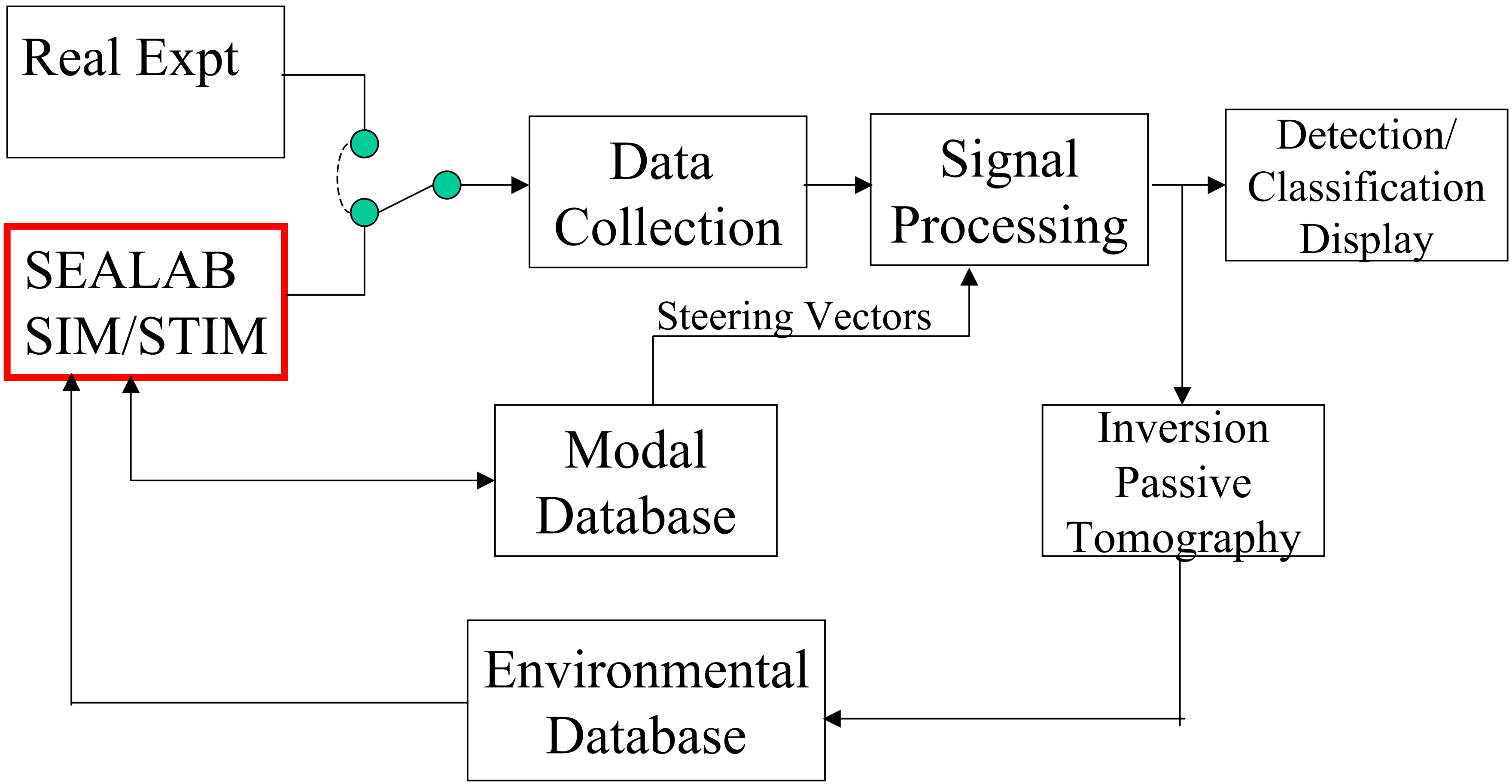
Computational Ocean Acoustics

- Ray Tracing
- Wavenumber Integration
- Normal Modes
- Parabolic Equation

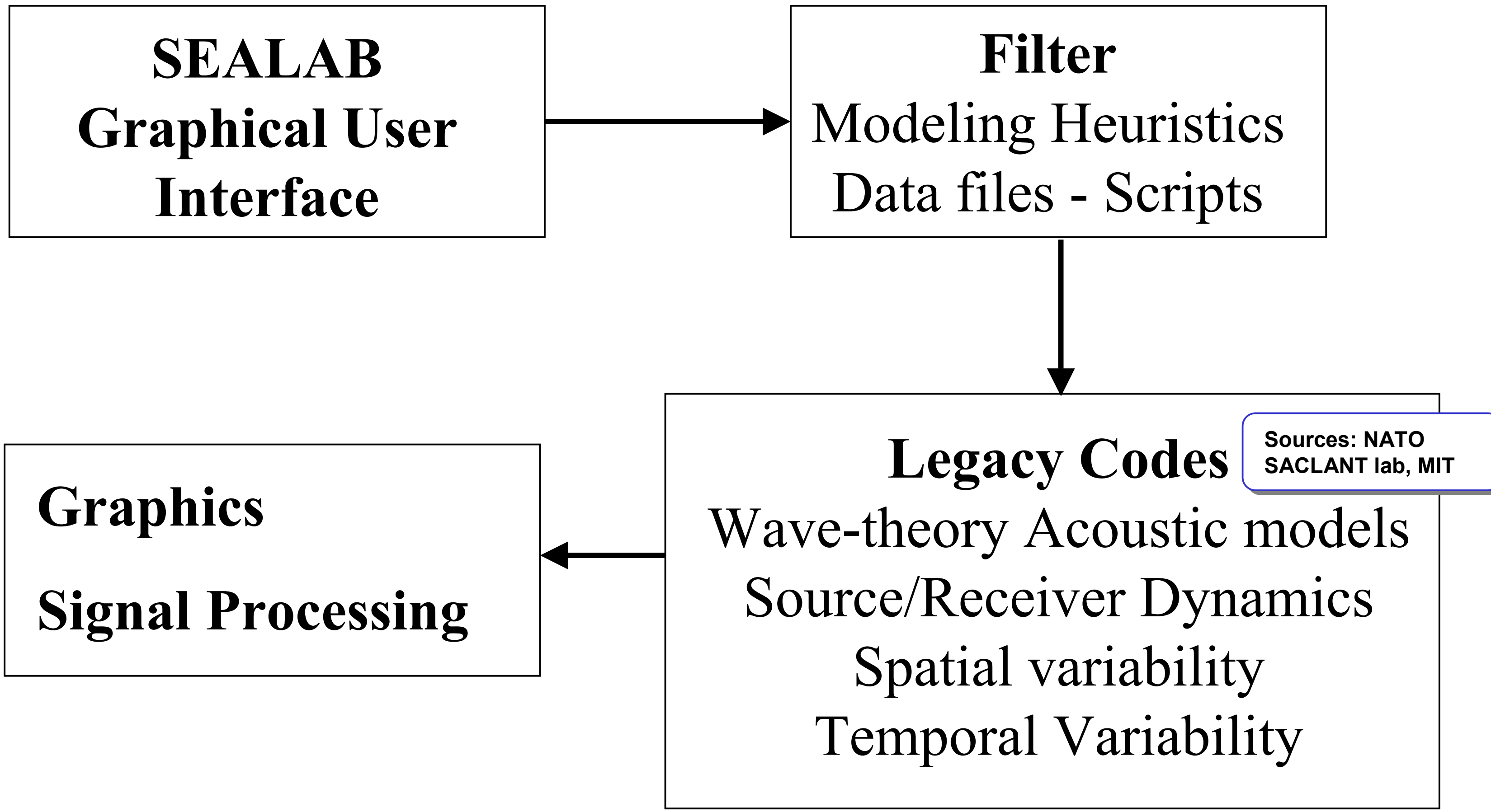
Normal Modes

- Modes for Range-Dependent Envir.
  - Coupled Modes (5.9)
  - One-way Coupled Modes
  - Adiabatic Modes
  - SEALAB Propagation Modeling Environment
- Modes in 3-D Environments
  - Continuously coupled modes
  - Adiabatic Approximation
  - 3-D Propagation in 2-D Environments
  - General 3-D Modal Propagation Framework
    - SEALAB Passive and Active Sonar Simulator

Simulation and Stimulation of Sonar Signal Processing Frameworks



SEALAB Architecture



SEALAB
MIT OE Acoustics Computing Environment

```
#!/usr/csh
#
# SEALAB Environment
#
 setenv SEALAB_ROOT /keel0/henrik/Vasa/Sealab
 setenv SCRIPTS ${SEALAB_ROOT}/scripts
 setenv SEALAB_BIN ${SEALAB_ROOT}/util/bin
