MASSACHUSETTS INSTITUTE OF TECHNOLOGY
DEPARTMENT OF OCEAN ENGINEERING

13.811 Advanced Structural Dynamics and Acoustics

Second Half - Problem Set 2 Solution

Problem 1

Using equation (2.52) from text and substituting the expression for p(x,y,0),

$$P(k_x, k_y) = P_o e^{-i\omega t} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} (e^{i2kx} + e^{-i2kx}) e^{-ik_x x} e^{-ik_y y} dxdy$$

$$= P_o e^{-i\omega t} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} (e^{-i(k_x - 2k)x} + e^{-i(2k + k_x)x}) e^{-ik_y y} dxdy$$

Using equation (1.5) from text,

$$P(k_x, k_y) = 4\pi^2 P_o e^{-i\omega t} [\delta(k_x - 2k) + \delta(k_x + 2k)] \delta(k_y) \quad (1)$$

Using equation (2.50) and substituting equation (1) above,

$$p(\mathrm{x,y,z}) = \frac{1}{4\pi^2}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}[4\pi^2 P_o e^{-i\omega t}\left[\delta(k_x - 2k) + \delta(k_x + 2k)\right]\delta(k_y)e^{ik_z z}]e^{ik_x x}e^{ik_y y}dk_x dk_y$$

$$= P_o e^{-i\omega t}\int_{-\infty}^{\infty}\left[\delta(k_x - 2k) + \delta(k_x + 2k)\right]e^{ik_x x}dk_x \int_{-\infty}^{\infty}\delta(k_y)e^{i(k_z z + k_y y)}dk_y$$

$$= P_o e^{-i\omega t}\int_{-\infty}^{\infty}\left[\delta(k_x - 2k) + \delta(k_x + 2k)\right]e^{ik_x x}dk_x I_1 \qquad (2)$$

Where

$$I_1 = \int_{-\infty}^{\infty}\delta(k_y)e^{i(k_z z + k_y y)}dk_y$$

Using sifting property of the delta function integral (equation (1.37) of text) and recognizing that $k_z = \sqrt{k^2 - k_x{}^2 - k_y{}^2}$,

$$I_1 \quad = \quad e^{i\sqrt{k^2-k_x{}^2}z} \tag{3}$$

Thus, substituting equation (3) above into (2) above,

$$p(\mathrm{x,y,z}) = P_o e^{-i\omega t}\int_{-\infty}^{\infty}[\delta(k_x-2k)+\delta(k_x+2k)]e^{ik_x x}e^{i\sqrt{k^2-k_x{}^2}z}dk_x$$

Again, using sifting property of the delta function integral (equation (1.37) of text),

$$p(\mathrm{x,y,z}) = P_o e^{-i\omega t}\left[e^{i2kx}e^{i\sqrt{-3}kz} + e^{-i2kx}e^{i\sqrt{-3}kz}\right]$$

$$= P_o e^{-i\omega t}e^{-\sqrt{3}kz}\cos(2kx)$$

Figure 1. shows the normalized pressure field $20\log\left|\frac{p}{P_o}\right|$ dB for normalized distance $0 \le \mathrm{kx} \le 10$ and $0 \le \mathrm{kz} \le 10$.

