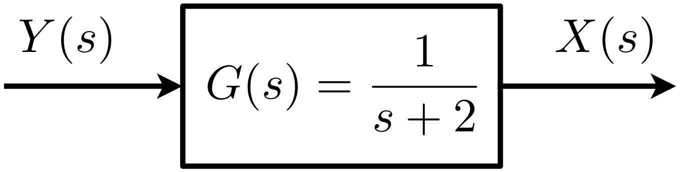
Today's goal

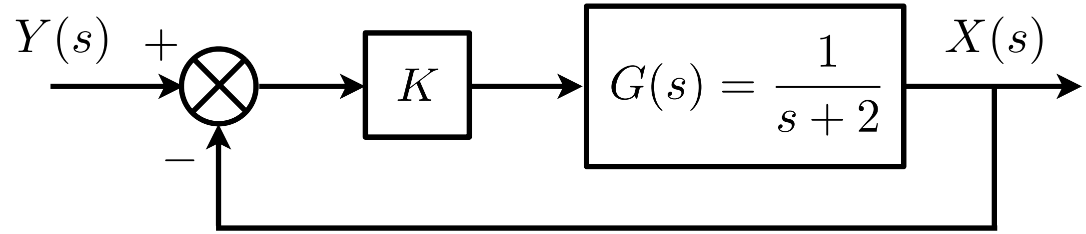
- Root Locus examples and how to apply the rules
  - single pole
  - single pole with one zero
  - two real poles
  - two real poles with one zero
  - three real poles
  - three real poles with one zero

- Extracting useful information from the Root Locus
  - transient response parameters
  - limit gain for stability

Root Locus definition

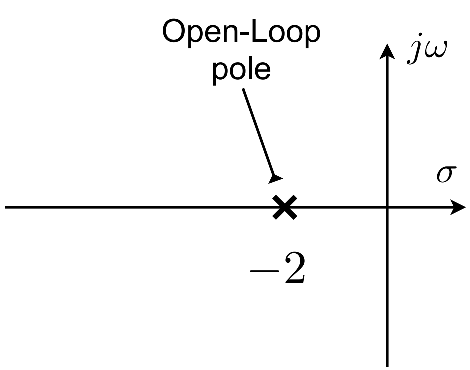
- Root Locus is the locus on the complex plane of **closed**-loop poles as the feedback gain is varied from 0 to ∞.

