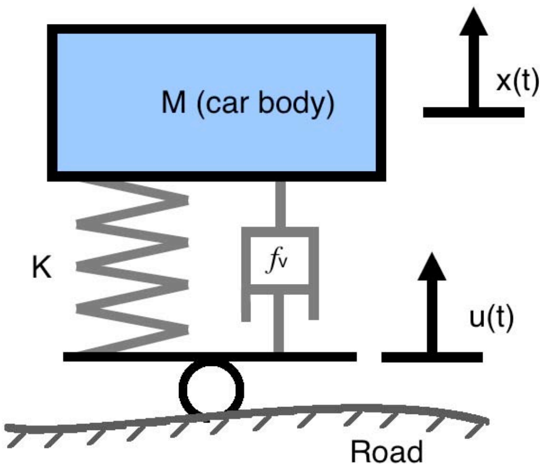
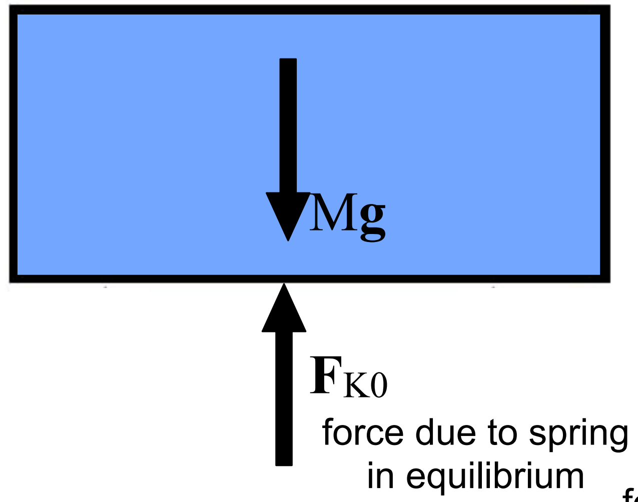
Car suspension model

Mass – spring – viscous damper system

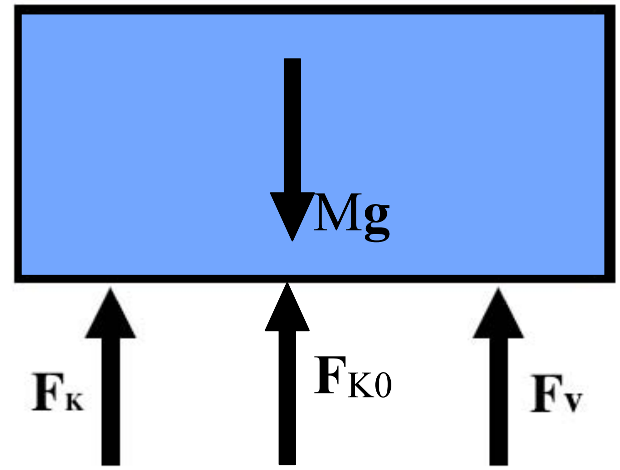


Model

Free body diagram (no motion)



Free body diagram



Force balance ➠

System ODE :
(2nd order ordinary linear differential equation)

$$\begin{cases} m\dfrac{d^2x}{dt^2} &= F_K + F_v \\ F_K &= -K(x-u) \\ F_v &= -f_v(\dot{x}-\dot{u}) \end{cases} \Longrightarrow m\frac{d^2x}{dt^2} = -Kx + Ku - f_v\dot{x} + f_v\dot{u}$$

Equation of motion:

$$\Longrightarrow m\frac{d^2x}{dt^2} + Kx + f_v\dot{x} = Ku + f_v\dot{u}$$

General Linear Time-Invariant (LTI) system

$$a_n \frac{d^n x}{dt^n} + a_{n-1} \frac{d^{n-1} x}{dt^{n-1}} + a_{n-2} \frac{d^n x}{dt^n} \cdots a_1 \frac{dx}{dt} + a_0 x = b_m \frac{d^m y}{dt^m} + b_{m-1} \frac{d^{m-1} y}{dt^{m-1}} + \cdots b_1 \frac{d^1 y}{dt^1} + b_0 u$$

nth-order Linear Ordinary Differential Equation (ODE)
with constant coefficients (time-invariant)

general solution:

$$x(t) = x_{\text{homogeneous}}(t) + x_{\text{forced}}(t)$$

➡ homogeneous solution: *y*=0 (no forcing term)

