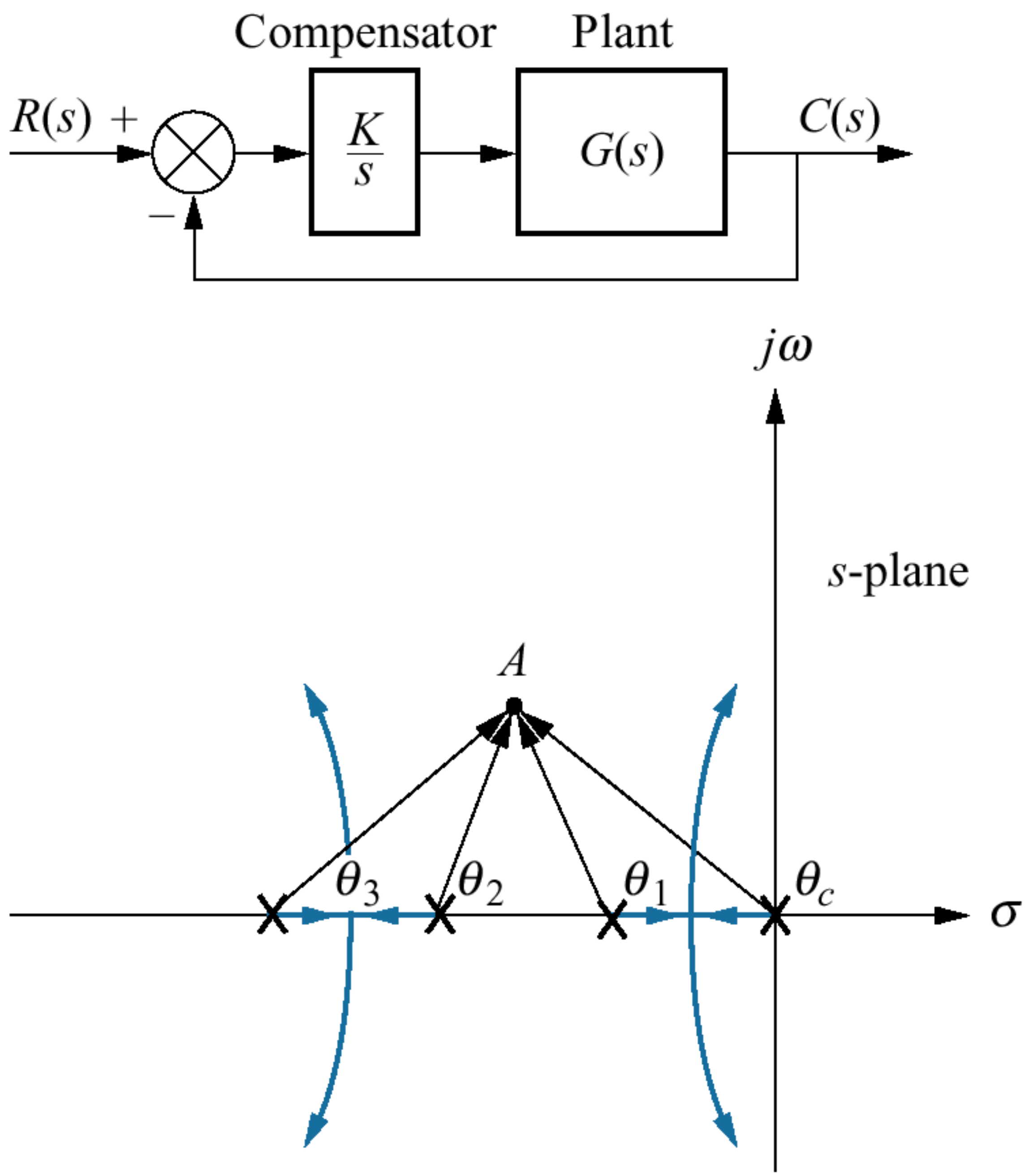
Improving the steady-state error: simple integrator



$$-\theta_1 - \theta_2 - \theta_3 - \theta_c \neq (2k+1)180°$$

(*b*)