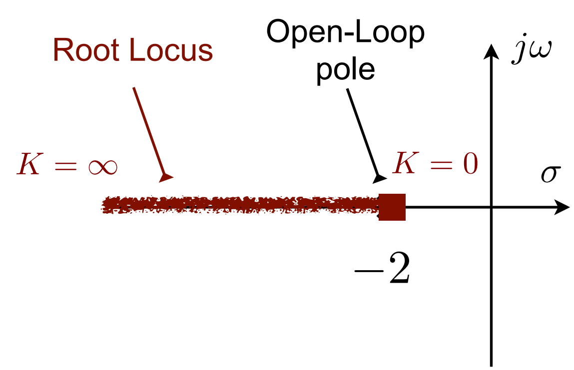




$$\left(\frac{X(s)}{Y(s)}\right)_{\mathrm{OL}} = \frac{1}{s+2}$$

$$\left(\frac{X(s)}{Y(s)}\right)_{\mathrm{CL}} = \frac{K}{s+2+K}$$

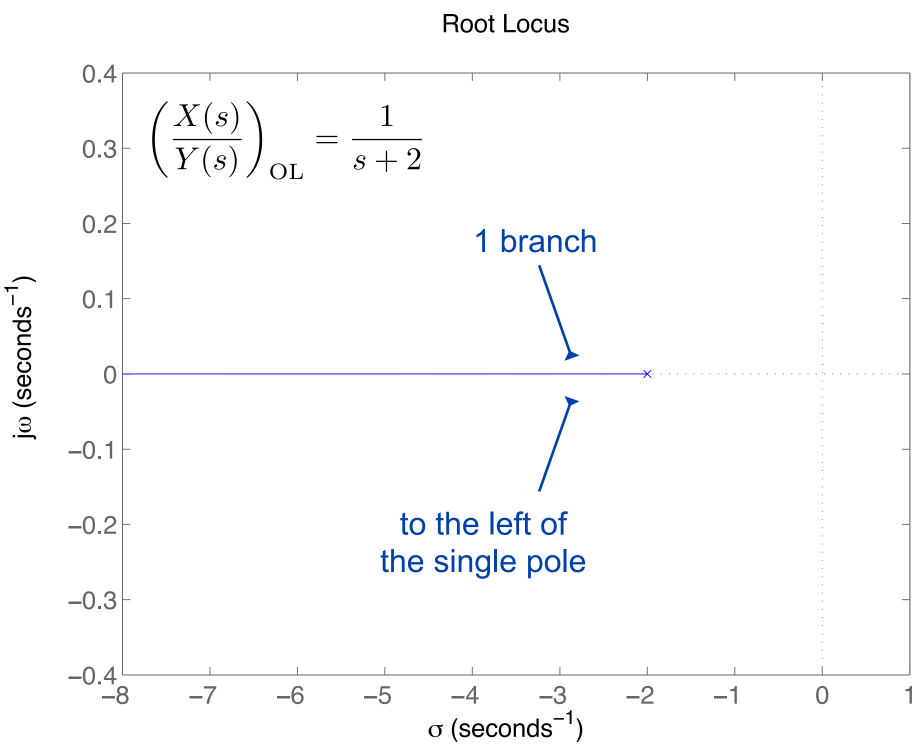


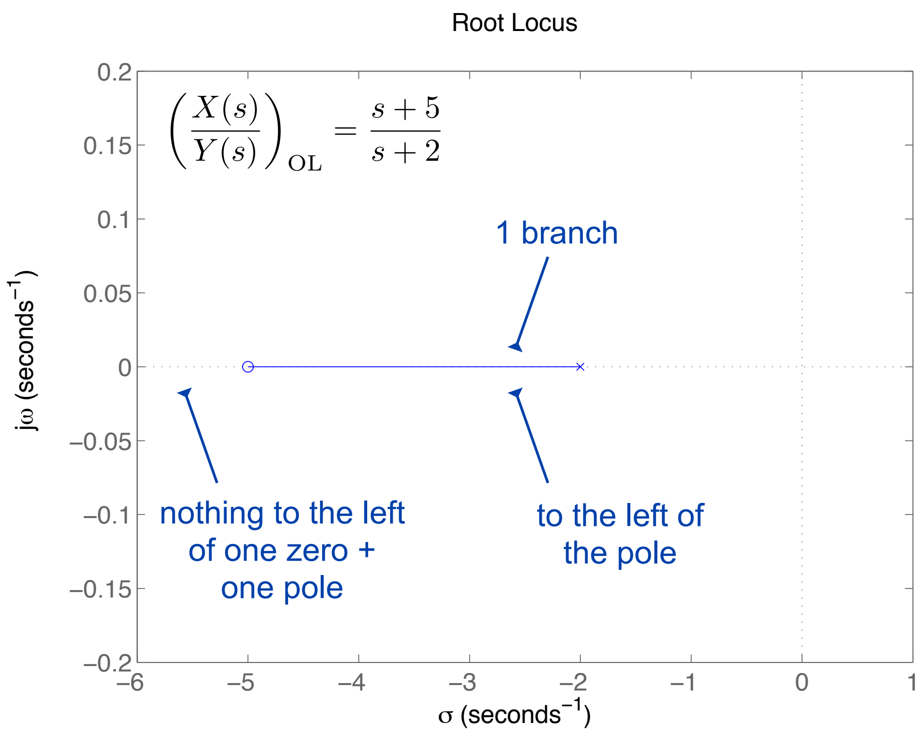


As K varies from 0 to ∞ ...

Root-locus sketching rules

- **Rule 1:** # branches = # poles
- **Rule 2:** always symmetric with respect to the real axis
- **Rule 3:** real-axis segments are to the left of an *odd* number of real-axis finite poles/zeros





Root-locus sketching rules

- **Rule 4:** begins at poles, ends at zeros

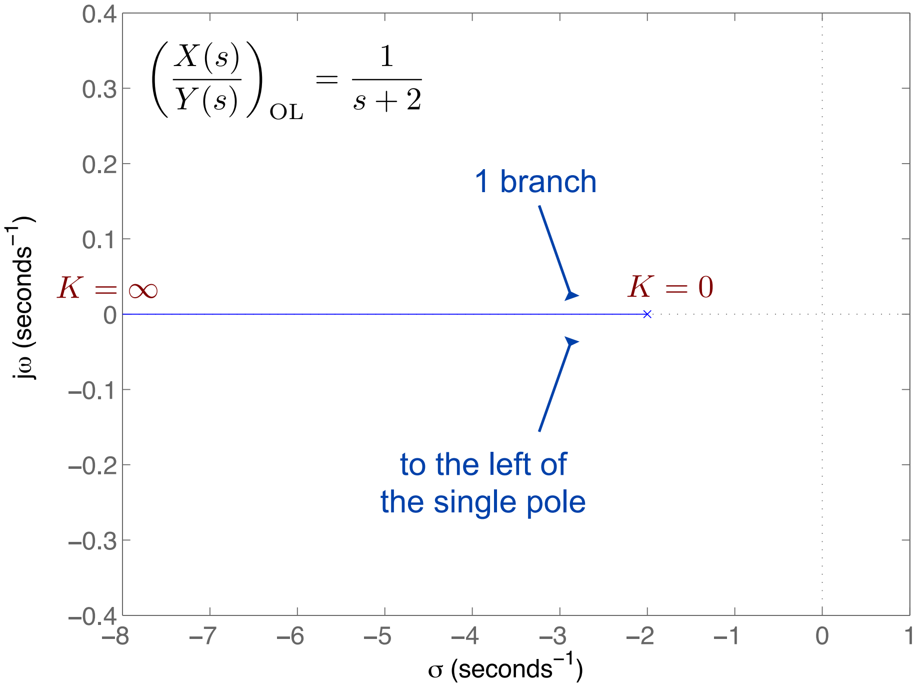
$$\left(\frac{X(s)}{Y(s)}\right)_{\mathrm{CL}} = \frac{K}{s + (K+2)}$$