➡ forced solution: a “guess” solution for the system behavior when y(t)≠0

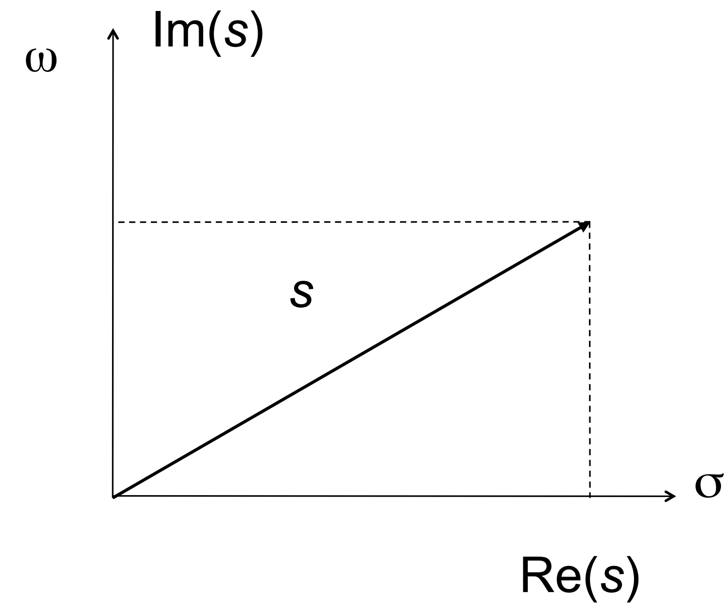
Homogeneous and forced solutions

Homogeneous solution:

$$x(t) = C_0 + C_1 e^{s_1 t} + C_2 e^{s_2 t} + \cdots + C_n e^{s_n t}$$

where in general

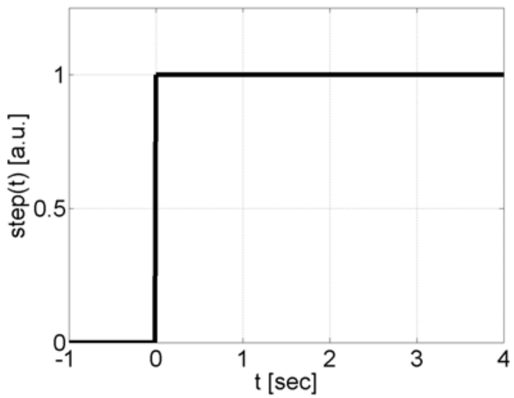
$s_i = \sigma_i + j\omega$ (complex number)



Forced solution: sometimes difficult to "guess" but for specific forces of interest, quite easy.

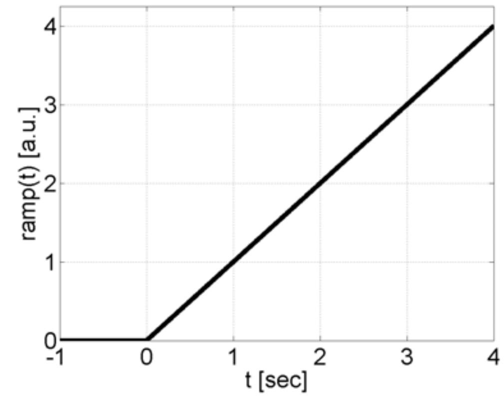
For example, if y(t)=constant, then y_{forced}=constant as well (but a different constant!)

