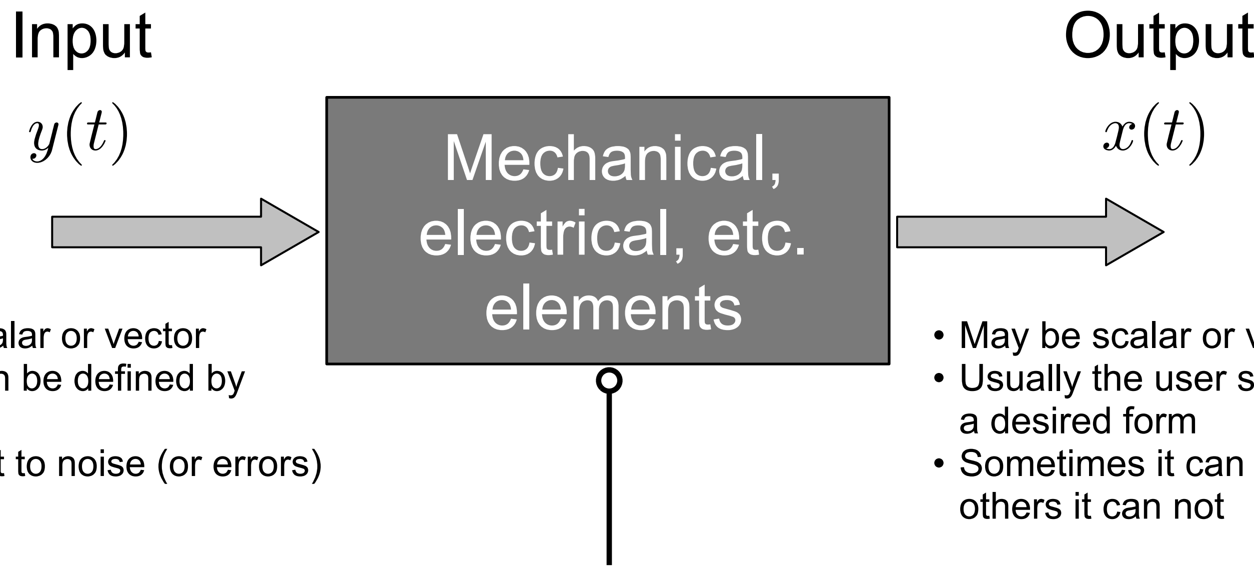
2.04A
System Dynamics and Control

Spring 2013

G. Barbastathis

System



- May be scalar or vector
- Usually can be defined by the user
- It is subject to noise (or errors)

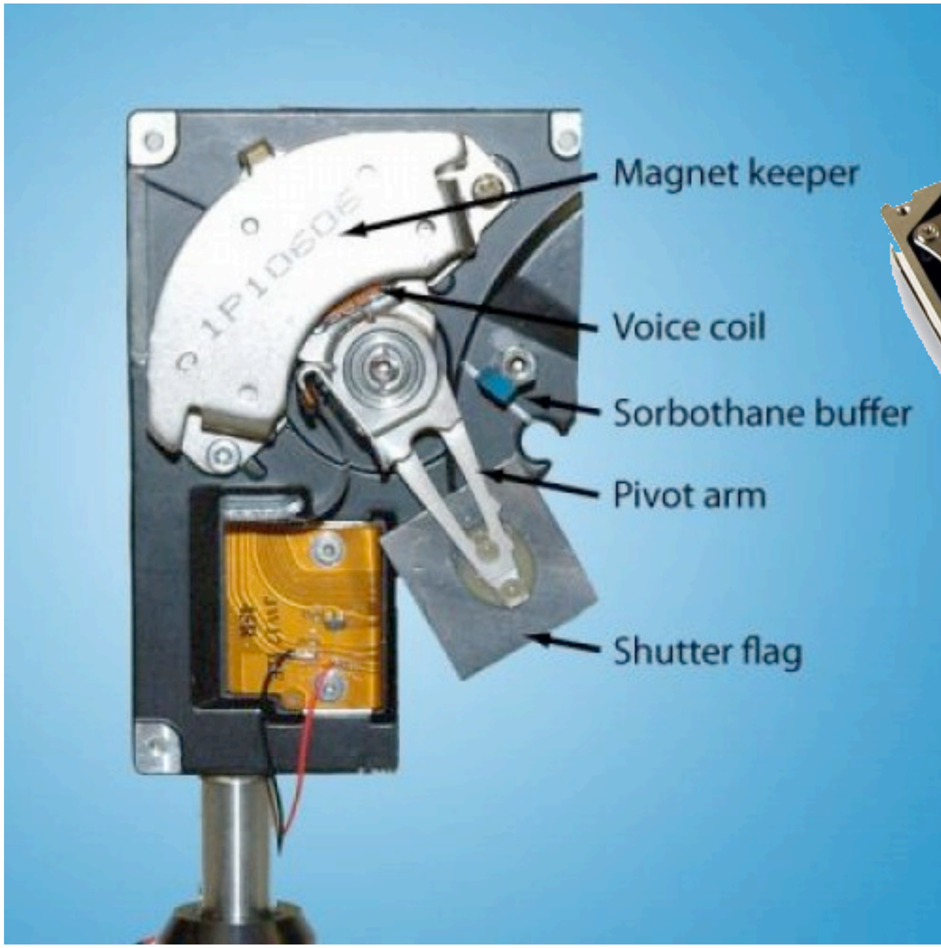
- May be scalar or vector
- Usually the user specifies a desired form
- Sometimes it can be measured, others it can not