Nise Figure 9.3

<u>Integrator as a Compensator:</u>

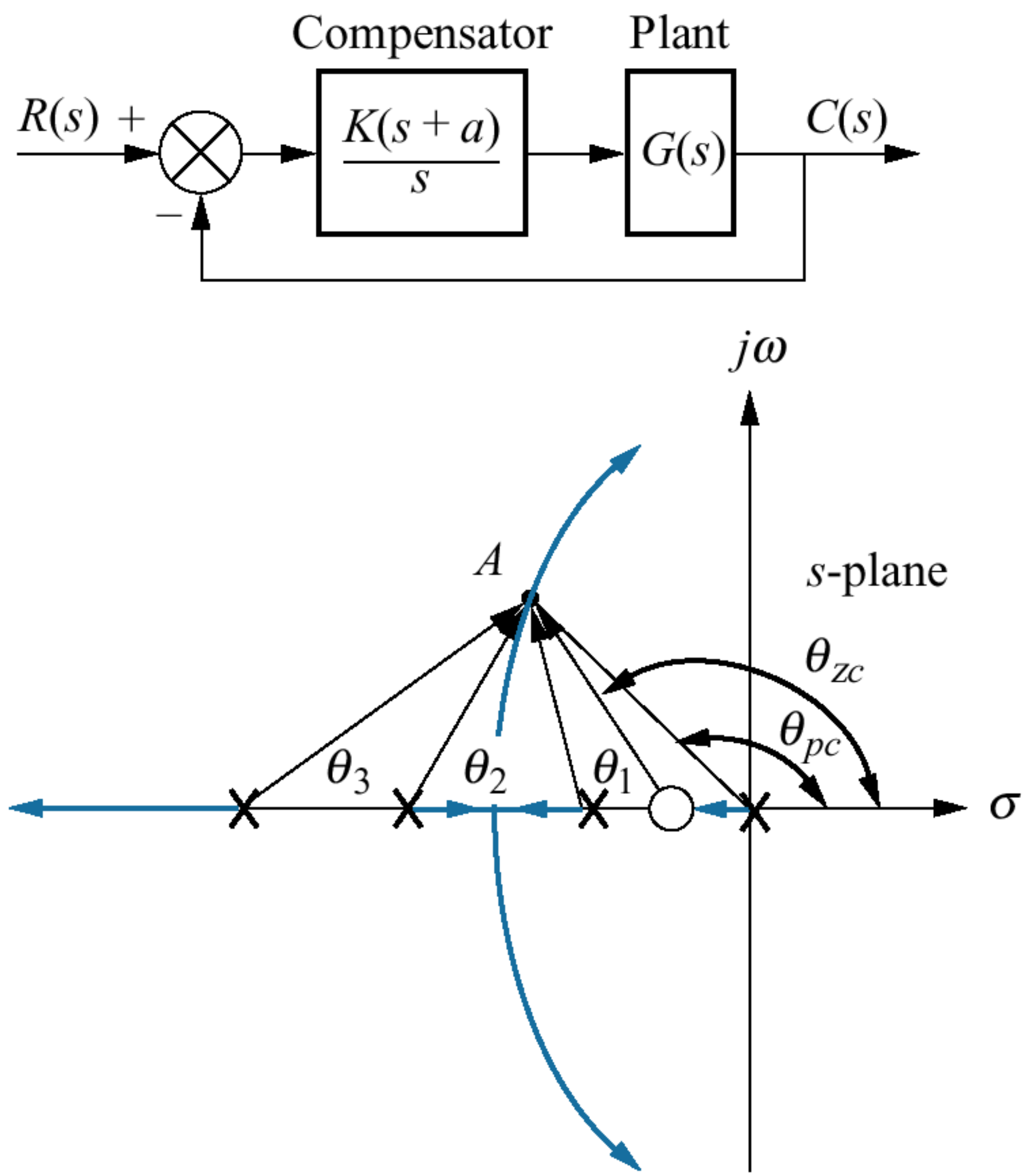
Eliminates the steady-state error, since it increases the system Type;

however, our desirable closed-loop pole A is no longer on the root locus;

this is because the new pole at *s*=0 changes the total angular contributions to A so that the 180° condition is no longer satisfied.

This means that our desirable transient response characteristics that would have been guaranteed by A are no longer available ☹

Improving the steady-state error: PI controller



$$-\theta_1 - \theta_2 - \theta_3 - \theta_{pc} + \theta_{zc} \cong (2k+1)180°$$

(c)

Nise Figure 9.3

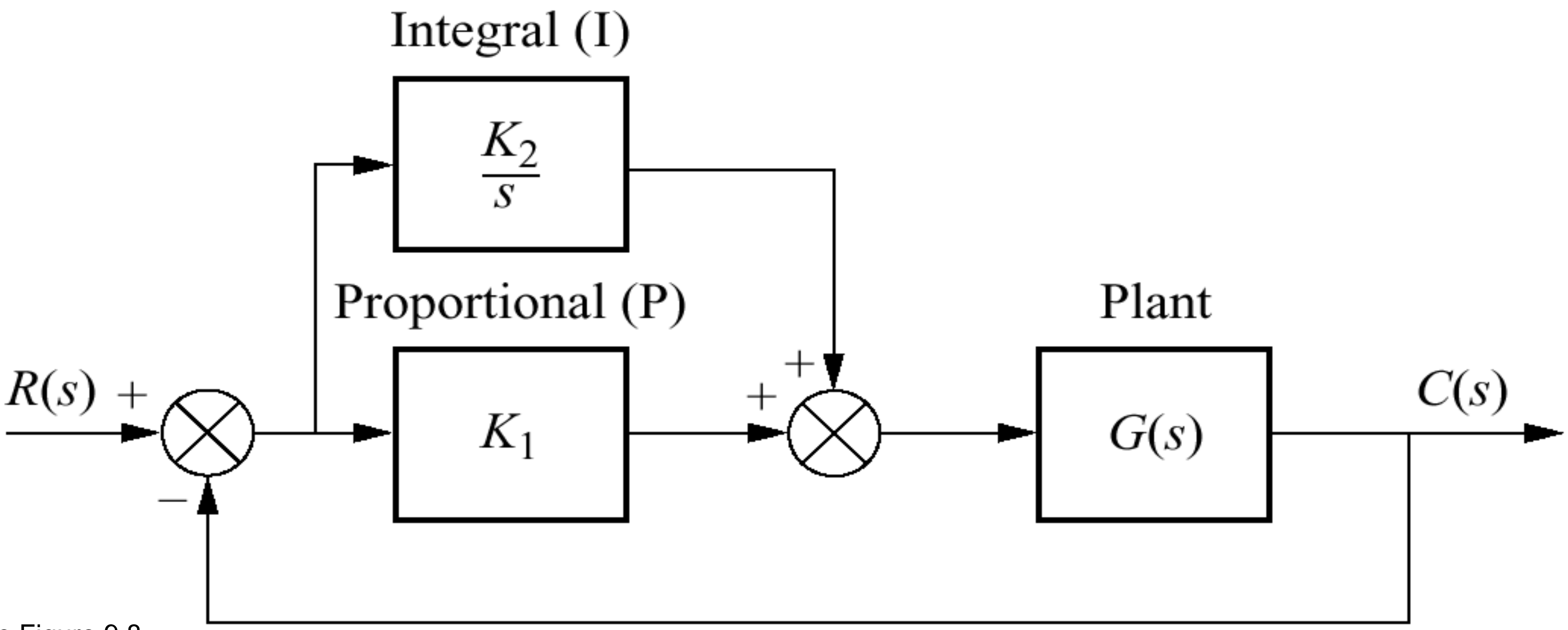
<u>Ideal Integral Compensator</u>
(or <u>Proportional-Integral Compensator</u>):