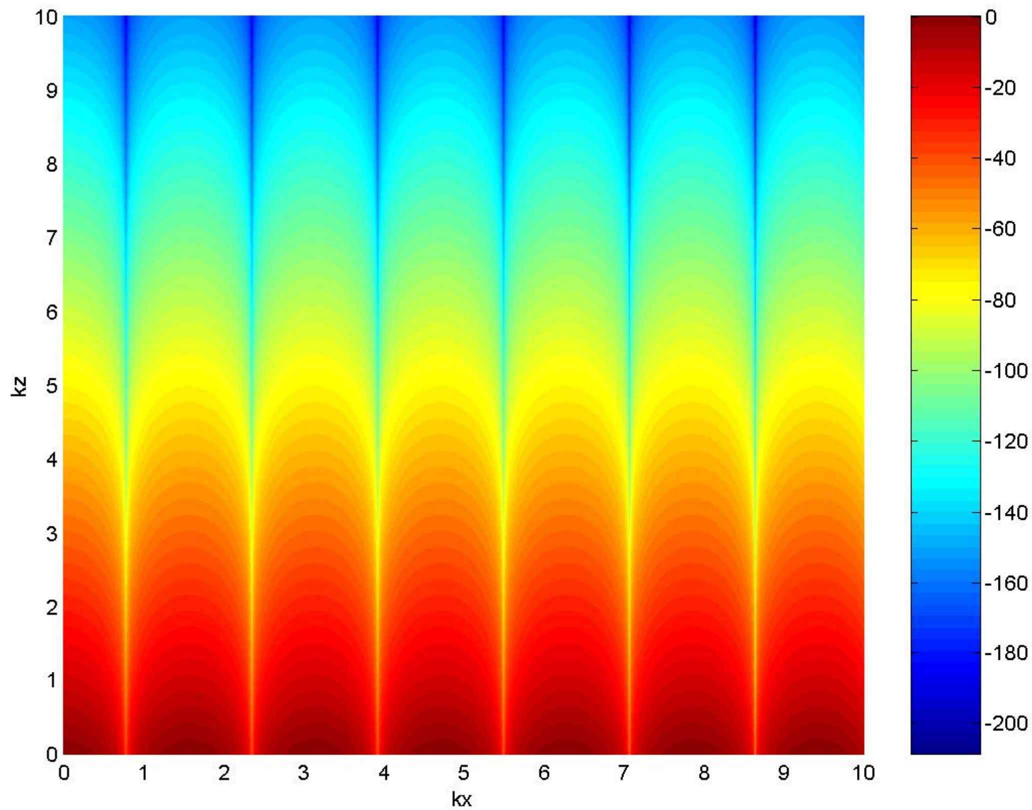

Figure 1. Normalized pressure field $20\log\left|\frac{p}{P_o}\right|$ in dB

for normalized distance $0 \le kx \le 10$ and $0 \le kz \le 10$.

Problem 2

Using equation (2.61) and let z = z’ = 0,

$$\dot{W}(k_x, k_y, 0) = \frac{k_z}{\rho_o ck} P(k_x, k_y, 0) \qquad (1)$$

By recognizing the definition in equation (2.56) and substituting (1) above into the expression,

$$P(k_x, k_y) \equiv P(k_x, k_y, 0)$$

$$P(k_x, k_y) \equiv \frac{\rho_o ck}{k_z} \dot{W}(k_x, k_y, 0) \qquad (2)$$

Using equation (2.50) and observing the form of the inverse 2-D Fourier Transform expression given in equation (1.17)

$$p(\mathrm{x}, \mathrm{y}, \mathrm{z}) = \mathscr{F}_x^{-1} \mathscr{F}_y^{-1} \left[P(k_x, k_y) e^{ik_z z} \right]$$

$$p(\mathrm{x}, \mathrm{y}, 0) = \mathscr{F}_x^{-1} \mathscr{F}_y^{-1} \left[P(k_x, k_y) \right] \qquad (3)$$

Substituting (2) above into (3)

$$p(\mathrm{x},\mathrm{y},0)=\mathscr{F}_x^{-1}\mathscr{F}_y^{-1}\left[\frac{\rho_o ck}{k_z}\dot{W}(k_x,k_y,0)\right] \tag{4}$$

From the 2-D Fourier transform expression provided in text for equation (2.76),

$$p(\mathrm{x},\mathrm{y},0)=\mathscr{F}_x^{-1}\mathscr{F}_y^{-1}\left[\frac{\rho_o ck}{k_z}Q_o e^{-ik_x x_o}e^{-ik_y y_o}\right]$$

From equation (2.72) and (2.74), and again recognizing that z = z' = 0,

$$p(\mathrm{x},\mathrm{y},0)=-iQ_o\rho_o ck\,\frac{e^{ik\left|\vec{r}-\vec{r}_o\right|}}{2\pi\left|\vec{r}-\vec{r}_o\right|}$$

Where

$$\left|\vec{r}-\vec{r}_o\right|=\sqrt{(x-x_o)^2+(y-y_o)^2}$$