| = L_0/2$. What is the velocity of the mass at time t_1?



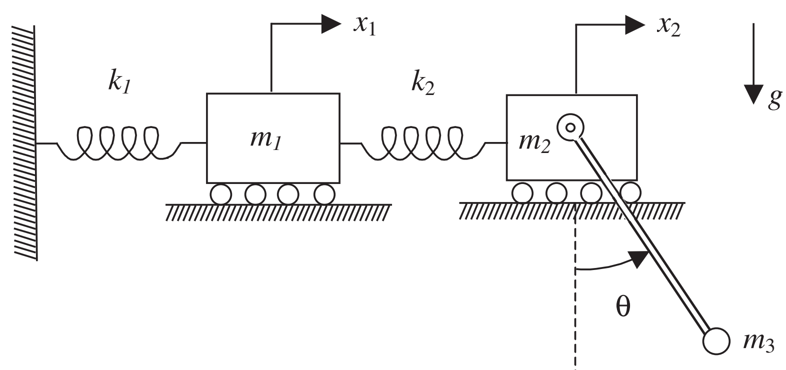
Problem 3

A particle of mass m_1 is attached to a massless rod of length L which is pivoted at O and is free to rotate in the vertical plane as shown below. A bead of mass m_2 is free to slide along the smooth rod under the action of a spring of stiffness k and unstretched length L_0. (a) Choose a complete and independent set of generalized coordinates. (b) Derive the governing equations of motion by applying momentum principles.



Problem 4 (adapted from Crandall et al., 2-35)

Consider the system shown below under the assumption that the pendulum arm connecting m_2 and m_3 is massless. By applying momentum principles, obtain the differential equations of motion for the generalized coordinates x_1, x_2 and θ.



Problem 5 (Doctoral Exam, 1999)

Two identical rods of length l, that have equal masses m attached at their ends, are clamped at an angle θ to a shaft as shown. (The shaft and the rods are in the same plane.) What reaction forces must the bearings be able to withstand, if the angle θ can be set anywhere from zero to 90° and the maximum angular velocity of the shaft is ω? (For simplicity, you may neglect the mass of the rods and ignore the effects of gravity.)