Includes a zero on the negative real axis but close to the integrator's pole at the origin. The zero

- has approximately the same angular contribution to A as the integrator's pole at the origin; therefore, the two cancel out;
- moreover, it contributes the same magnitude to the pole at A, so A is reached with the same feedback gain K.

The net effect is that *we have fixed the steady-state error without affecting the transient response* ☺

Implementing the PI controller



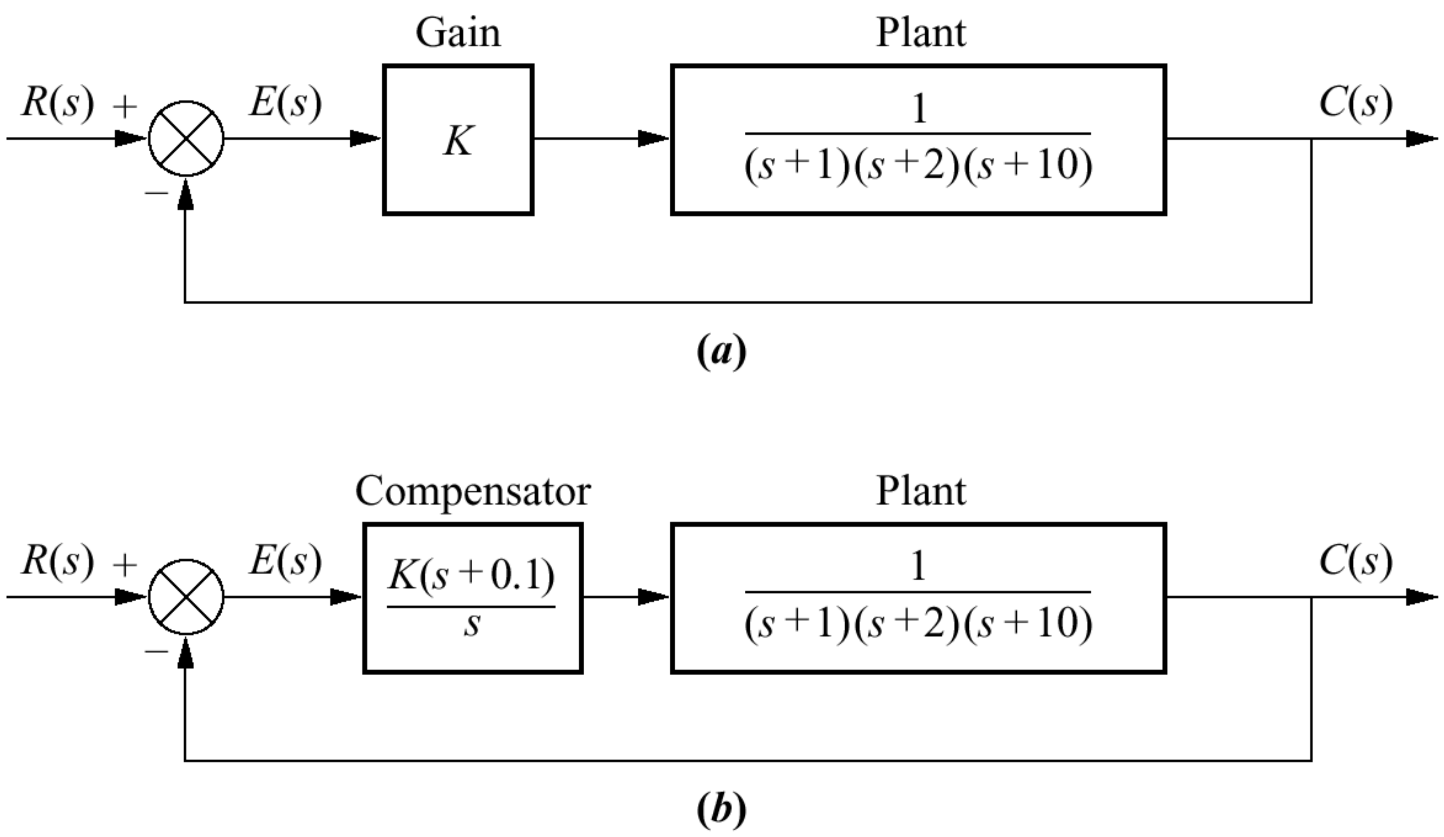
Nise Figure 9.8

$$\text{PI compensator TF:} \qquad K_1 + \frac{K_2}{s} = \frac{K_1\left(s + \frac{K_2}{K_1}\right)}{s} = \frac{K(s+z)}{s}$$

$$\text{where} \qquad K \equiv K_1, \qquad z \equiv \frac{K_2}{K_1}$$

Another implementation is the "lag compensator," which you can learn about in more advanced classes (e.g., 2.14.)