- Described by ordinary differential equations (ODEs) operating on the input ("dynamics")
- If the ODEs operate on the output as well, then we refer to it as a *feedback* system
- Can be *linear* or *nonlinear*

➡ The purpose of "control" is to ensure that the output waveform resembles the waveform desired by the user, despite the system's dynamics and disturbances by noise

➡ Usually, control requires feedback

Example1: Hard Disk Drive (HDD)



Speed Control
Head-Disk Tracking

Hard Disk Drives

Example 2: The Segway

http://www.segway.com/

Speed Control
Stability Control

Example 3: Manufacturing Automation

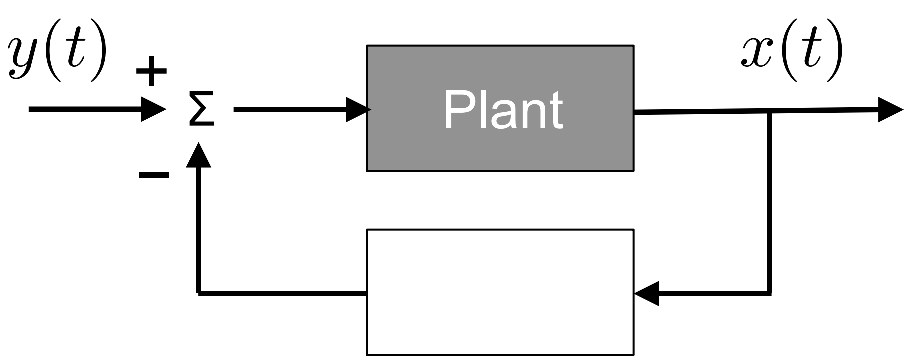
http://www.youtube.com/watch?v=iwIzPjS5L6w