 setenv MCM_BIN "${SEALAB_ROOT}/mcm/bin"
 setenv MODEL_BIN  "${SEALAB_ROOT}/model/bin"
 setenv PASSIVE_BIN  "${SEALAB_ROOT}/pas/bin"
 setenv ACTIVE_BIN  "${SEALAB_ROOT}/act/bin"
#
# OASES and CSNAP environment
#
setenv OASES_ROOT /keel0/henrik/Oases
setenv OASES_SH  $OASES_ROOT/bin
setenv OASES_BIN $OASES_ROOT/bin/i386-linux-linux
setenv OASES_LIB $OASES_ROOT/lib/i386-linux-linux
setenv CON_BWCOL COL
setenv CON_PACKGE MTV
setenv CON_DEVICE X11
#
# OASES3D
#
setenv SCATT_BIN /keel0/henrik/Scatt/bin/i386-linux-linux
#
set path=( . $SEALAB_BIN $ACTIVE_BIN $PASSIVE_BIN $MODEL_BIN \
             $MCM_BIN $SCRIPTS  $OASES_SH $OASES_BIN $SCATT_BIN $path)
#
# MTV environment
#
setenv MTV_WRB_COLORMAP "ON"
setenv MTV_COLORMAP jet
setenv MTV_PSFONT helvetica
setenv MTV_PRINTER_CMD "lpr"
setenv MTV_PSCOLOR "ON"
#
#  PRINTER
#
setenv PRINTER lp1
```

SEALAB Initialization on Linux boxes:

- source /keel0/henrik/Vasa/Sealab/scripts/slbinit

Running SEALAB

Transmission Loss

- sealab –t

Passive Sonar

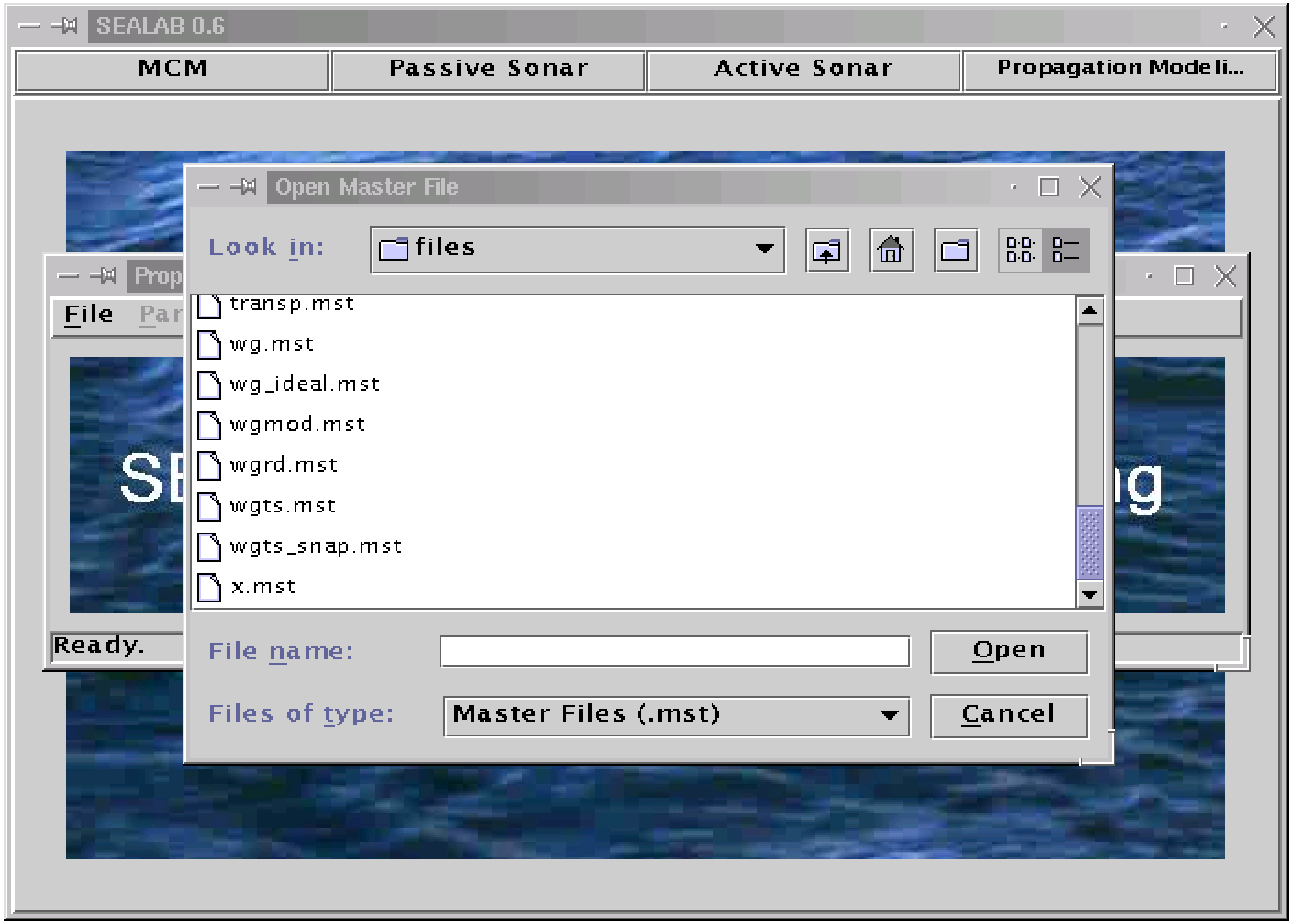
- sealab –p

Active Sonar

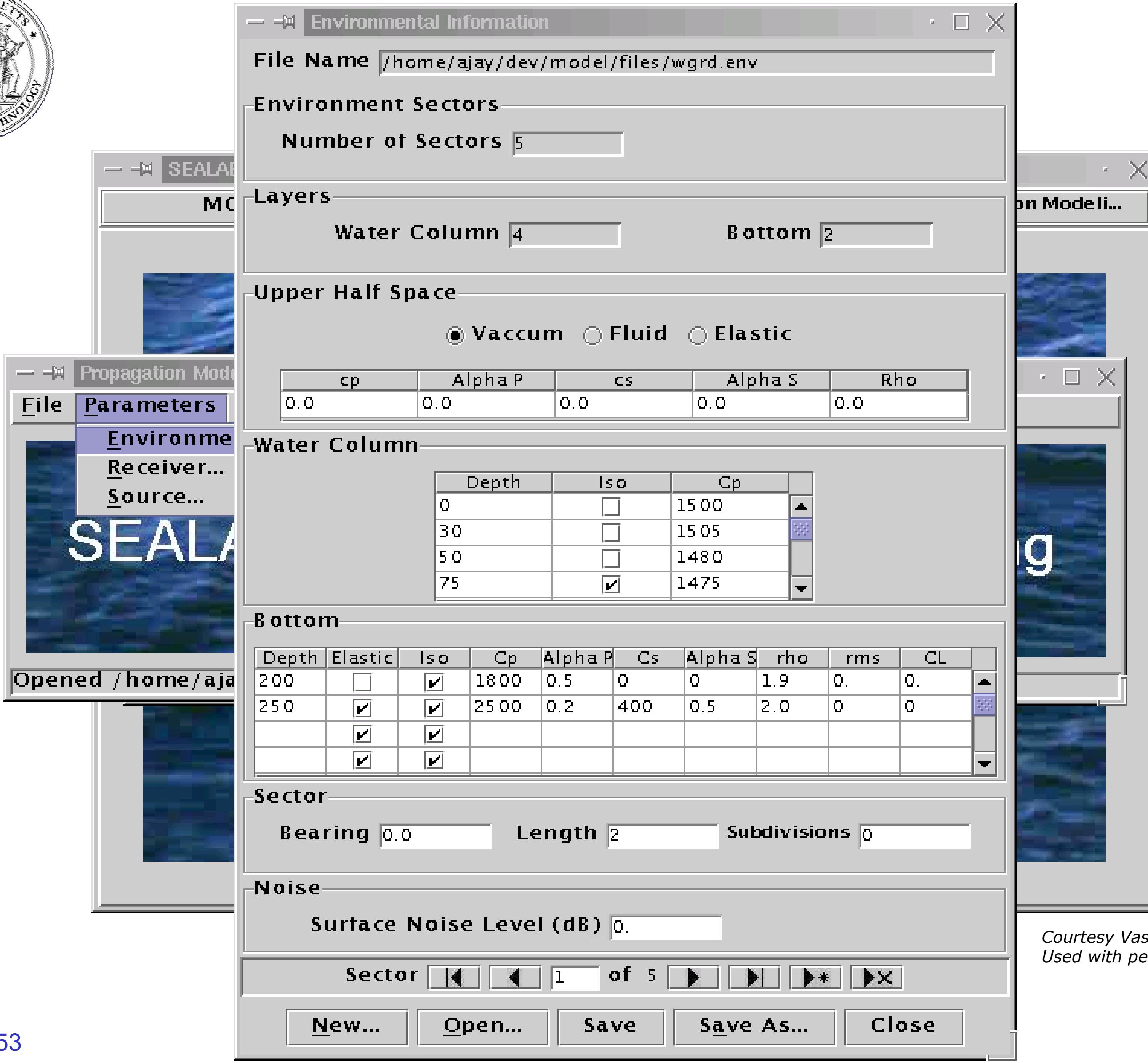