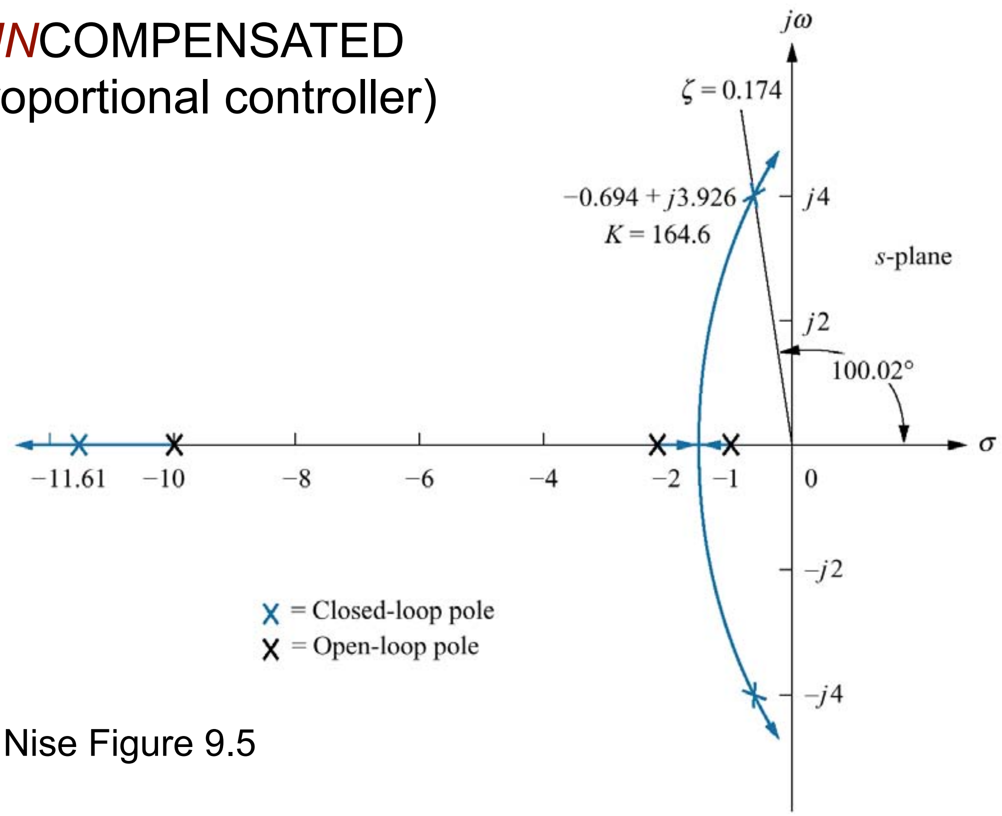
Example (Nise 9.1)



Nise Figure 9.4

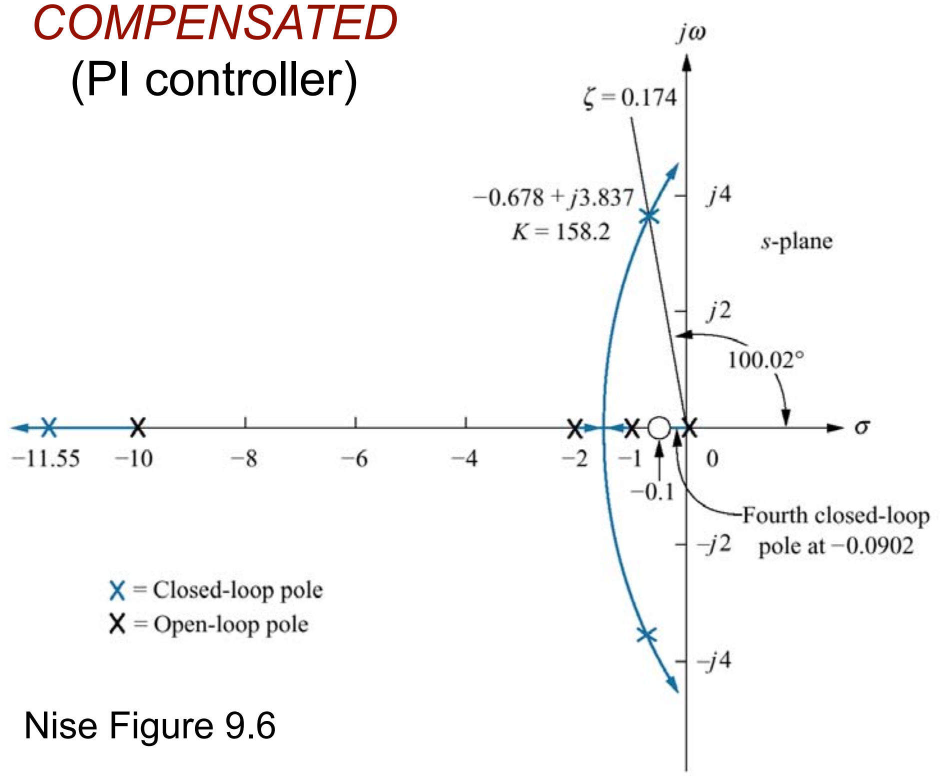
Steady-state and transients with the PI controller

*UN*COMPENSATED
(Proportional controller)