Commonly used input functions



Step function
(*aka* Heaviside)

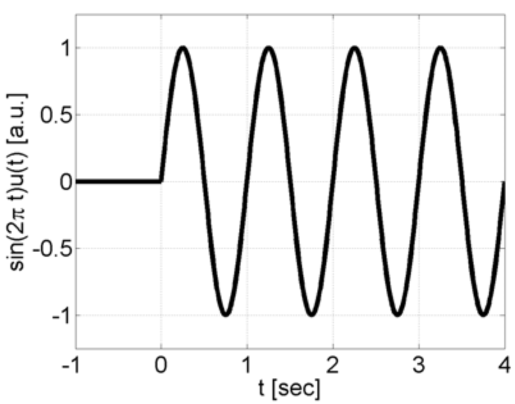
$$\text{step}(t) = \begin{cases} 0, & t < 0; \\ 1, & t \geq 0. \end{cases}$$



Ramp
function

$$\text{ramp}(t) = \begin{cases} 0, & t < 0; \\ t, & t \geq 0. \end{cases}$$

$$= t\,\text{step}(t)$$

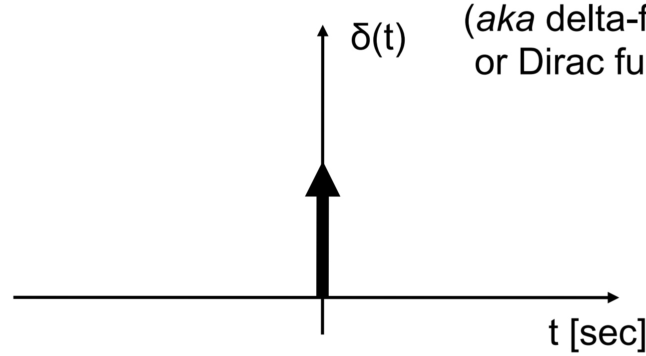


Sinusoidal
function

$$f(t) = \begin{cases} 0, & t < 0; \\ \sin(\omega t), & t \geq 0. \end{cases}$$

$$= \sin(\omega t)\,\text{step}(t)$$

Impulse
(*aka* delta-function
or Dirac function)



1^{st} order system

$$M\dot{v} + bv = f(t)$$

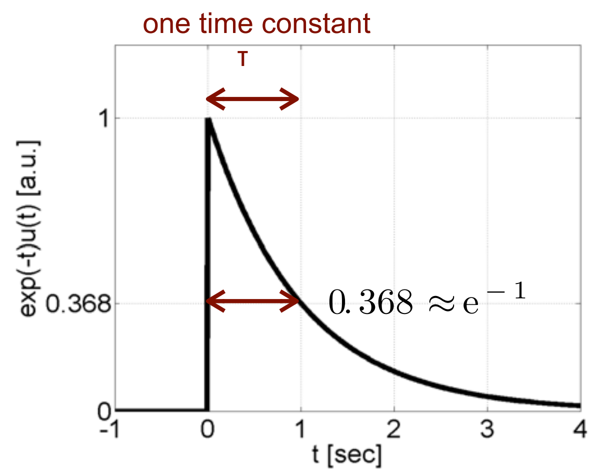
mass viscous damping force

Impulse response:
equivalent to setting an initial condition v(t=0)