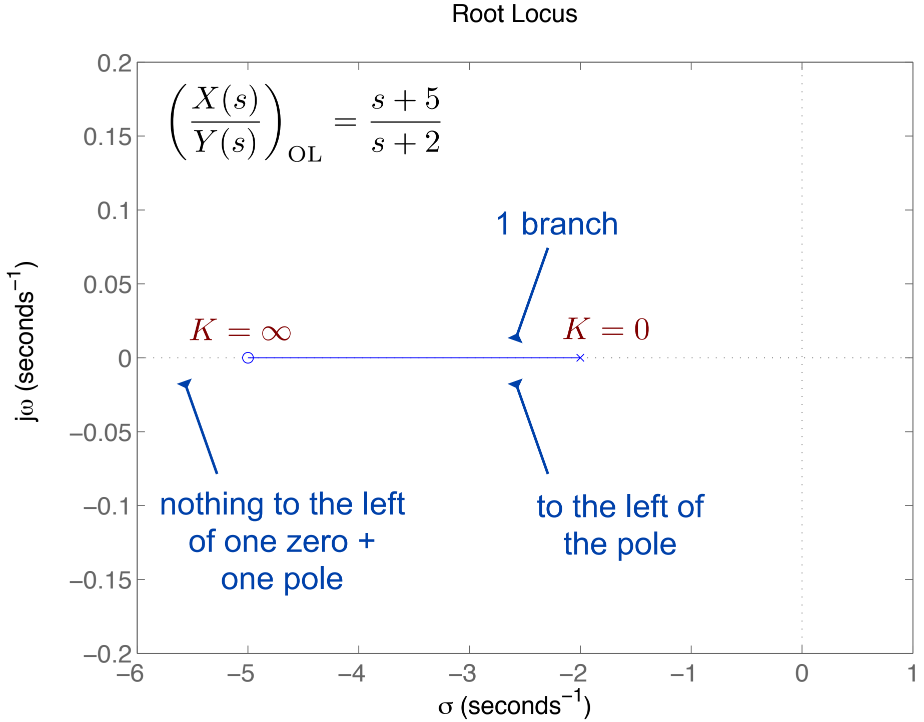
$$\begin{pmatrix}\text{closed–loop}\\ \text{pole}\end{pmatrix} = -(K+2) \to -\infty, \quad \text{as} \quad K \to \infty$$

We say that this TF has a "**zero at infinity**"

$$\left(\frac{X(s)}{Y(s)}\right)_{\mathrm{CL}} = \frac{K(s+5)}{(K+1)\,s + (5K+2)}$$

$$\begin{pmatrix}\text{closed–loop}\\ \text{pole}\end{pmatrix} = -\frac{5K+2}{K+1} \to -5, \quad \text{as} \quad K \to \infty$$



