2.032 DYNAMICS Fall 2004

Problem Set No. 4

Out: Wednesday, October 6, 2004

Due: Wednesday, October 13, 2004 *at the beginning of class*

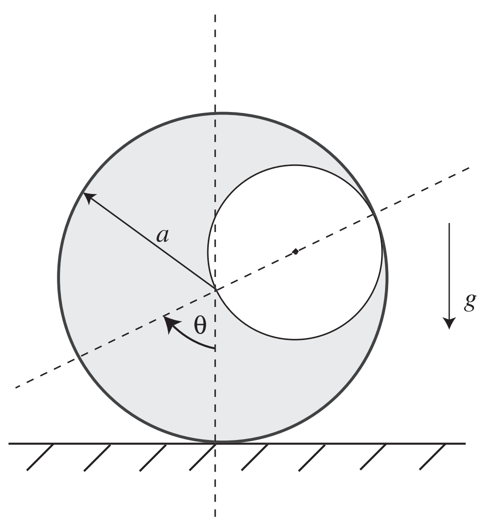
Problem 1

A rigid circular cylinder of radius a has a hole of radius $\frac{1}{2}a$ cut out. Assume that the cylinder rolls without slipping on the floor.

(i) Compute the kinetic energy and the potential energy of the cylinder using the generalized coordinate θ defined below.

(ii) By suitably approximating the kinetic and potential energy expressions in (i), deduce the frequency of small rocking oscillations of the cylinder about the equilibrium position $\theta = 0$.

(iii) Use the potential to plot trajectories qualitatively on the $(\theta, \dot{\theta})$ phase plane.



Problem 2

A billiard ball, initially at rest, is given a sharp impulse by a cue. The cue is held horizontally a distance h above the centerline. The ball leaves the cue with a speed v_0 and eventually acquires a final speed of $\frac{9}{7}v_0$. Show that $h = \frac{4}{5}R$, where R is the radius of the ball.

Problem 3

Determine the principal centroidal moments of inertia for the following homogeneous bodies:

(a) a sphere of radius R,

(b) a circular cone of height h and base radius R.

Problem 4 (adapted from Crandall et al., 4-14)

The uniform rod of length L and mass M is pivoted, without friction, to the shaft OA, which revolves in fixed bearings at the steady rate ω_0. The rod is constrained to remain in a plane through OA which rotates with the shaft.

(a) Formulate an equation of motion for $\theta(t)$.

(b) For each value of ω_0 there is at least one stationary angle θ_0 which the rod can maintain while steadily precessing at the rate ω_0. Find all stationary configurations as functions of ω_0.

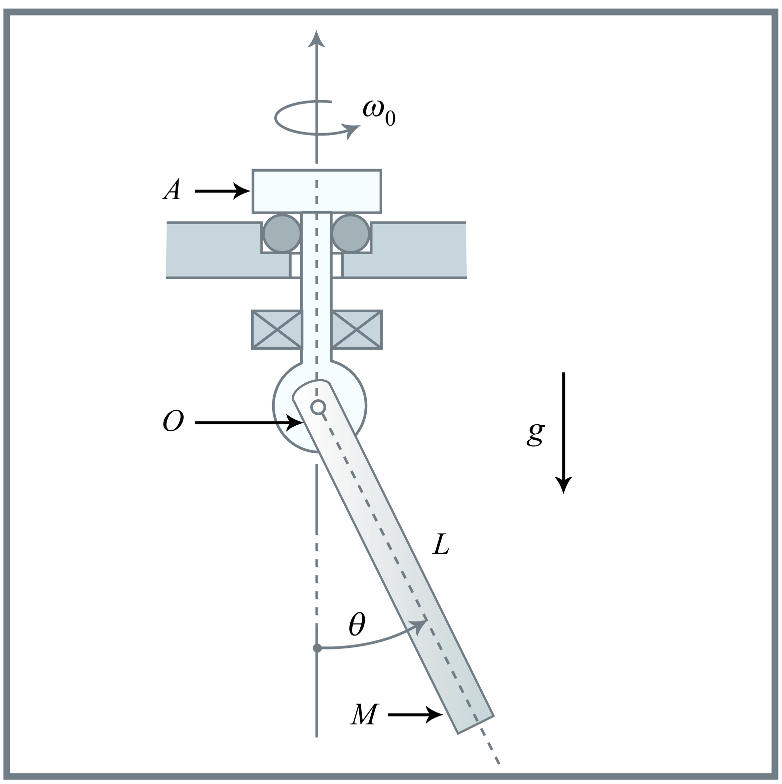


Figure by OCW. After Problem 4-14 in Crandall, S. H., et al. *Dynamics of Mechanical and Electromechanical Systems*. Malabar, FL: Krieger, 1982.