- sealab –a

SEALAB

Propagation Modeling



Courtesy Vasa Associates. Used with permission.



Courtesy Vasa Associates.
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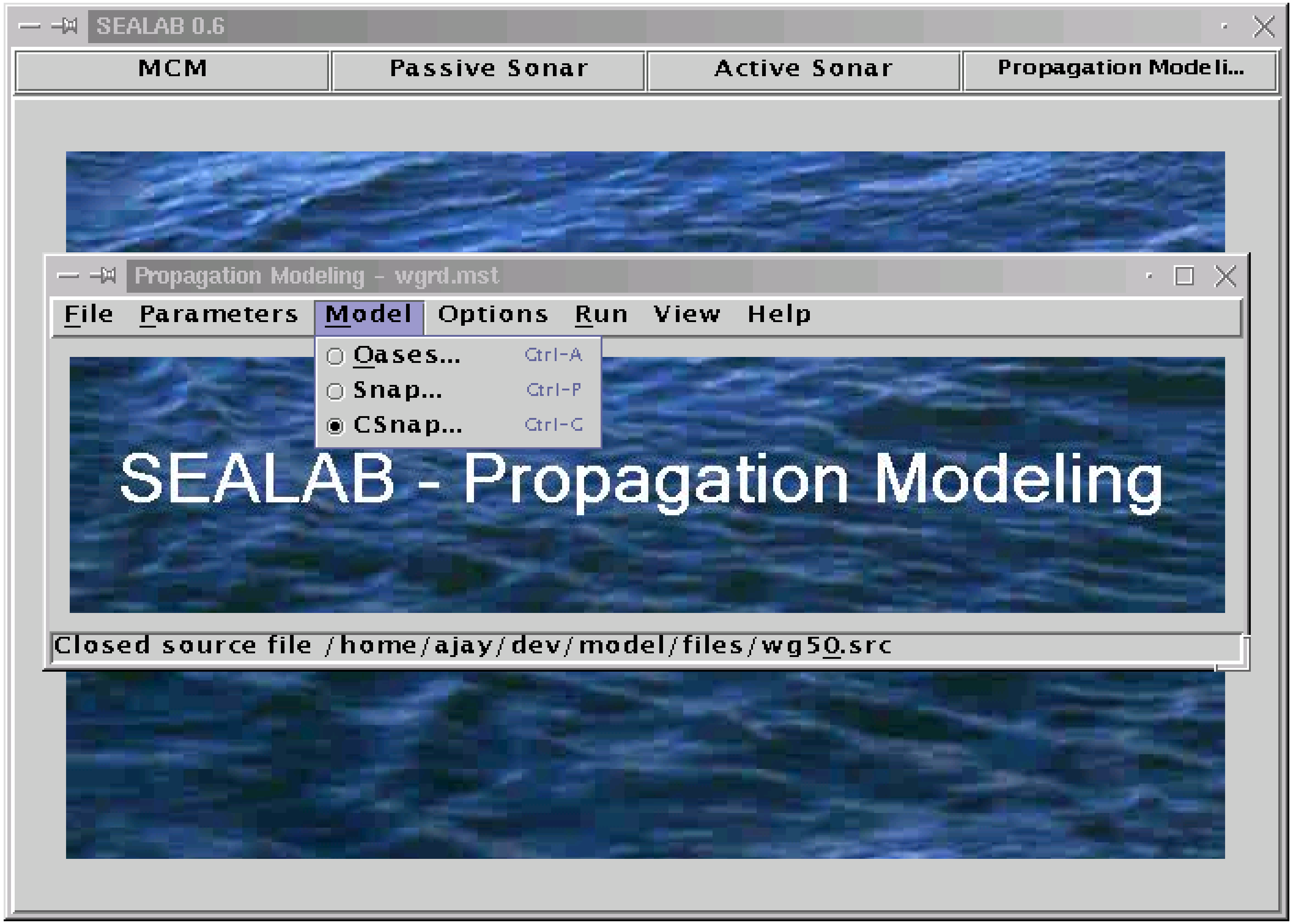
SEALAB