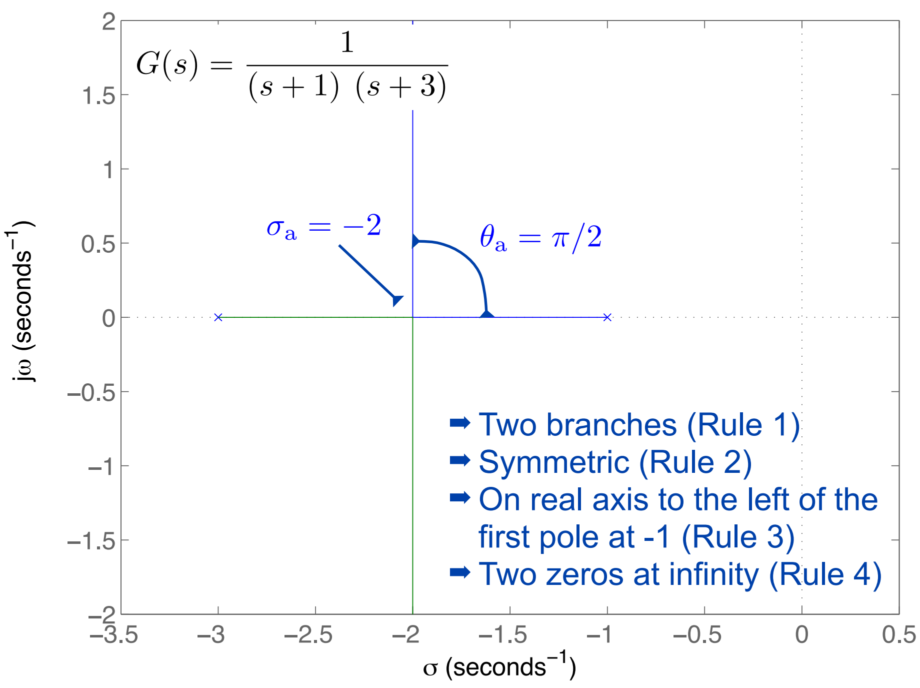


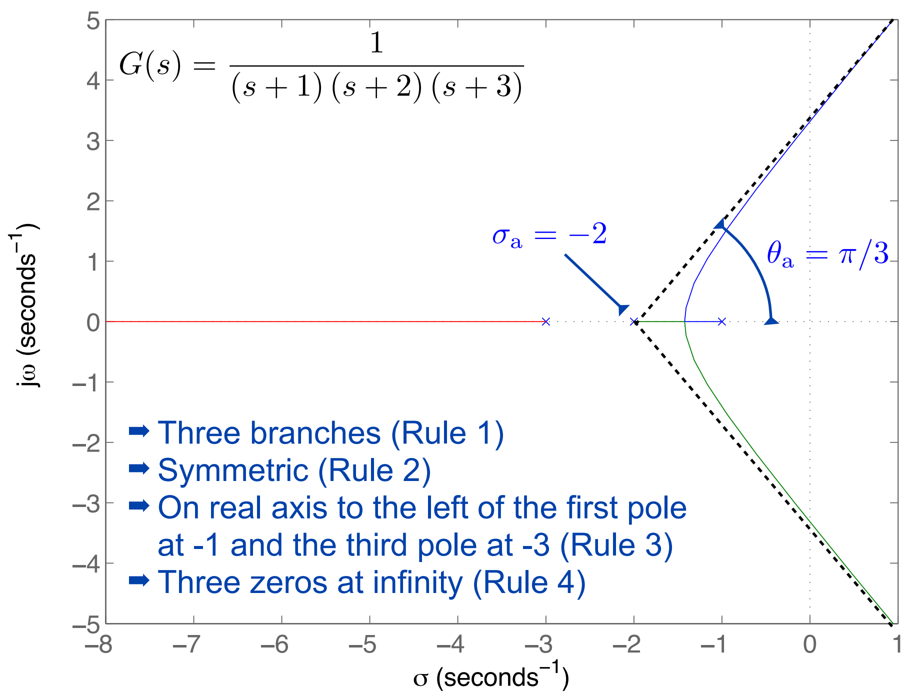
Root Locus sketching rules

- **Rule 5:** Real-axis intercept and angle of asymptote

$$\sigma_a = \frac{\sum \text{finite poles} - \sum \text{finite zeros}}{\#\text{finite poles} - \#\sum \text{finite zeros}}$$

$$\theta_a = \frac{(2m+1)\,\pi}{\#\text{finite poles} - \#\sum \text{finite zeros}}$$