System Classifications

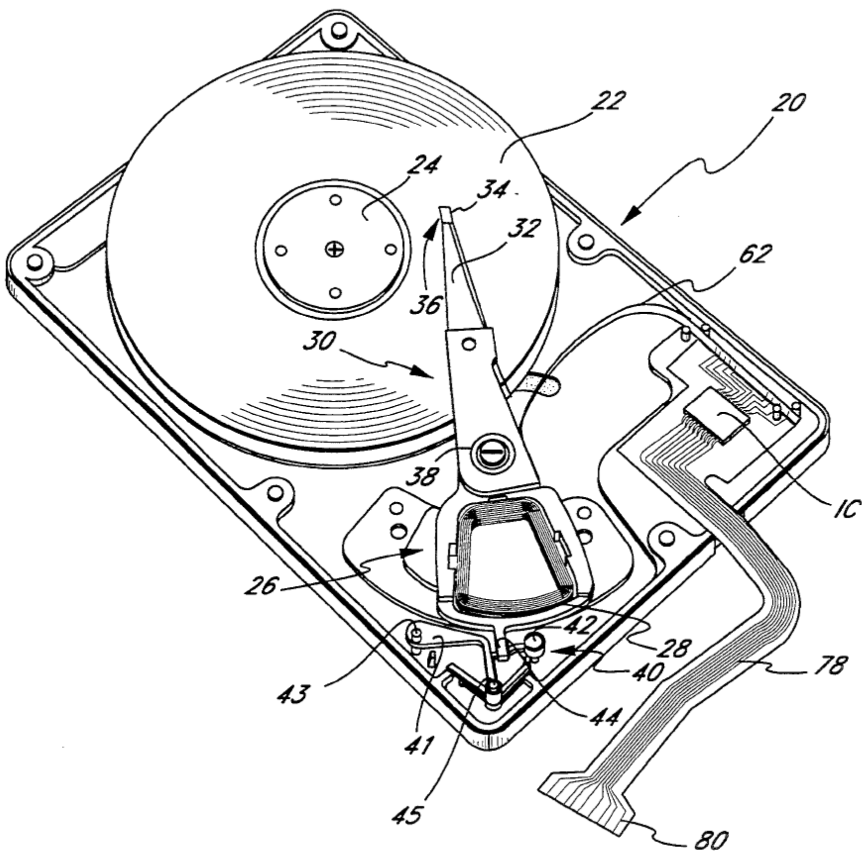
- Single vs Multiple Inputs / Single vs Multiple Outputs
  - SISO (single input - single output)
  - SIMO
  - MISO
  - MIMO
- Feed-forward *vs* feedback
- Linear vs nonlinear

Feedforward vs feedback

- Feedforward: acts without taking the output into account
  - Example: your dishwasher does not measure the cleanliness of plates during its operation
- Feedback: the output is specified by taking the input into account (somehow)



HDD Control System



Karman Tam et al, US Patent 5,412,809

Source: Figure 1 of patent US 5412809 A

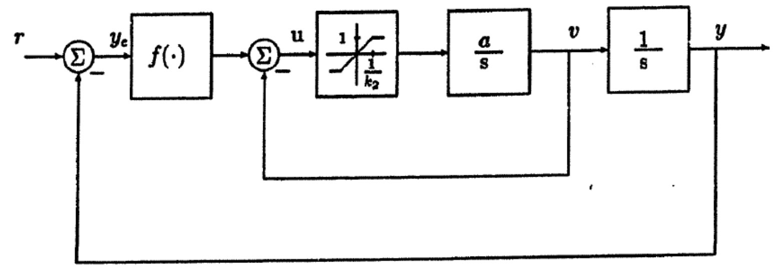
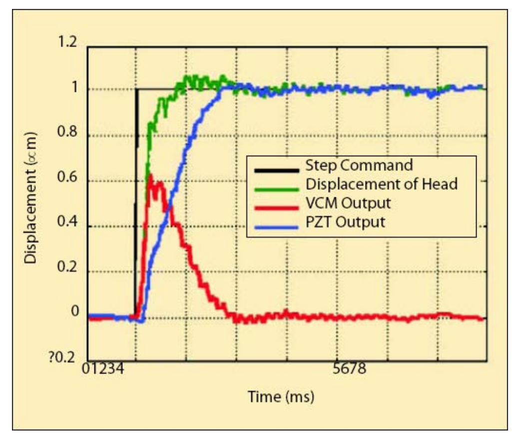


Figure 3.3: Proximate Time-Optimal Servomechansim (PTOS).

Michael L. Workman, PhD thesis, Stanford University, 1987.