$$v(t=0) = v_0$$

$$v(t) = v_0 e^{-t/\tau}, \qquad t \geq 0$$

$$\text{time constant} \quad \tau = \frac{M}{b}$$



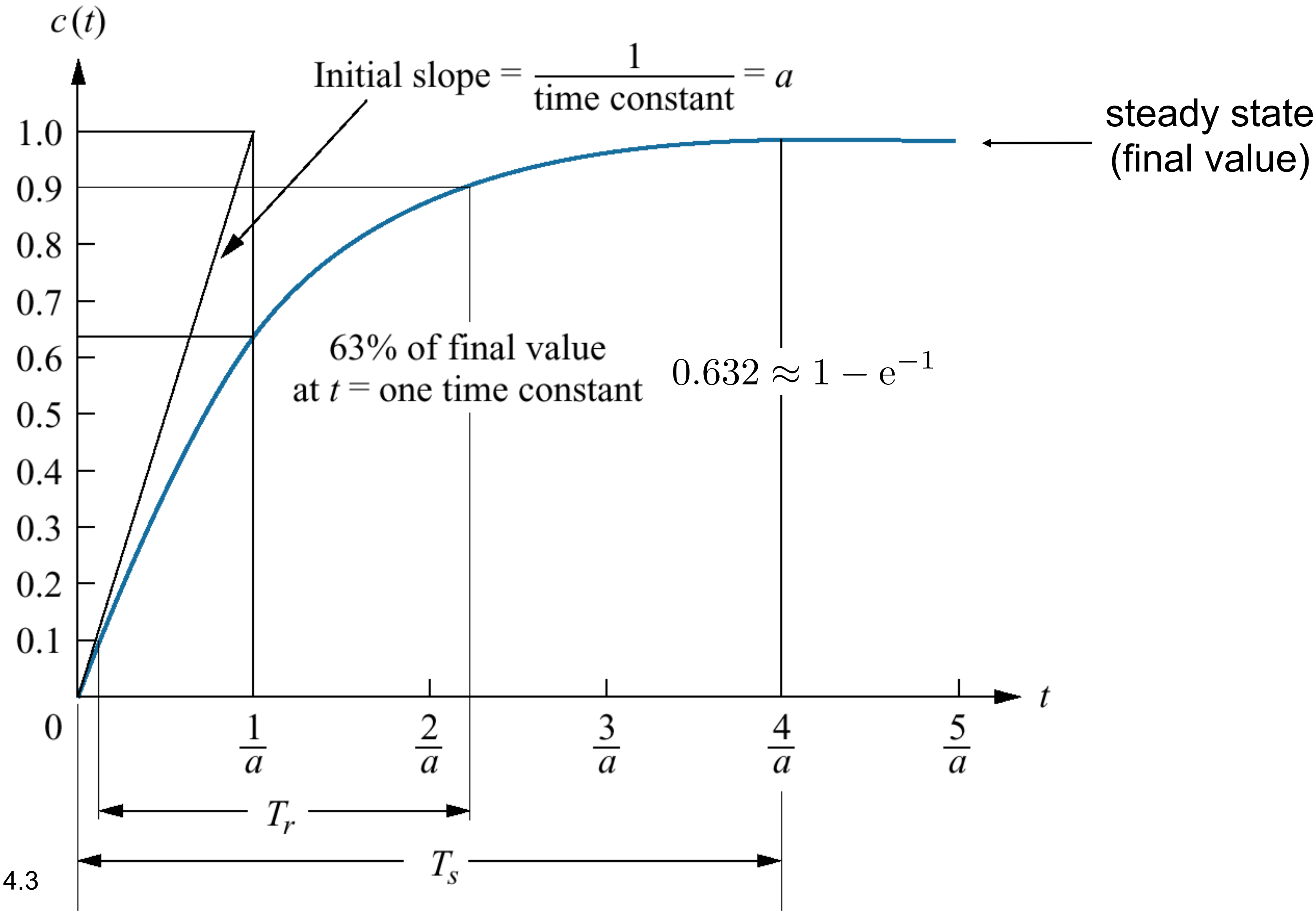
Step response:
f(t) is the "step function" (or Heaviside function)

$$f(t) = F_0 \, \text{step}(t) = \begin{cases} 0, & t < 0; \\ F_0, & t \geq 0. \end{cases}$$