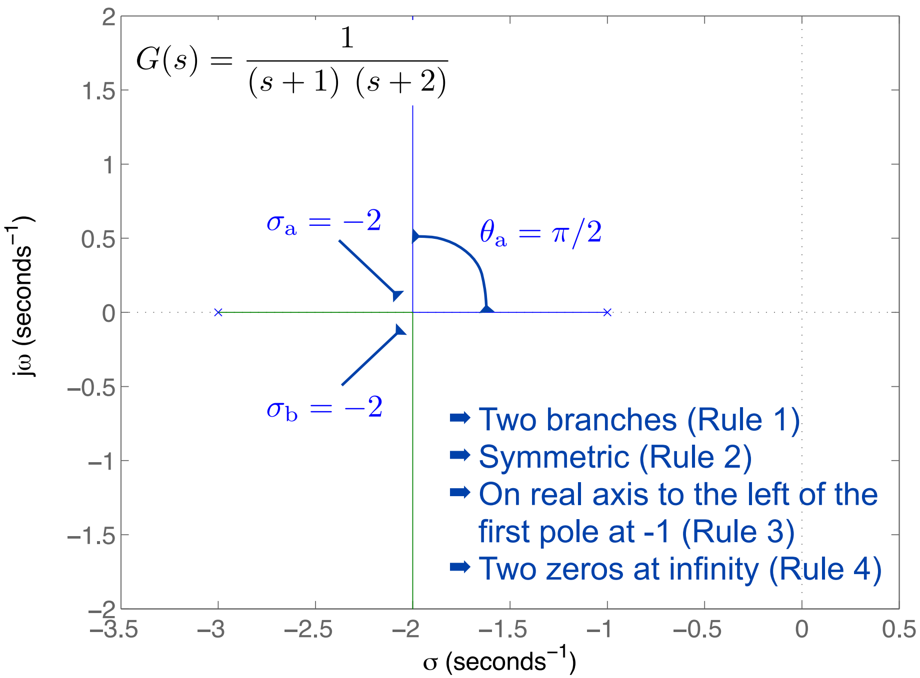




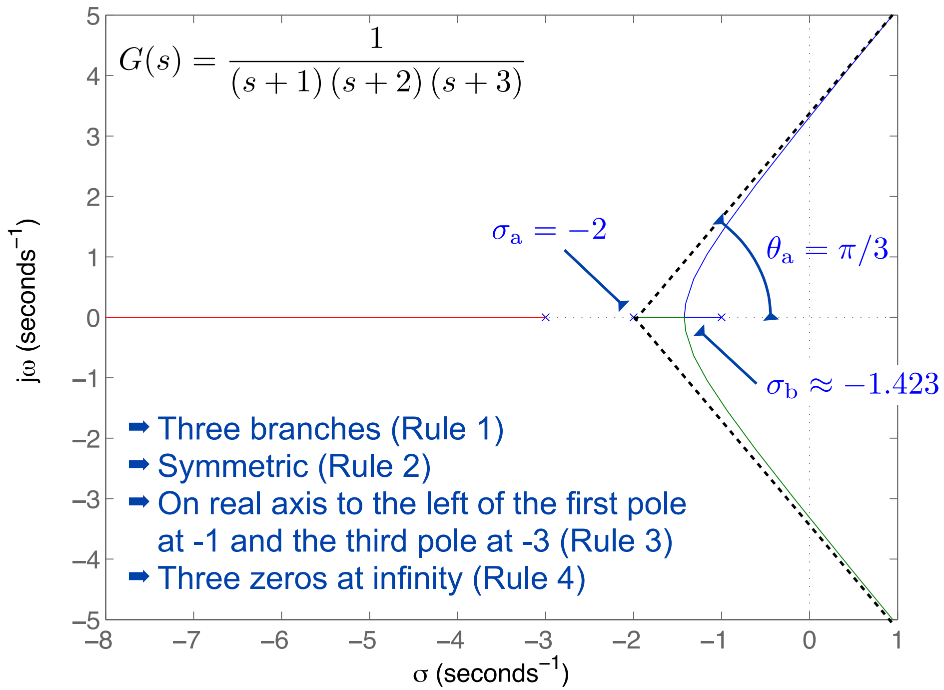
Root Locus sketching rules

- **Rule 6:** Real axis breakaway and break-in points σb

$$\text{Solve} \quad \sum_n \frac{1}{\sigma_b - z_n} = \sum_q \frac{1}{\sigma_b - p_q}$$



➡ Two branches (Rule 1)
➡ Symmetric (Rule 2)
➡ On real axis to the left of the first pole at -1 (Rule 3)
➡ Two zeros at infinity (Rule 4)

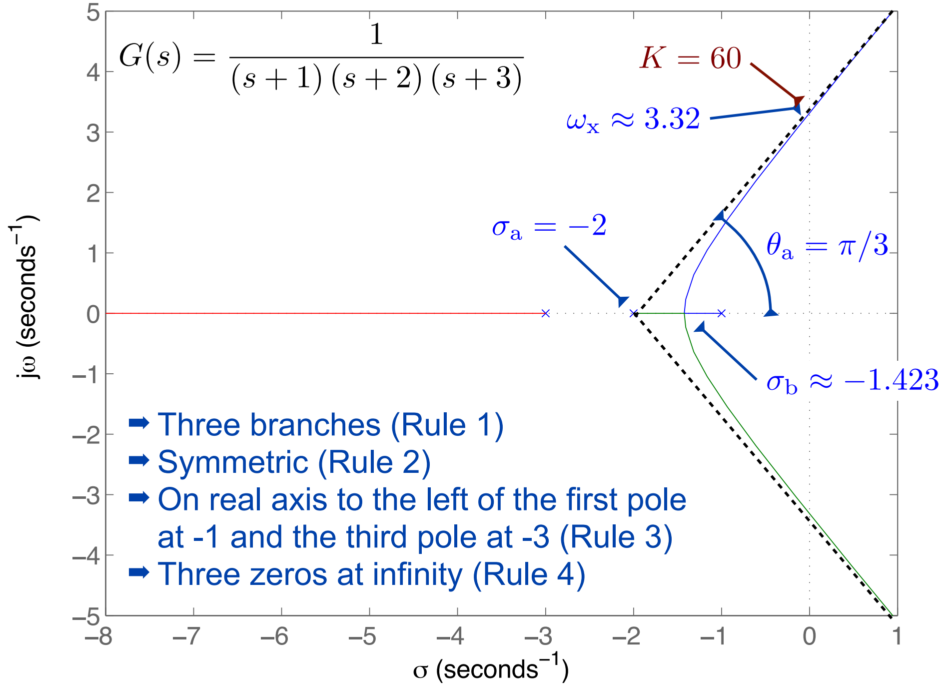


➡ Three branches (Rule 1)
➡ Symmetric (Rule 2)
➡ On real axis to the left of the first pole at -1 and the third pole at -3 (Rule 3)
➡ Three zeros at infinity (Rule 4)