Propagation Modeling



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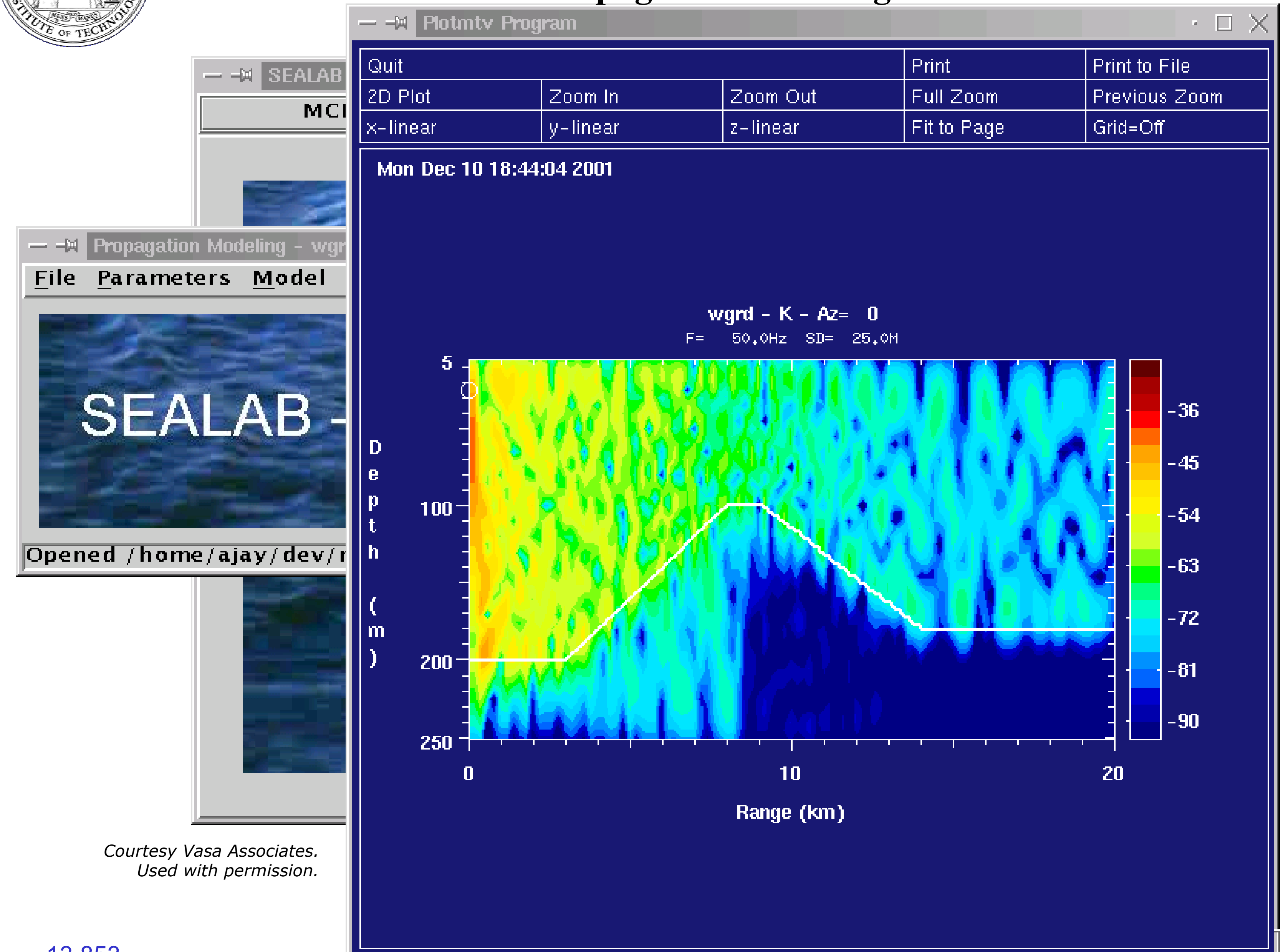
SEALAB

Propagation Modeling



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Normal Modes for 3-D Varying Environments

Horizontal Refraction Equations

3-D Helmholtz equation

$$\rho \nabla \cdot \left(\frac{1}{\rho} \nabla p \right) + \frac{\omega^2}{c^2(x,y,z)}\, p = -\delta(x)\,\delta(y)\,\delta(z - z_s)\ ,$$

Laterally Homogeneous Density

$$\rho(x,y,z) \simeq \rho(z)$$

[See Jensen Fig. 5.19a]

$$\rho \frac{\partial}{\partial x}\left(\frac{1}{\rho}\frac{\partial p}{\partial x}\right) + \rho \frac{\partial}{\partial y}\left(\frac{1}{\rho}\frac{\partial p}{\partial y}\right) + \rho \frac{\partial}{\partial z}\left(\frac{1}{\rho}\frac{\partial p}{\partial z}\right) + \frac{\omega^2}{c^2(x,y,z)}\, p = -\delta(x)\,\delta(y)\,\delta(z - z_s)\ .$$

Factorized Solution

$$p(x,y,z) = \sum_m \Phi_m(x,y)\, \Psi_m(x,y,z)\ ,$$

Orthogonality Operator

$$\int (\cdot)\, \frac{\Psi_n(x,y,z)}{\rho}\, dz\ ,$$

Continuous Mode Coupling Equation

$$\frac{\partial^2 \Phi_n}{\partial x^2} + \frac{\partial^2 \Phi_n}{\partial y^2} + k_{rn}^2(x,y)\,\Phi_n + \sum_m A_{mn}\Phi_m + \sum_m 2B_{mn}\frac{\partial \Phi_m}{\partial x} + \sum_m 2C_{mn}\frac{\partial \Phi_m}{\partial y} = -\delta(x)\,\delta(y)\,\delta(z-z_s)\,,$$