$$v(t) = \frac{F_0}{b}\left(1 - e^{-t/\tau}\right), \qquad t \geq 0$$

$$v(t=0) = 0$$

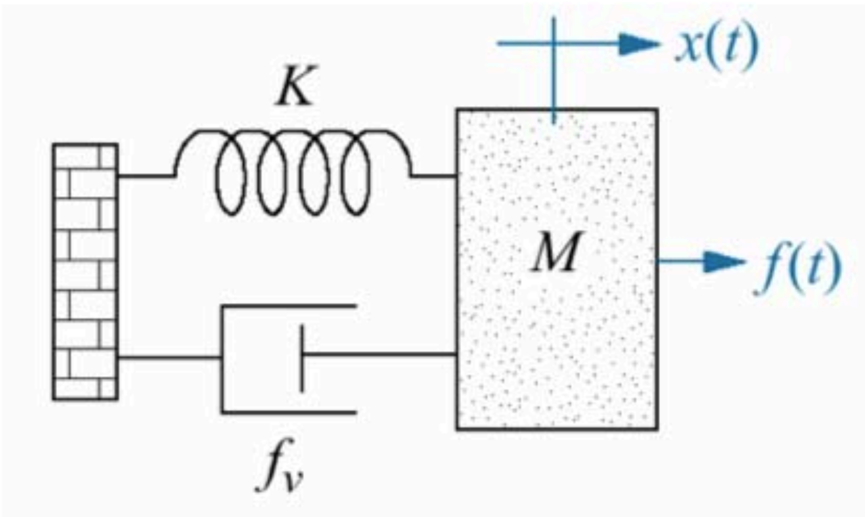
1^{st} order system: step response



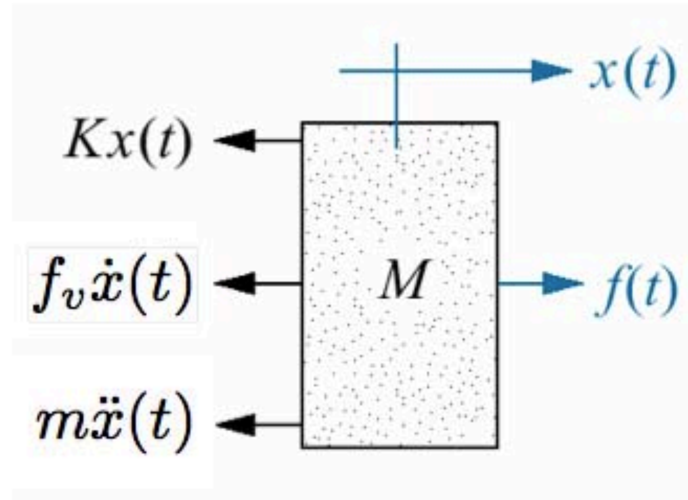
Nise Figure 4.3

Mass – spring – viscous damper system

How is this different than the car suspension system?



