2.032 DYNAMICS Fall 2004

Problem Set No. 10

Out: Wednesday, November 24, 2004

Due: Wednesday, December 1, 2004 *at the beginning of class*

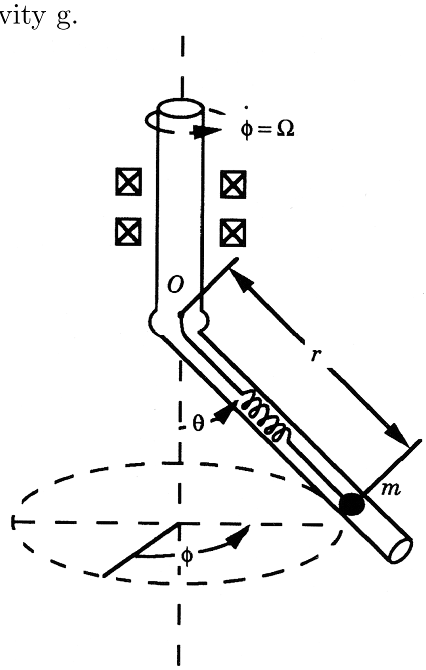
Problem 1

The system below consists of a massless hollow cylindrical tube, joined to a vertical shaft at the point O. The tube is fixed in θ-direction and $\theta = \pi/4$. Inside the tube, moves without friction a mass m which is connected to O through a spring of stiffness k and neutral length of r_0. Assume that the shaft is rotating with angular velocity $\dot{\phi}$ about its axis. Using r, ϕ as generalized coordinates:

(a) Reduce the problem to a one-degree-of-freedom problem for r, that has only potential active forces.

(b) Find the equilibria for the reduced system and investigate the stability using Dirichlet theorem.

(c) Sketch the trajectories on the $(r,\dot{r})$ phase plane. Select all parameters to be equal to one, including gravity g.



Problem 2 (adapted from PhD Qualifying Exam 2003)

Reconsider Problem 2 of PS No. 5. Using the same notation,

(i) Derive the differential equation describing the motion of the bead on the ring.

(ii) Find equilibrium positions θ_0 for the bead and investigate the stability of these positions at various speeds Ω.

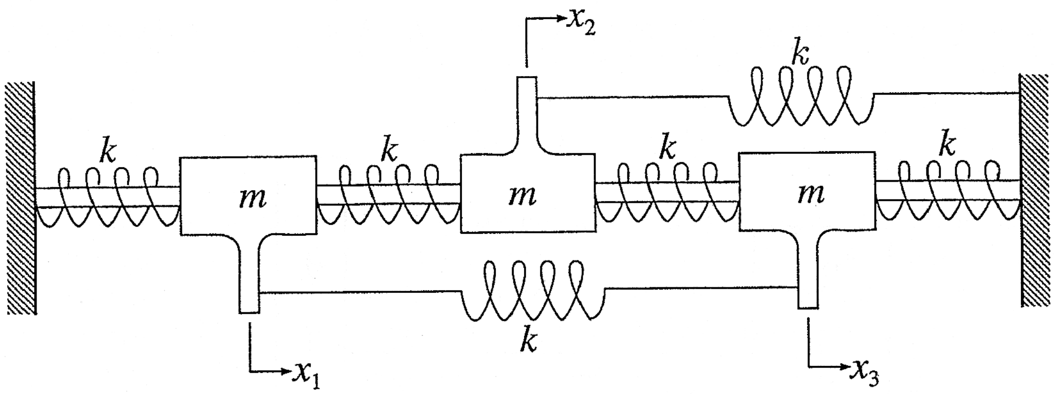
(iii) Draw a stability diagram showing all solution branches (and their stability properties) for $0 < \Omega < \infty$.

(iv) Draw the phase plane of solution trajectories at representative values of Ω.

(v) Instead now consider a ring inclined at 120° to the vertical, so that C is below O. *Without any calculations,* state how many equilibrium positions you expect and how their stability will vary with Ω.

Problem 3

Three equal masses m slide without friction on a rigid horizontal rod. Six identical springs with spring constant k are attached to the masses as shown in the sketch below. Identify the natural modes and natural frequencies of this system.



Problem 4

A square plate of mass m and side $2a$ is constrained to remain in the plane of the sketch. Its moment of inertia about an axis perpendicular to the plane through the center of the square is $I = \frac{2}{3}ma^2$. The plate is supported in a gravity-free environment by the four equal springs shown. It is desired to formulate matrix equations of small motions for the three degrees of freedom, using the generalized coordinates x, y, and $a\theta$.

(a) Obtain matrix equations of motion of the form

$$[M]\{\ddot{x}\} + [K]\{x\} = 0$$

and the three natural-mode solutions.

(b) Construct the modal matrix Φ and evaluate

$$[\Phi]^t[M][\Phi], \quad [\Phi]^t[K][\Phi].$$

Verify that the quotients of the diagonal elements of the resulting square matrices in (b) give the squares of the natural frequencies of the three modes.