Root Locus sketching rules

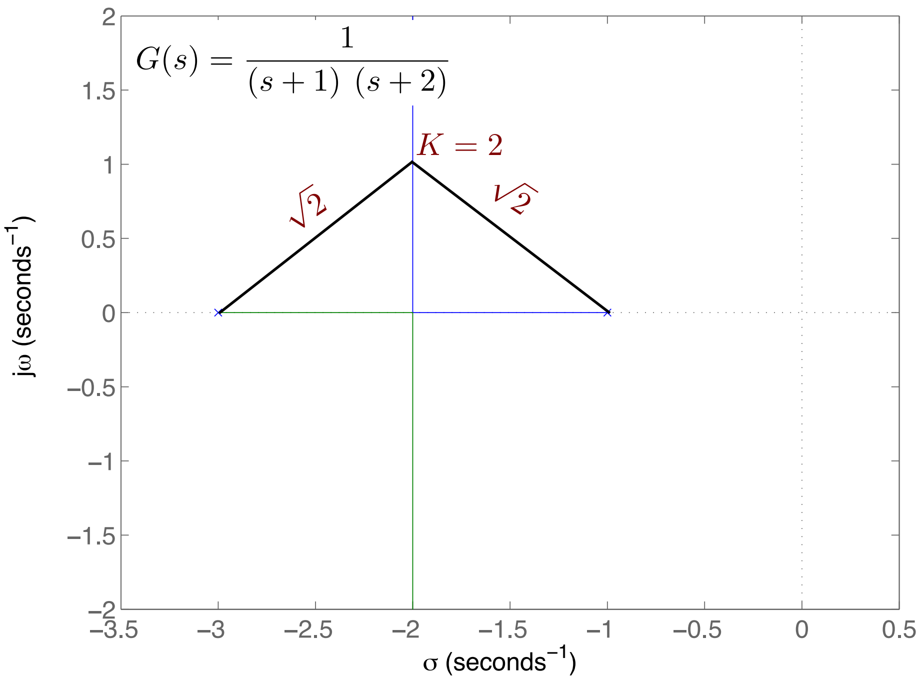
- **Rule 7:** Imaginary axis crossings

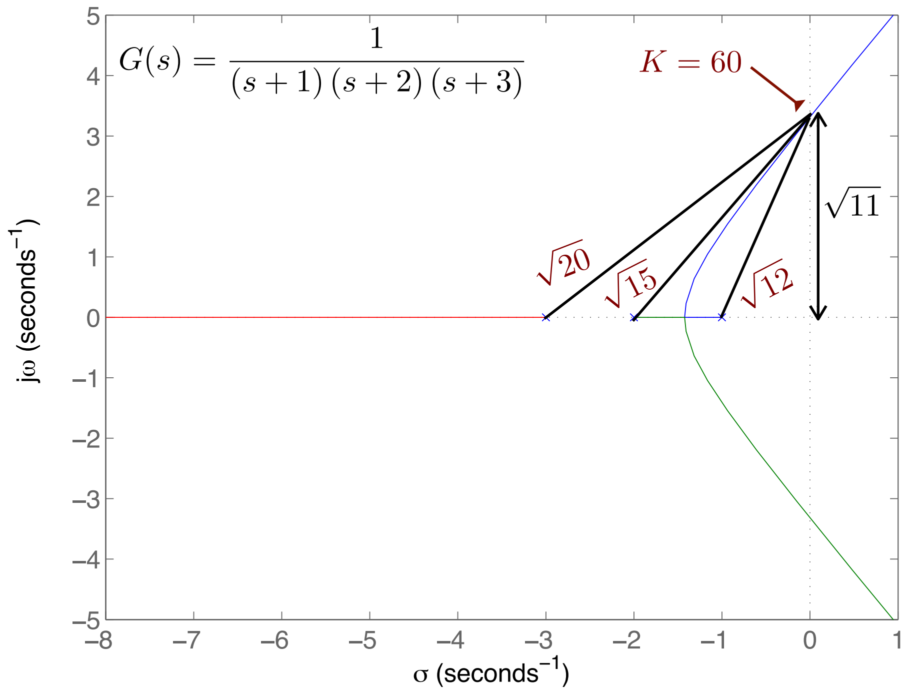
$$\text{Solve} \quad KG\left(j\omega_{\mathrm{x}}\right) = -1$$



What else is the Root Locus telling us

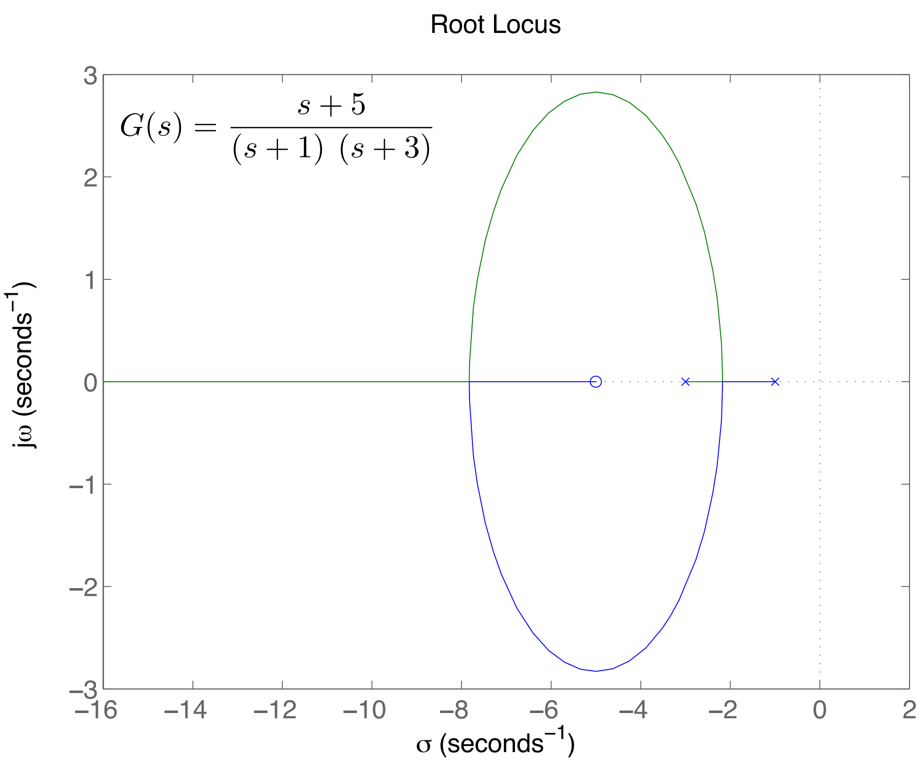
- Gain = product of distances to the poles