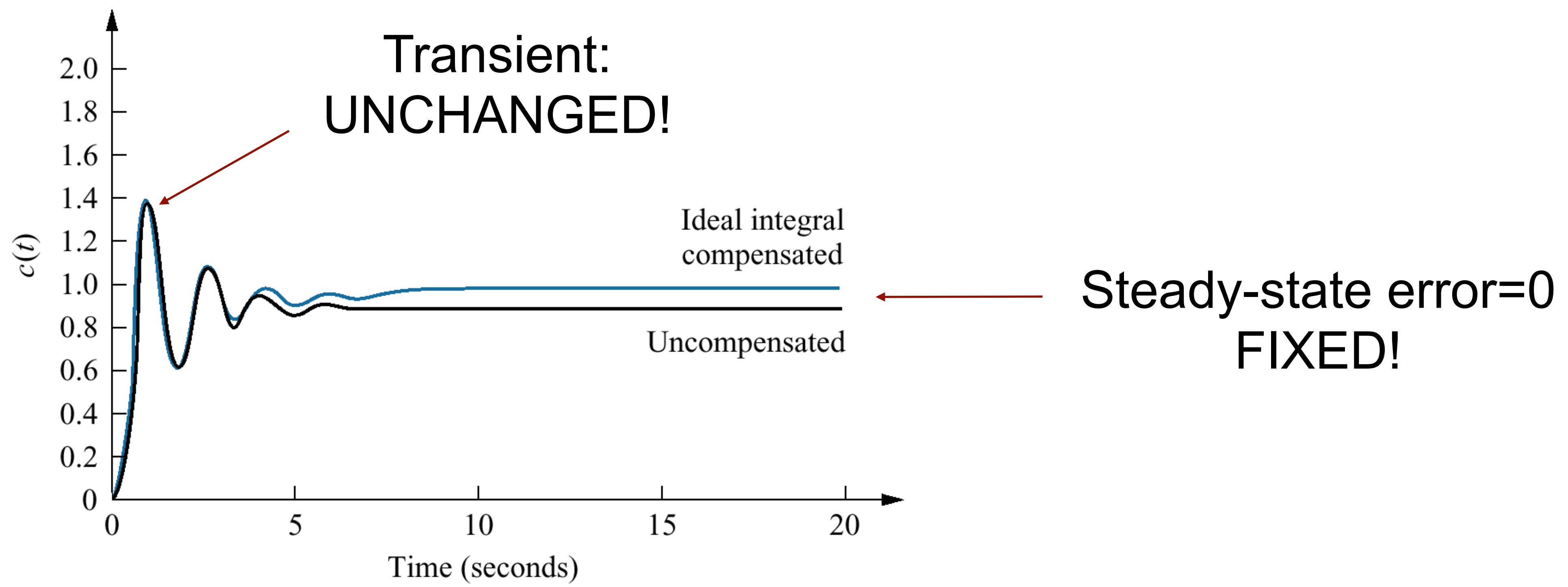


Nise Figure 9.5

COMPENSATED
(PI controller)