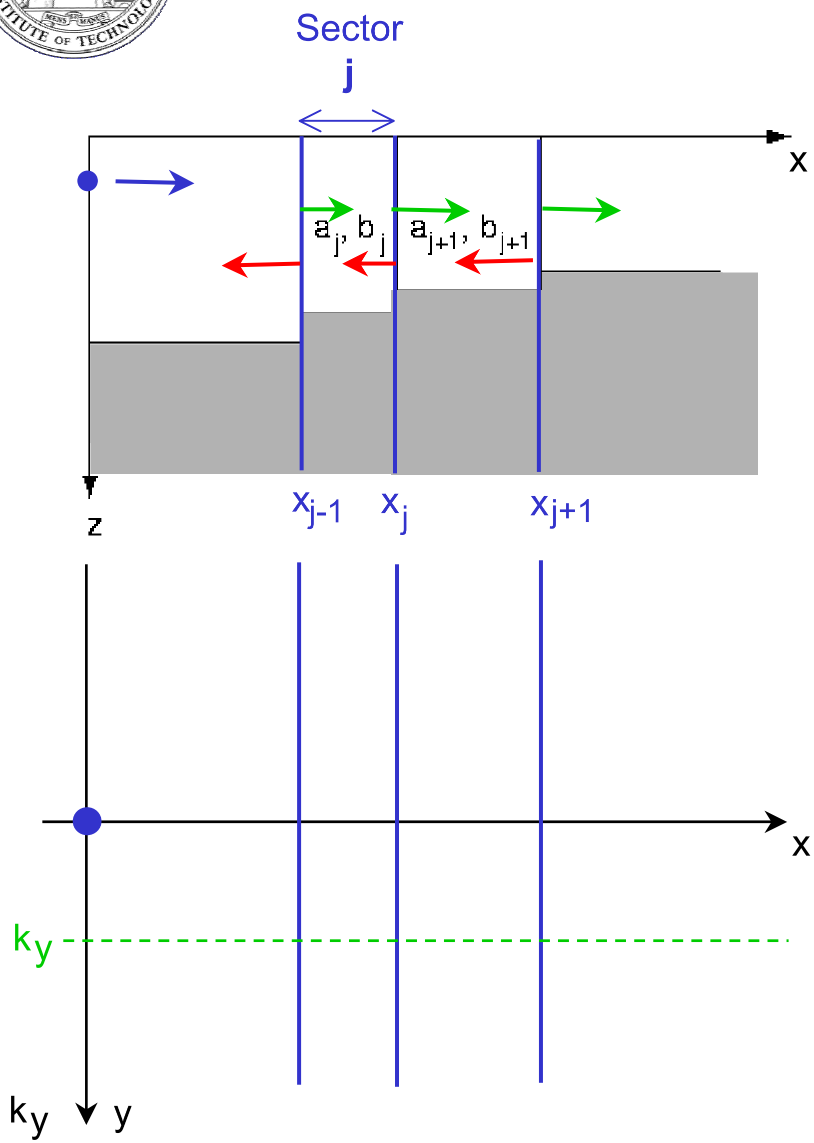
$$\begin{aligned}
A_{mn} &= \int \left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}\right)\Psi_m \frac{\Psi_n}{\rho}\,dz\,,\\
B_{mn} &= -B_{nm} = \int \frac{\partial \Psi_m}{\partial x}\frac{\Psi_n}{\rho}\,dz\,,\\
C_{mn} &= -C_{nm} = \int \frac{\partial \Psi_m}{\partial y}\frac{\Psi_n}{\rho}\,dz\,.
\end{aligned}$$

Adiabatic Approximation - Ignore Coupling

$$\frac{\partial^2 \Phi_n}{\partial x^2} + \frac{\partial^2 \Phi_n}{\partial y^2} + k_{rn}^2(x,y)\,\Phi_n = -\Psi_n(z_s)\,\delta(x)\,\delta(y)\,.$$

Solution Techniques

- Ray tracing
- Parabolic Equation
- ‘Layer Method’ – Direct Global Matrix



Discrete Mode Coupling

3-D Helmholtz equation

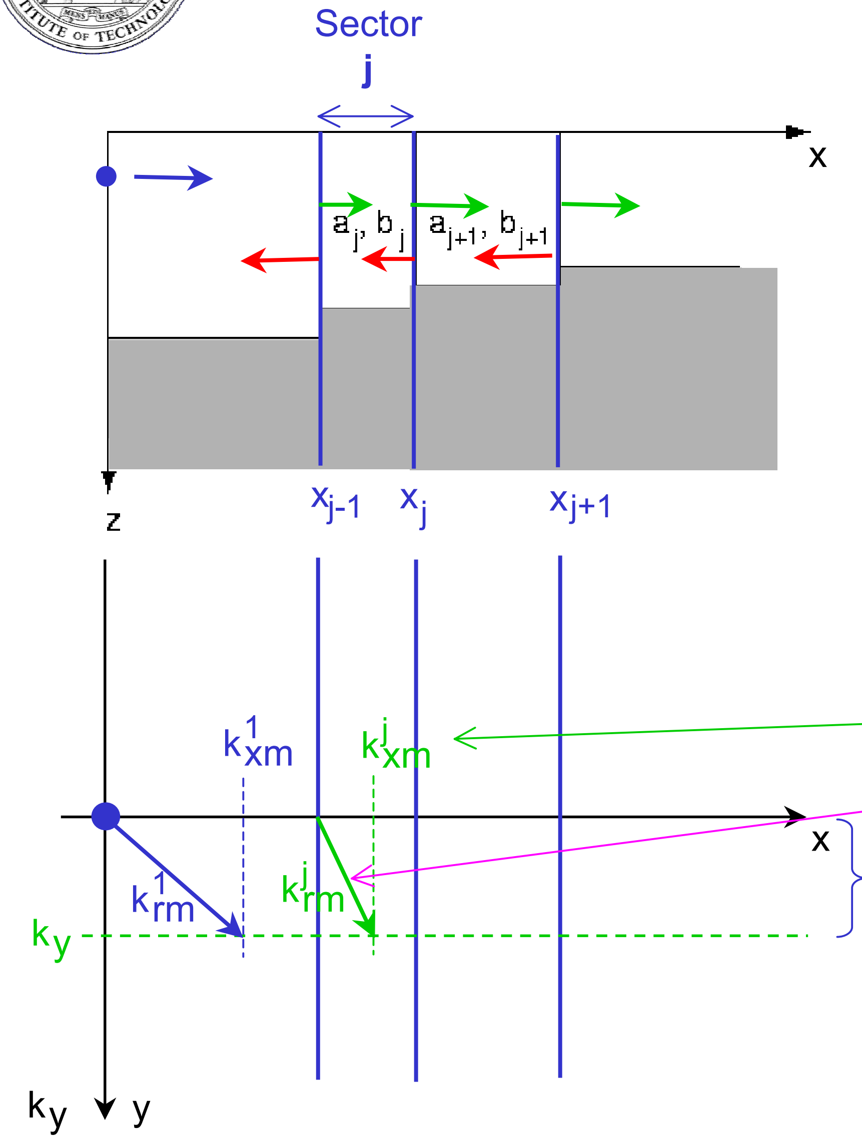
$$\rho \nabla \cdot \left(\frac{1}{\rho} \nabla p \right) + \frac{\omega^2}{c^2(x,y,z)} p = -\delta(x)\,\delta(y)\,\delta(z - z_s) \;,$$