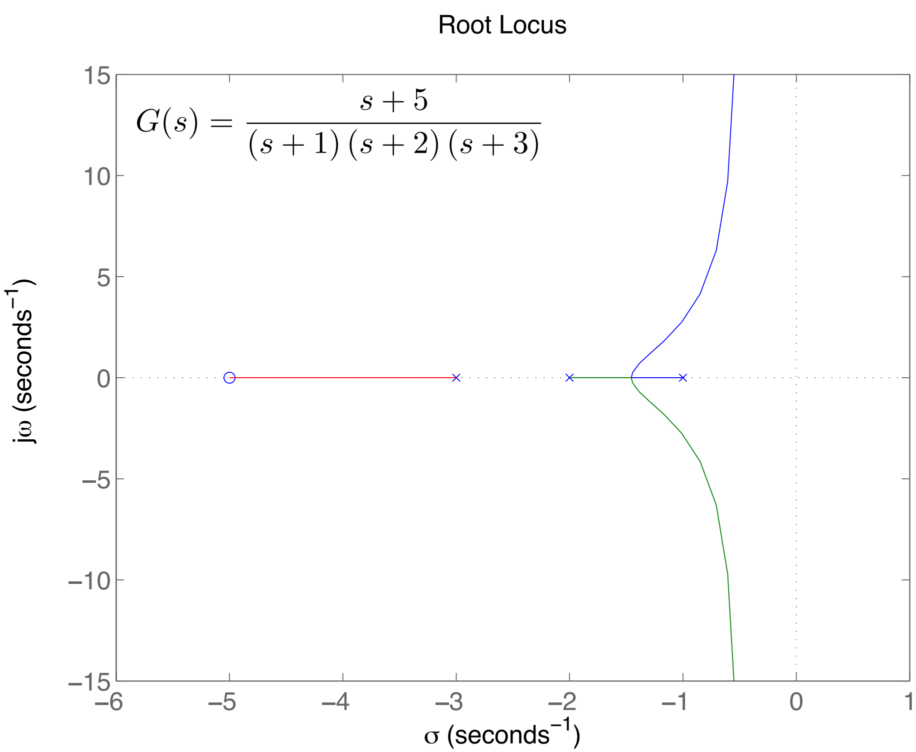




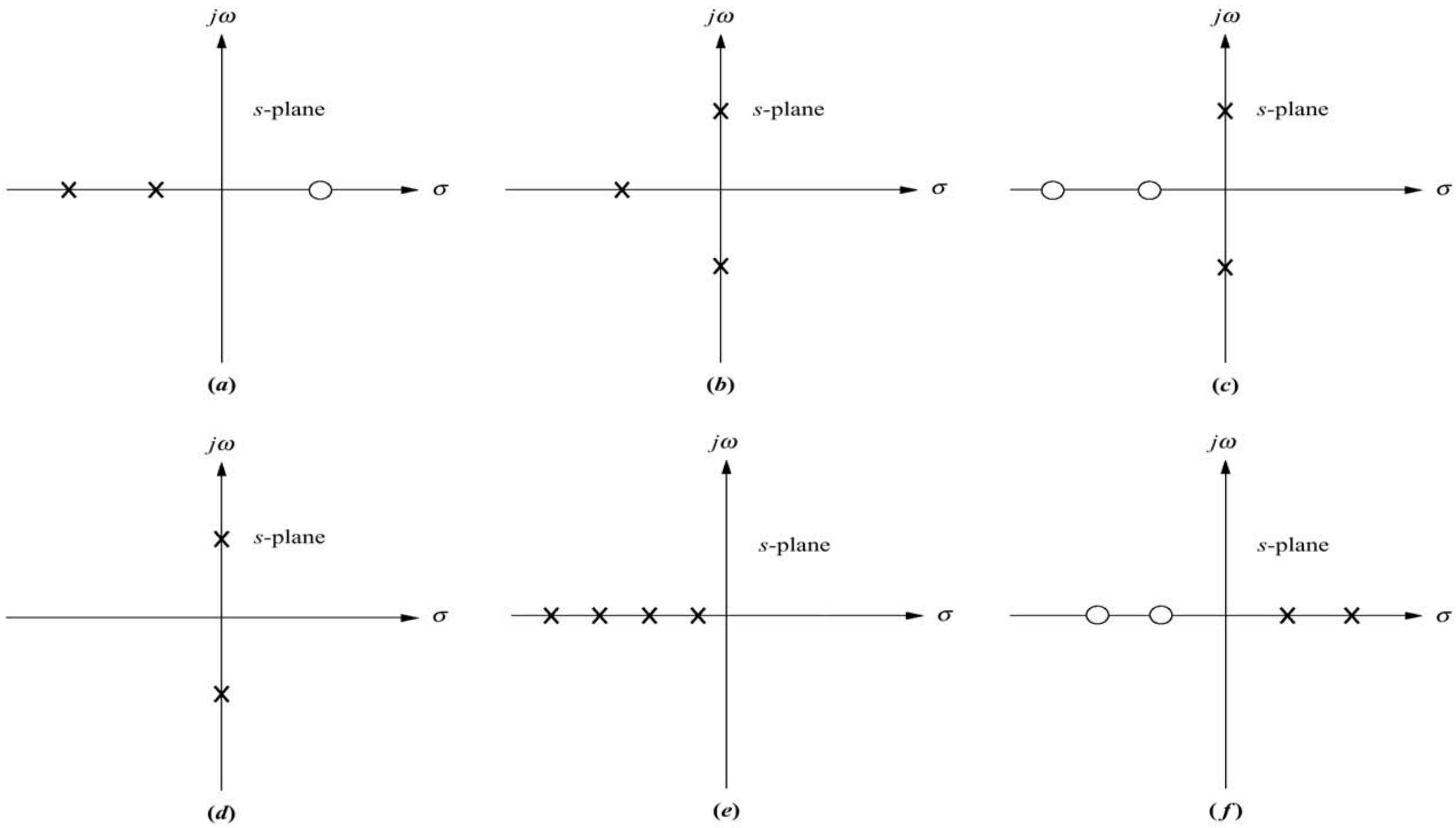
The zeros are "pulling" the Root Locus

- Because of Rule 4
- Therefore, adding a zero makes the response
  - faster
  - stable