Nise Figure 9.6



Nise Figure 9.7

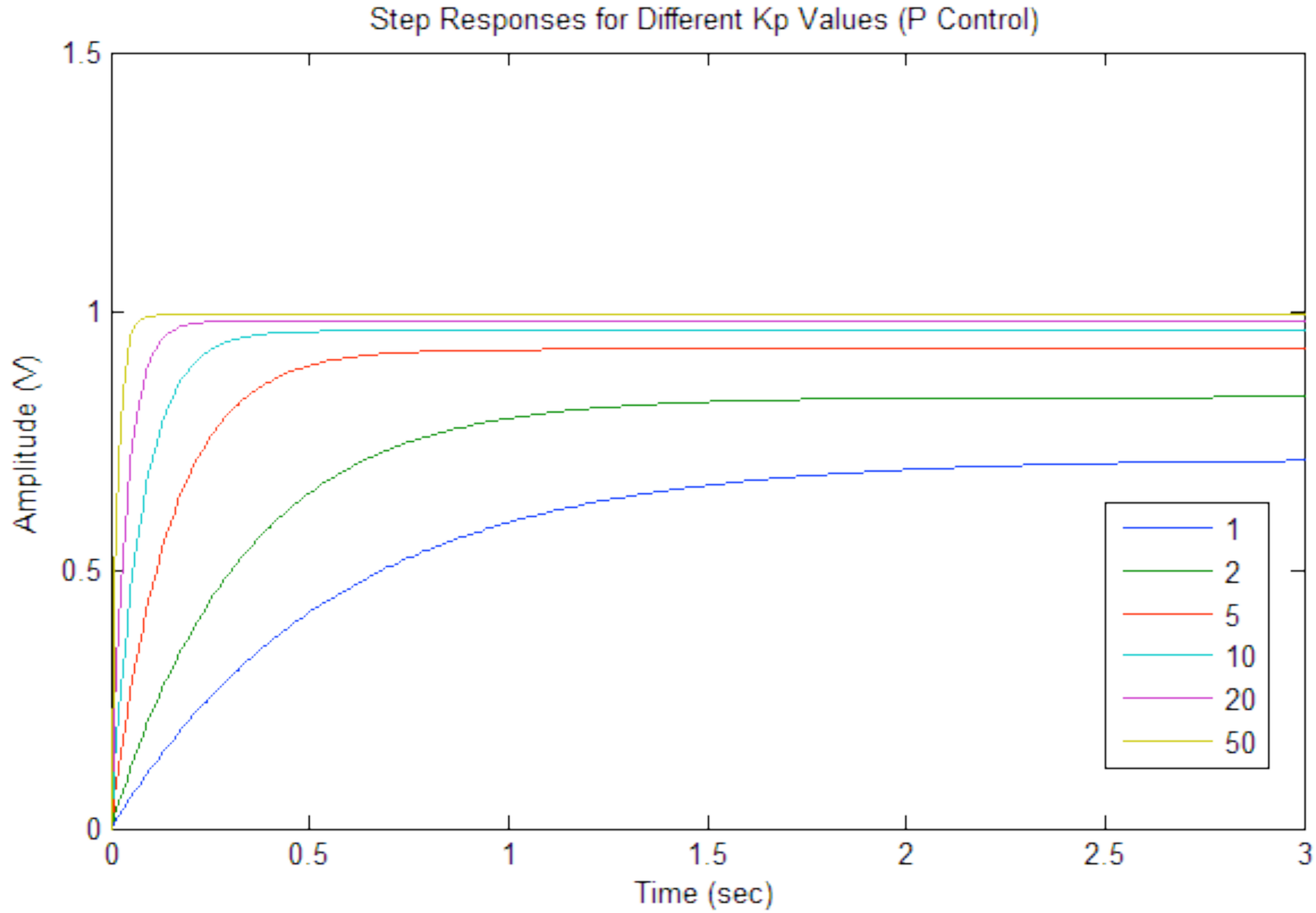
In-class experiments

- Elimination of steady-state error through the use of *integral* (I), and *proportional plus integral* (PI) control.
  - **Experiment #1:** Pure Integral Control & PI Control
  - **Experiment #2:** Compare your results with a Simulink Simulation

- Hand in:
  - Properly annotated plots showing your results.
  - Comments and discussions on your observations and results. (How do the P, I, and PI control actions look and feel like?)

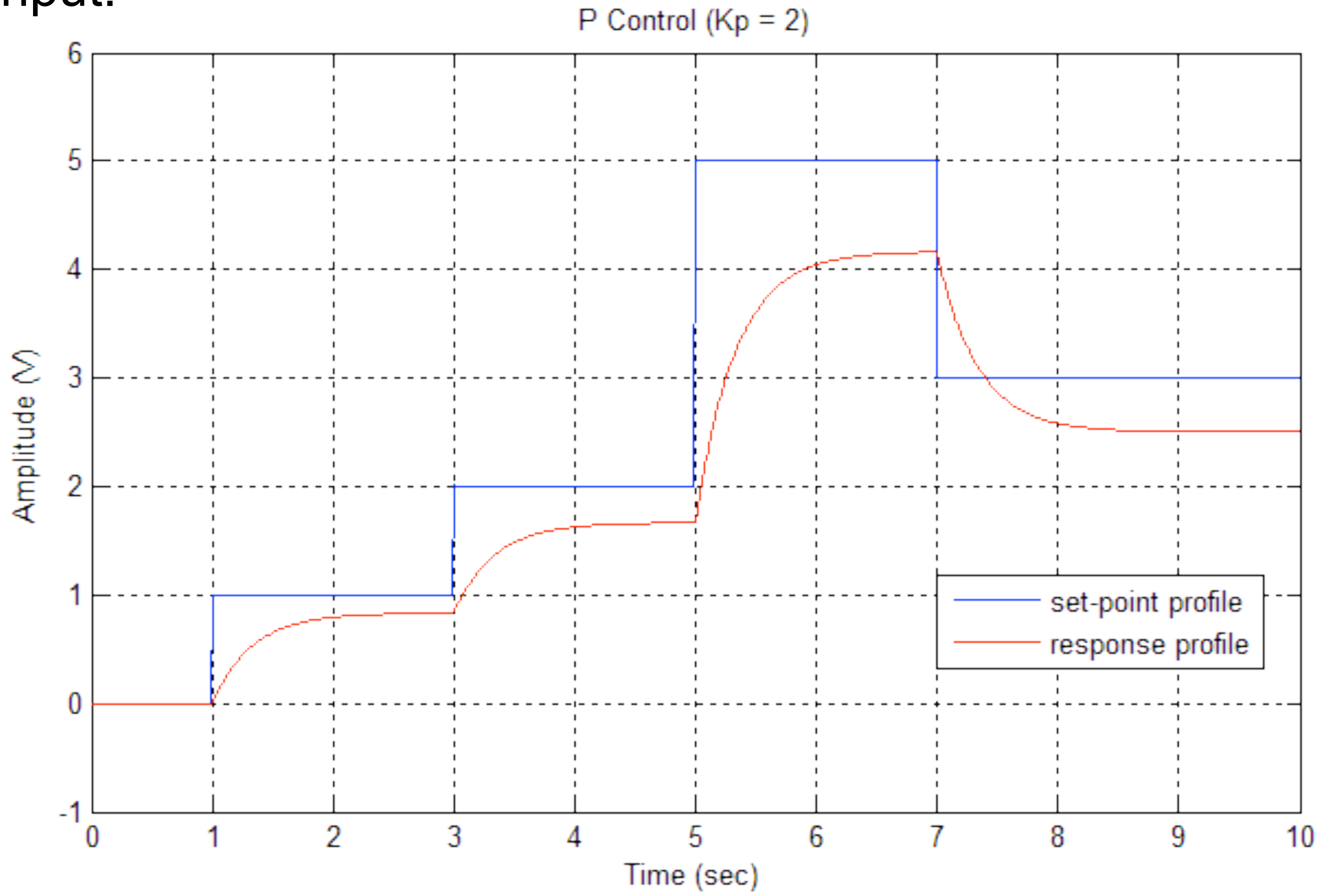
(Another) Limitation of P-control

- Steady-state error
- Kp is limited by the saturation limit of the system
- Large Kp may amplify noise and disturbances and lead to instability