2-D Environment

$$\begin{aligned} c(x,y,z) &= c(x,z) \\ \rho(x,y,z) &= \rho(x,z) \end{aligned}$$

Fourier Transform

$$\begin{aligned} f(x,y) &= \int_{-\infty}^{\infty} f(x,k_y)\, e^{ik_y y}\, dk_y \;, \\ f(x,k_y) &= \frac{1}{2\pi} \int_{-\infty}^{\infty} f(x,y)\, e^{-ik_y y}\, dy \;, \end{aligned}$$



2-D Separated Helmholtz Equation

$$\rho \frac{\partial}{\partial x}\left(\frac{1}{\rho}\frac{\partial p}{\partial x}\right) + \rho \frac{\partial}{\partial z}\left(\frac{1}{\rho}\frac{\partial p}{\partial z}\right) + \left[\frac{\omega^2}{c^2(x,z)} - k_y^2\right] p(x,z;k_y) = \frac{-\delta(x)\,\delta(z-z_s)}{2\pi}\,.$$

Modal Equation

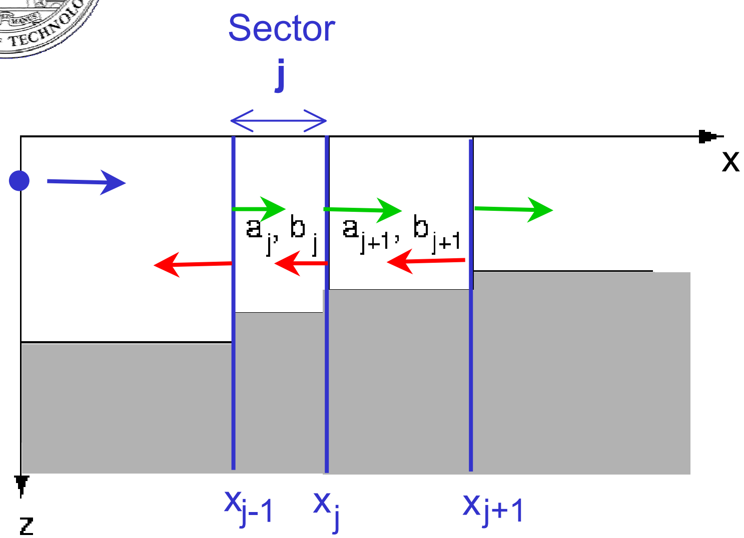
$$\rho(z)\frac{d}{dz}\left[\frac{1}{\rho(z)}\frac{d\Psi_m(z;k_y)}{dz}\right] + \left[\frac{\omega^2}{c^2(z)} - \boxed{(k_y^2 + k_{xm}^2(k_y))}\right]\Psi_m(z;k_y) = 0\,,$$

Eigenvalues and Eigenfunctions

$$\boxed{k_{xm}^2(k_y)} = \boxed{k_{rm}^2} - \boxed{k_y^2}$$

$$\Psi_m(z;k_y) = \Psi_m(z)$$

Eigenvalue Problem Independent of k_y



Coupled Mode Solution - Sector j

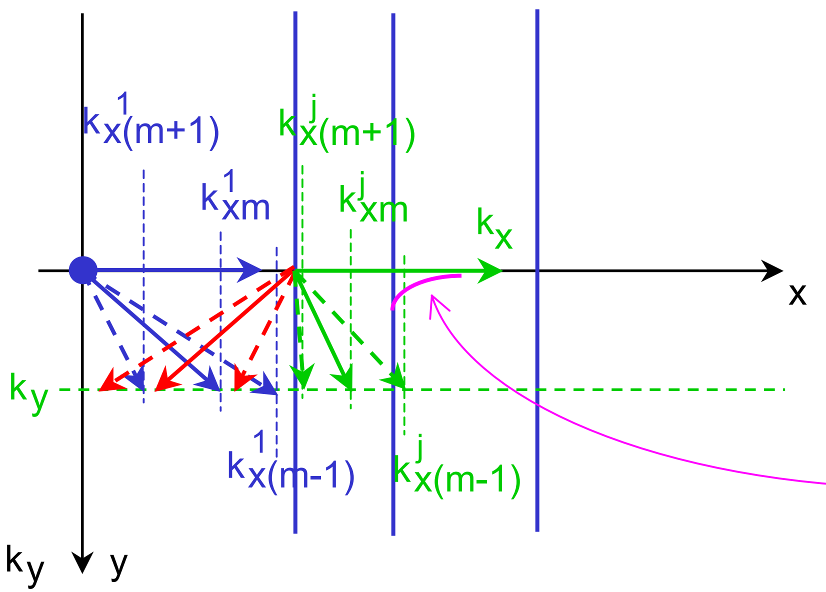
$$\tilde{p}^j(x, z; k_y) = \sum_{m=1}^{M} \left[a_m^j e^{ik_{xm}^j(k_y)(x-x_{j-1})} + b_m^j \, e^{ik_{xm}^j(k_y)(x_j - x)} \right] \Psi_m^j(z) \, ,$$