Model



Force balance

System ODE

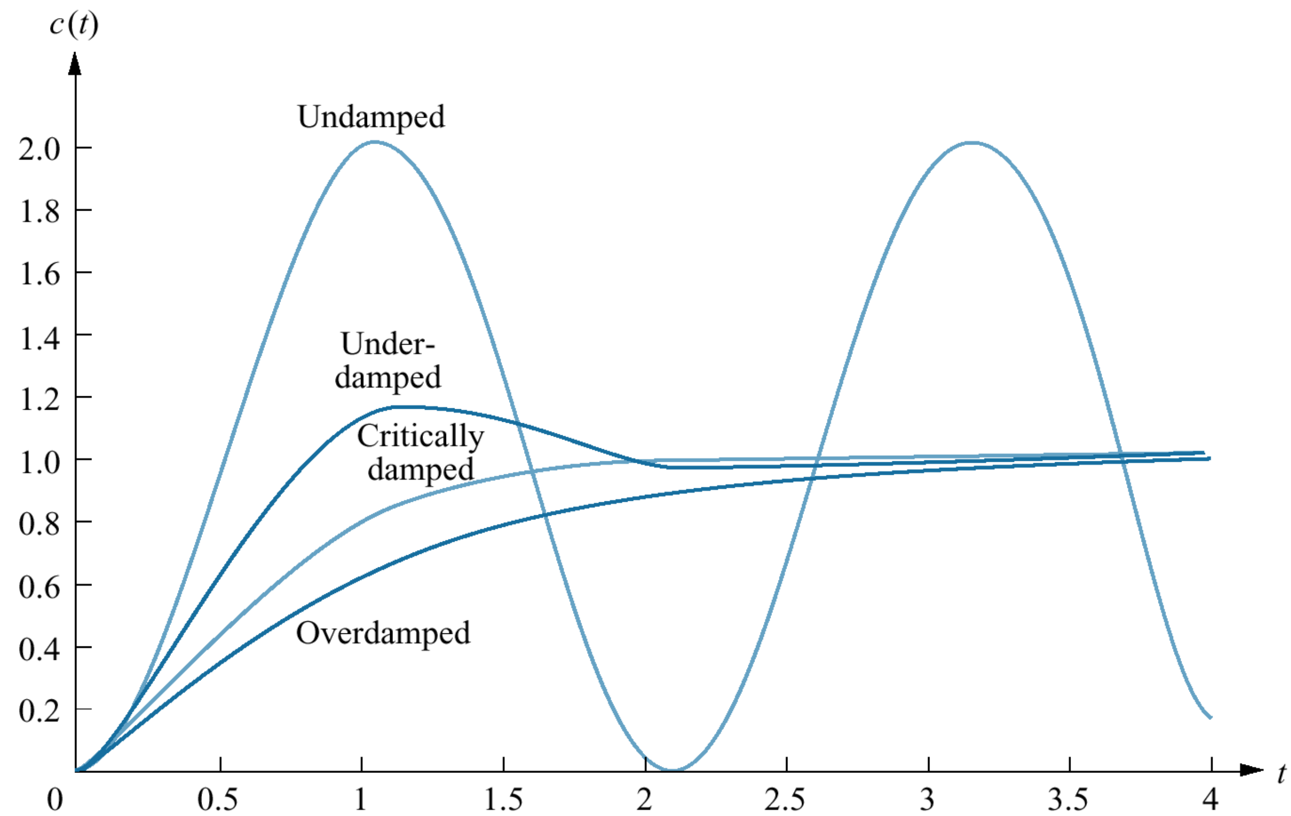
(2nd order ordinary <u>linear</u> differential equation)

$$M\ddot{x}(t) + f_v\dot{x}(t) + Kx(t) = f(t)$$

Equation of motion

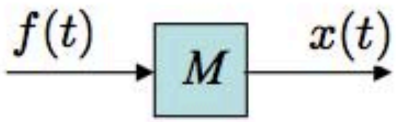
Nise Figure 2.15a/2.16a

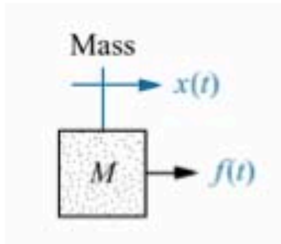
2^{nd} order system: step response



Mechanical system components: translation

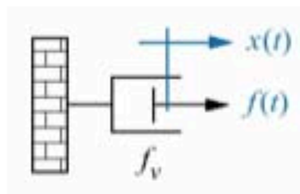
- Mass





- Component input: **force** $f(t)$
- Component output: **position** $x(t)$
- Component ODE (Newton's law): $M\dfrac{\mathrm{d}^2x(t)}{\mathrm{d}t^2} \equiv M\ddot{x}(t) = f(t)$

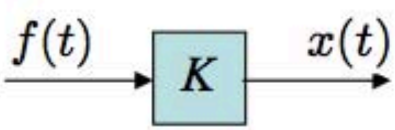
- Damper (friction)
  - viscous

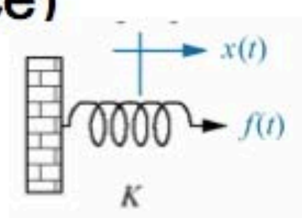


- Component input: **force** $f(t)$
- Component output: **position** $x(t)$
- Component ODE: $f_v\dot{x}(t) = f(t)$

  - Coulomb → Component ODE: $f_c\mathrm{sgn}[\dot{x}(t)] = f(t)$
  - drag → Component ODE: $f_d\,|\dot{x}(t)|\,\dot{x}(t) = f(t)$

- Spring (compliance)





- Component input: **force** $f(t)$
- Component output: **position** $x(t)$
- Component ODE (Hooke's law): $Kx(t) = f(t)$

Nise Table 2.4