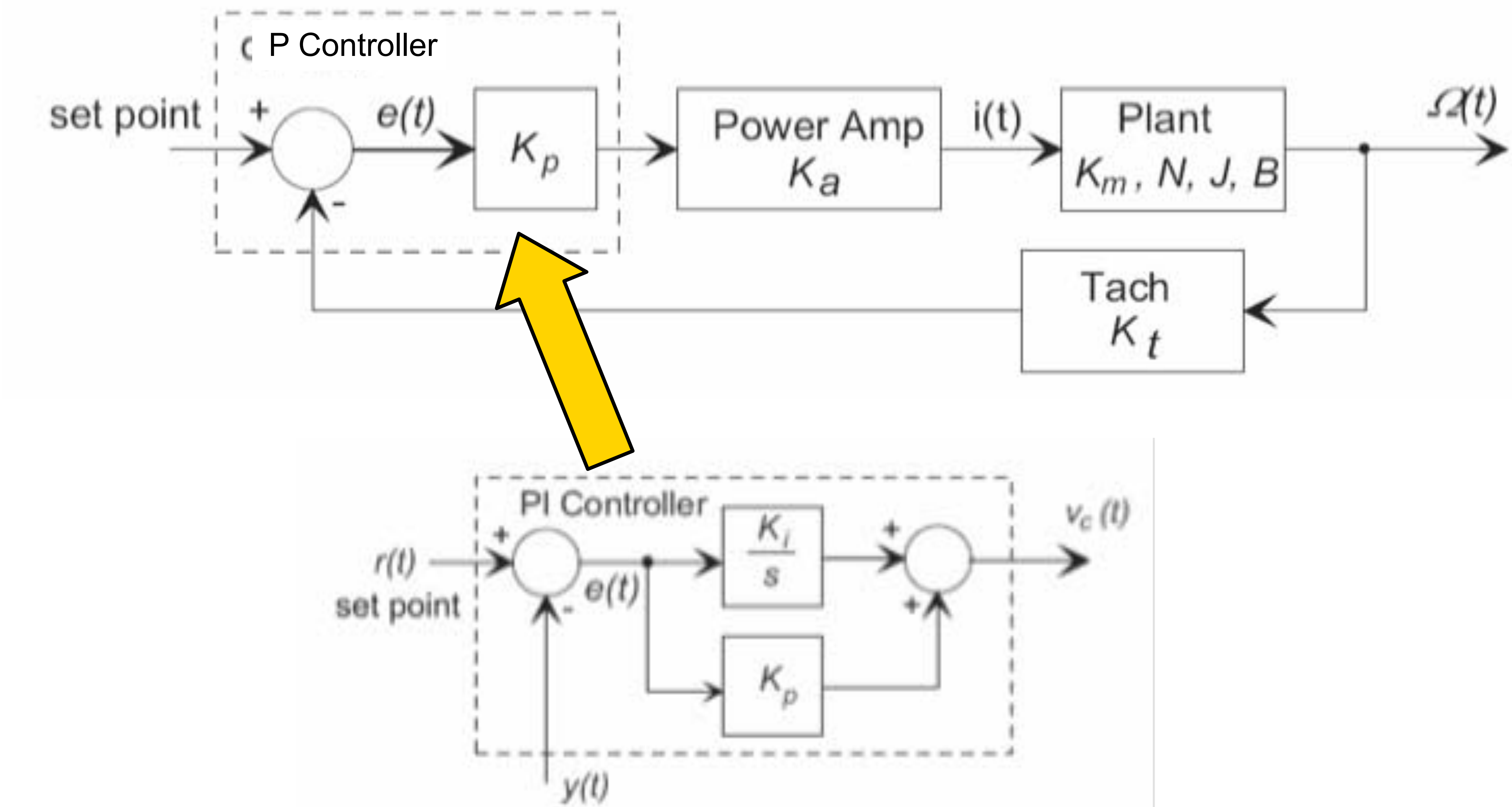
P Control Steady-Sate Errors

- In real world a set-point profile is often more complex than a simple step input.