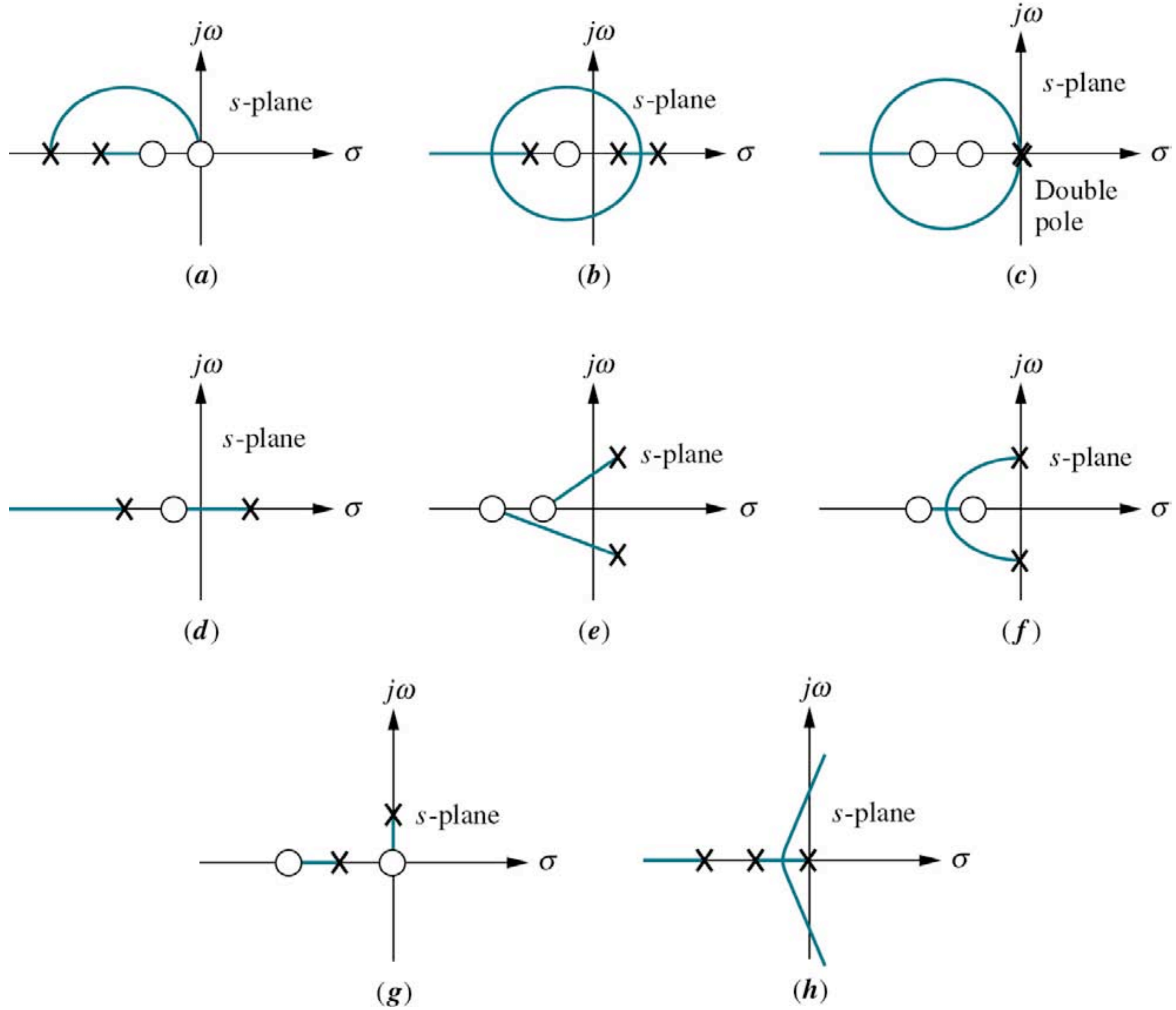


Practice 1: Sketch the Root Locus



Nise Figure P8.2

Practice 2:
Are these Root Loci valid? If not, correct them



Nise Figure P8.1

2.04A Systems and Controls
Spring 2013