2.04A Learning Objectives

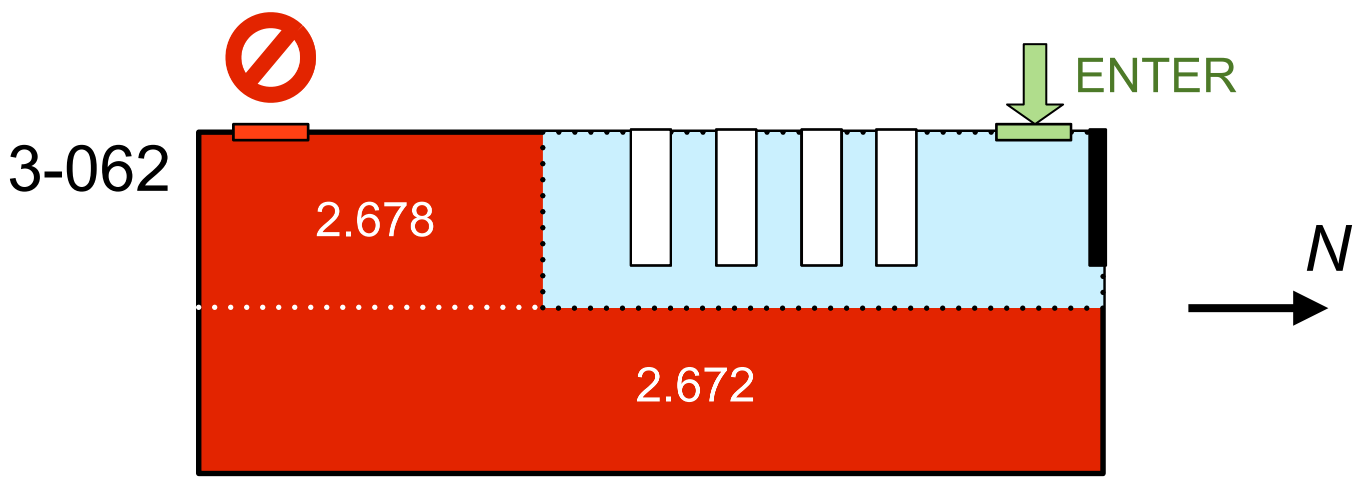
- Learn the process of modeling linear time-invariant (LTI) dynamical systems in dual domains: in the time domain using ordinary differential equations and in the Laplace domain (s-domain).
- Understand the behavior of LTI systems qualitatively and quantitatively, both in the transient and steady-state regimes, and appreciate how it impacts the performance of electro-mechanical systems.
- Introduce feedback control and understand, using the s-domain primarily, how feedback impacts transient and steady-state performance.
- Learn how to design proportional, proportional-integral, proportional-derivative, and proportional-integral-derivative feedback control systems meeting specific system performance requirements.
- Introduce qualitatively the frequency response of LTI systems and how it relates to the transient and steady-state system performance.

What you need

- 8.01 and 8.02
  - basic behavior of mechanical and electrical elements
- 18.03
  - Linear ordinary differential equations (ODEs) and systems of ODEs
  - Laplace transforms
- 2.003/2.03
  - from a physical description of system, derive the set of ODEs that describe it
- We will review these here as necessary; but please refer back to your materials from these classes, anticipating the topics that we cover

Lab Rules - IMPORTANT!!

- You must stay within the designated 2.04A/2.004 lab space
- No working on other classes (e.g. your 2.007 project) allowed in the machine shop



Lab: Equipment Overview

Please read the equipment description before before Thu PM lab!

Framework for system control

Mechanical, Electrical, Fluid, Thermal

Time Varying / Frequency Dependent } Input → **Dynamic System** → Output { Time Varying / Frequency Dependent

Modeling

- Real Physical System
  - ***Multi-Domain Ideal Elements***
  - ***Network Representations***
- Idealized Representation: Lumped Model

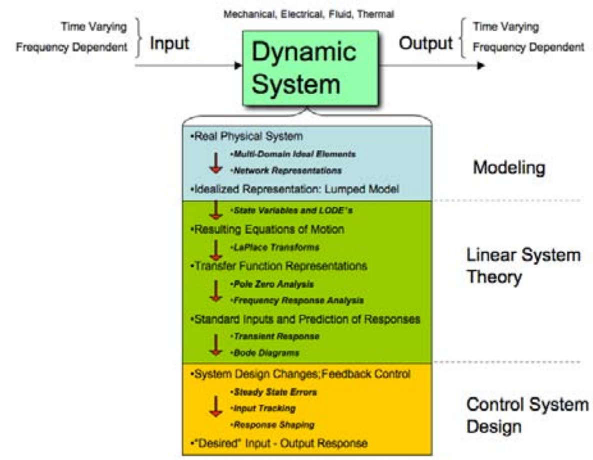
Linear System Theory

  - ***State Variables and LODE's***
- Resulting Equations of Motion
  - ***Laplace Transforms***
- Transfer Function Representations
  - ***Pole Zero Analysis***
  - ***Frequency Response Analysis***
- Standard Inputs and Prediction of Responses
  - ***Transient Response***
  - ***Bode Diagrams***