PI Controller

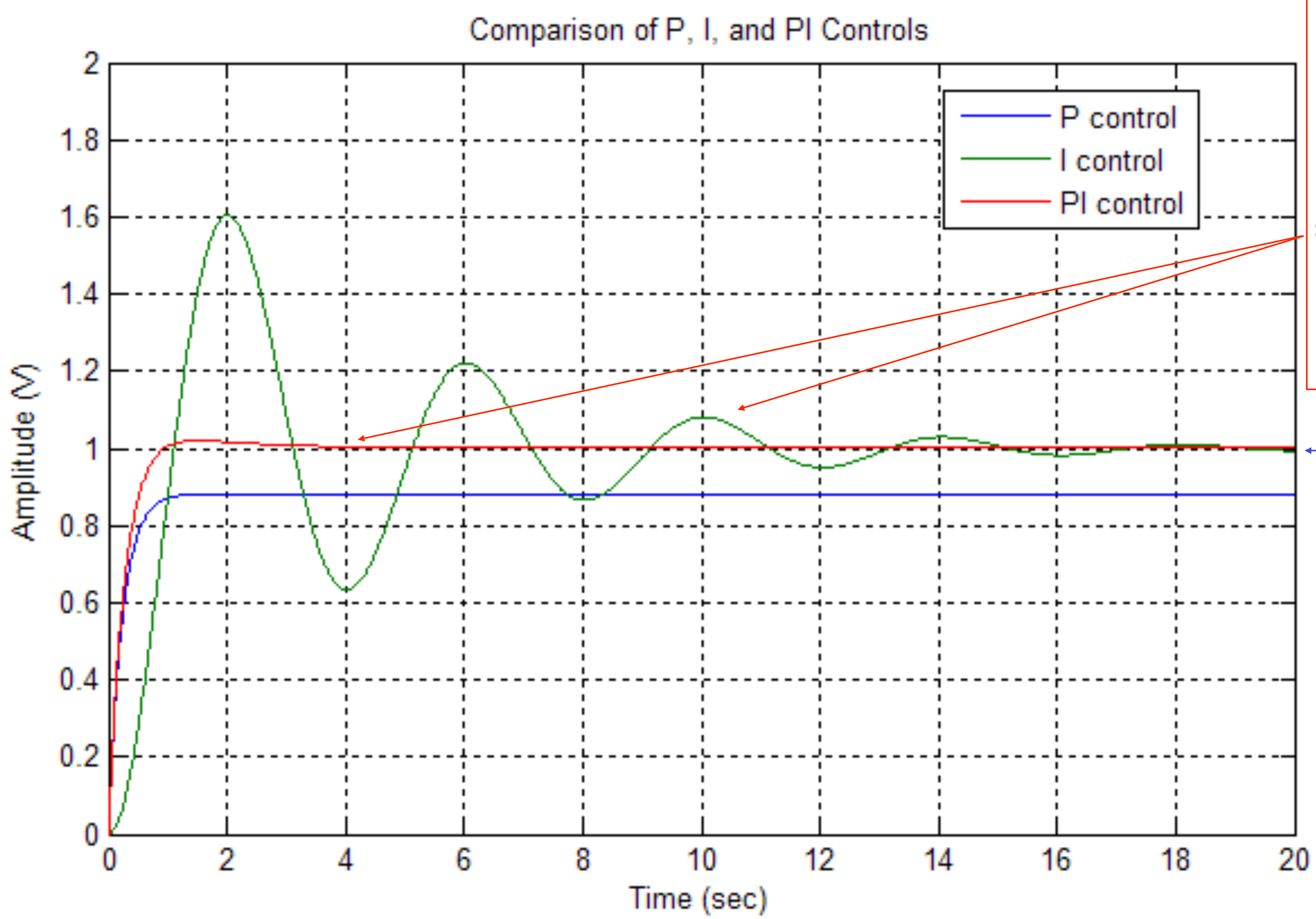
$$G_c(s) = K_p$$



$$G_c(s) = K_p + \frac{K_i}{s}$$

P, I, and PI Comparison

- P Control: steady-state error
- I Control: overshoot, longer transient, integrator windup



$$s^2 + 2\zeta\omega_n s + \omega_n^2 \Leftrightarrow ms^2 + cs + k$$

$$\Rightarrow \begin{cases} \omega_n = \sqrt{\dfrac{k}{m}} \\ \zeta = \dfrac{c}{2\sqrt{km}} \end{cases}$$

Set-point

Comparison of Closed-Loop Transfer Functions

Let $K = K_a K_m \left(\frac{1}{N}\right)$

P Control:

$G_c(s) = K_p$

$$G_{cl}(s) = \frac{V_t(s)}{R(s)} = \frac{K_p K K_t}{Js + B + K_p K K_t}$$

$$R(s) = \frac{A}{s} \longrightarrow V_{ss} = \lim_{s \to 0} = \frac{A}{S} G_{cl}(s) = A \frac{K_p K K_t}{Js + B + K_p K K_t}$$

PI Control:

$G_c(s) = K_p + \frac{K_i}{s}$

$$G_{cl}(s) = \frac{V_t(s)}{R(s)} = \frac{(K_p s + K_i) K K_t}{Js^2 + (B + K_p K K_t)s + K_i K K_t}$$

A real zero

Complex pole pair

$s^2 + 2\zeta\omega_n s + \omega_n^2$

$$R(s) = \frac{A}{s} \longrightarrow V_{ss} = \lim_{s \to 0} \left(\frac{A}{s} G_{cl}(s) \right) = A \frac{V_t(s)}{R(s)} = \frac{(K_p s + K_i) K K_t}{Js^2 + (B + K_p K K_t)s + K_i K K_t} = A$$

Procedures

- **EXP1**: Connect the computer-based controller same as Lab 5, install one magnet
  - Make function generator to produce a DC with offset = 1.0 V. Set $K_p = 0$, $K_i = 0.5$ and 0.2. Make sure power amplifier break is turned on and record system response for 4 to 5 seconds for each case.
  - Set $K_p = 2$, and $K_i = 1$ and record your response data. Compute steady-state error. Compare this result with pure integral control and pure proportional control from Lab 5. Change K_p and K_i both to 3, repeat experiment. Discuss your results, pay attention to the motion of flywheel when the in square wave drops down to "0". What effect does a integrator have on system performance?
- **EXP2:**
  - Modify the controller design in your Simulink model from Lab 5 to have a form of $K_p + K_i s$; set K_p to 2 and K_i to 1 and run simulation for 5 seconds.
  - Comments and discussions on your observations and results. (How do the P, I, and PI control actions look and feel like?)

System Parameters

- $J_{eq} = 0.03$ N-m^2.
- $B_{eq} = 0.014$ N-m-s/rad (lab average).
- $K_a = 2.0$ A/v.
- $K_m = 0.0292$ N-m/A (lab average).
- $K_t = (0.016 \frac{\text{v}}{\text{rev/min}})(60 \frac{\text{s}}{\text{min}})(\frac{1}{2\pi} \frac{\text{rev}}{\text{rad}}) = 0.153$ v/(rad/s).
- $N = \frac{44}{180} = 0.244$