Control System Design

- System Design Changes;Feedback Control
  - ***Steady State Errors***
  - ***Input Tracking***
  - ***Response Shaping***
- "Desired" Input - Output Response

Example: Car Suspension



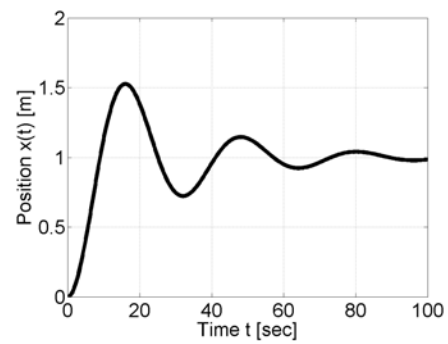
- System modeling

Hardware

Model: ordinary differential equation (ODE) or other mathematical representation

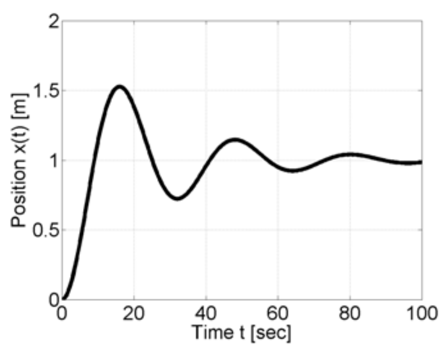
- System dynamics (as is)

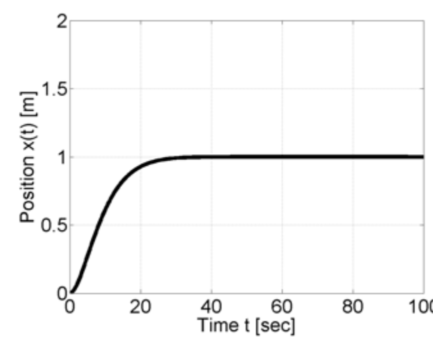
Model



Response

- System Control





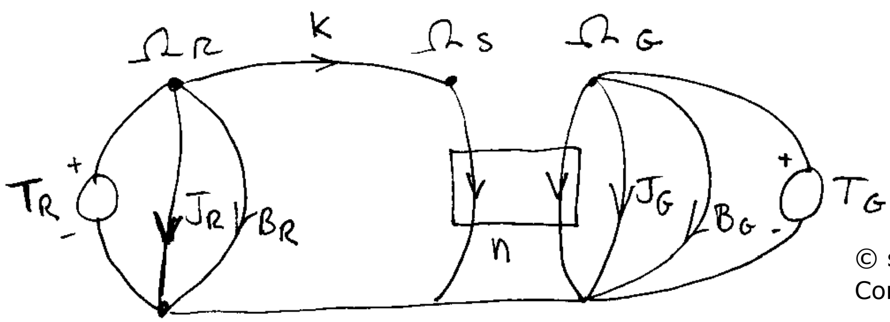
Desired response

Physical realization of systems

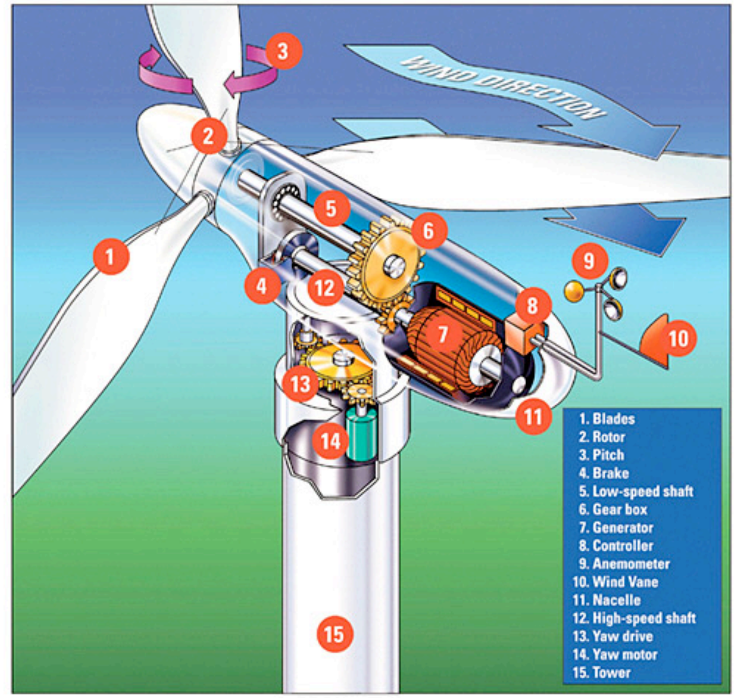
- Mechanical
- Electrical
- Fluid
- Thermal
- Electromechanical
- Mechano-fluid
- Electro-thermal
- Electromechanicalfluidthermal

Complex Interconnected Systems?

- Combine Mechanical, Electrical Fluid and Thermal
- Common Modeling Method
  - Linear, Lumped Parameter
- Circuit-Like Analysis:

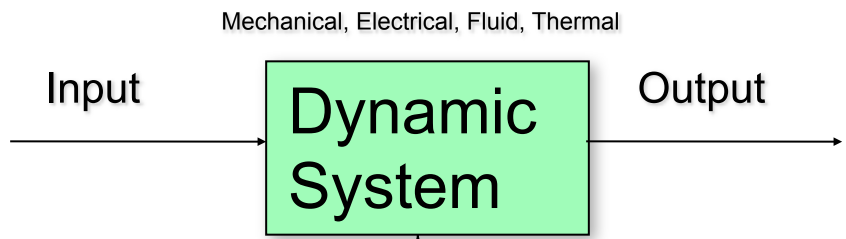


- Common Analytical Tools
  - Linear System Theory
- Powerful Design Tools
  - Feedback Control



2.12, 2.14, 2.151

A simple modeling example



- Real Physical System
  - ***Multi-Domain Ideal Elements***
  - ***Network Representations***
- Idealized Representation: Lumped Model

Modeling

  - ***State Variables and LODE's***
- Resulting Equations of Motion
  - ***LaPlace Transforms***
- Transfer Function Representations
  - ***Pole Zero Analysis***
  - ***Frequency Response Analysis***
- Standard Inputs and Prediction of Responses
  - ***Transient Response***
  - ***Bode Diagrams***

Linear System Theory

- System Design Changes;Feedback Control
  - ***Steady State Errors***
  - ***Input Tracking***
  - ***Response Shaping***
- "Desired" Input - Output Response

Control System Design

Linear Systems

- Suppose

- The system is linear iff

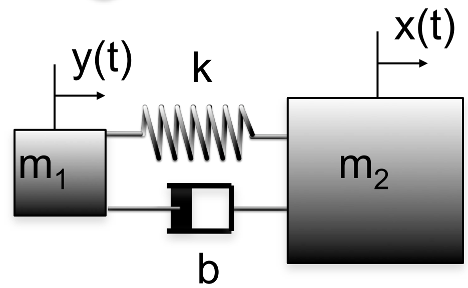
$a, b:$ constant scalar or tensors

- Corollary

and

Modeling

Hardware



Idealized Lumped Parameter Model

mathematical representation

$$\ddot{x} + a_2\dot{x} + a_1 x = b_2\dot{y} + b_1 y$$

ODE

or

$$\begin{vmatrix} \dot{x}_1 \\ \dot{x}_2 \end{vmatrix} = \begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix} \begin{vmatrix} x_1 \\ x_2 \end{vmatrix} + \begin{vmatrix} b_1 \\ b_2 \end{vmatrix} y(t)$$

State Space

or

$$\frac{X(s)}{Y(s)} = \frac{2\zeta\omega_n s + \omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2}$$

time domain
➡ equation(s) of motion

Laplace domain
➡ transfer function