Source Contribution

$$\tilde{p}^1(x, z; k_y) = \frac{i}{4\pi \, \rho(z_s)} \sum_{m=1}^{\infty} \Psi_m^1(z_s; k_y) \, \Psi_m^1(z; k_y) \, \frac{e^{ik_{xm}^1(k_y)|x|}}{k_{xm}^1(k_y)} .$$

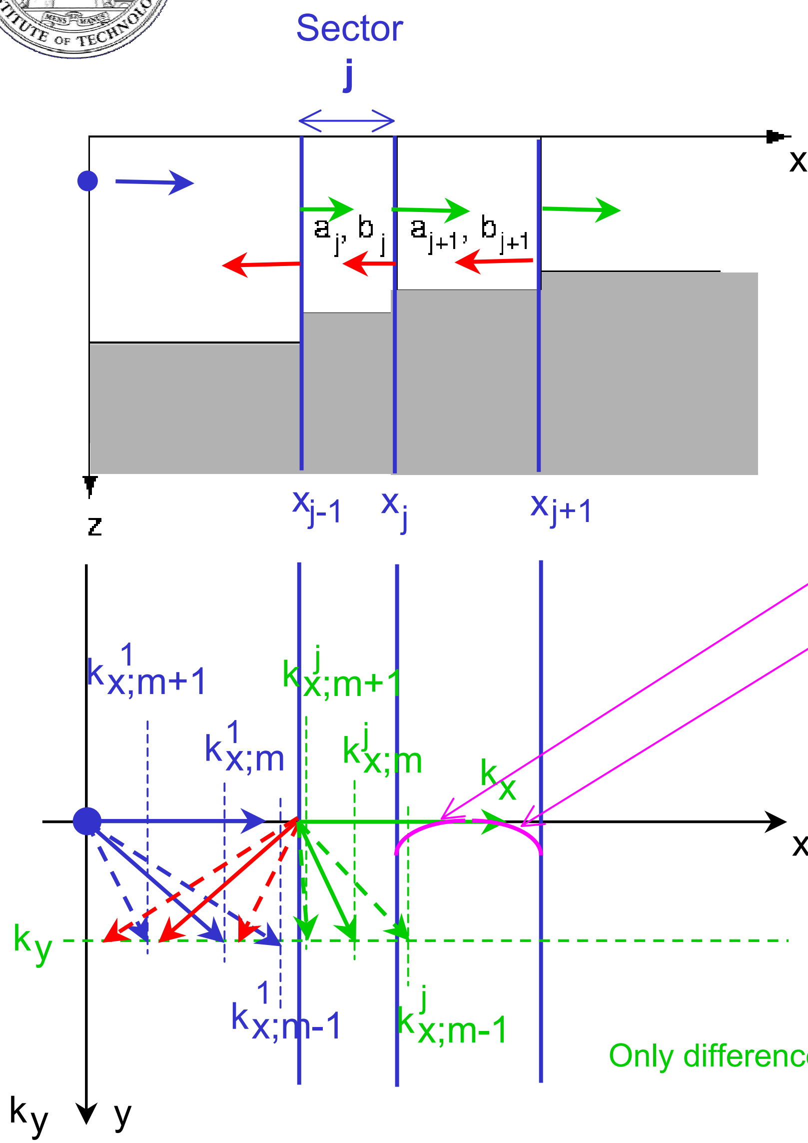
Propagation Wavenumber

$$\begin{aligned} k_{xm}^j(k_y) &= \sqrt{(k_{rm}^j)^2 - k_y^2} \\ &= \begin{cases} \sqrt{(k_{rm}^j)^2 - k_y^2}, & |k_y| \le k_{rm}^j \\ i\sqrt{k_y^2 - (k_{rm}^j)^2}, & |k_y| > k_{rm}^j \end{cases} \end{aligned}$$



Evanescent Modes

Pressure Continuity - Interface j



$$p^{j+1}(x^j; k_y) = p^j(x^j; k_y))$$

Matrix Notation

$$\mathbf{a}^{j+1} + \mathbf{E}_2^{j+1}\,\mathbf{b}^{j+1} = \widetilde{\mathbf{C}}^j(k_y)\left(\mathbf{E}_1^j\,\mathbf{a}^j + \mathbf{b}^j\right) ,$$

$$\begin{aligned}
\mathbf{E}_1^j &= \operatorname{diag}(e^{ik_{xm}^j(k_y)(x-x_{j-1})}) \\
\mathbf{E}_2^j &= \operatorname{diag}(e^{ik_{xm}^j(k_y)(x_j-x)}) \\
\widetilde{\mathbf{C}}_{lm}^j(k_y) &= \int \frac{\Psi_l^{j+1}(z)\,\Psi_m^j(z)}{\rho_{j+1}(z)}\,dz\,.
\end{aligned}$$

Normal Velocity Continuity - Interface j

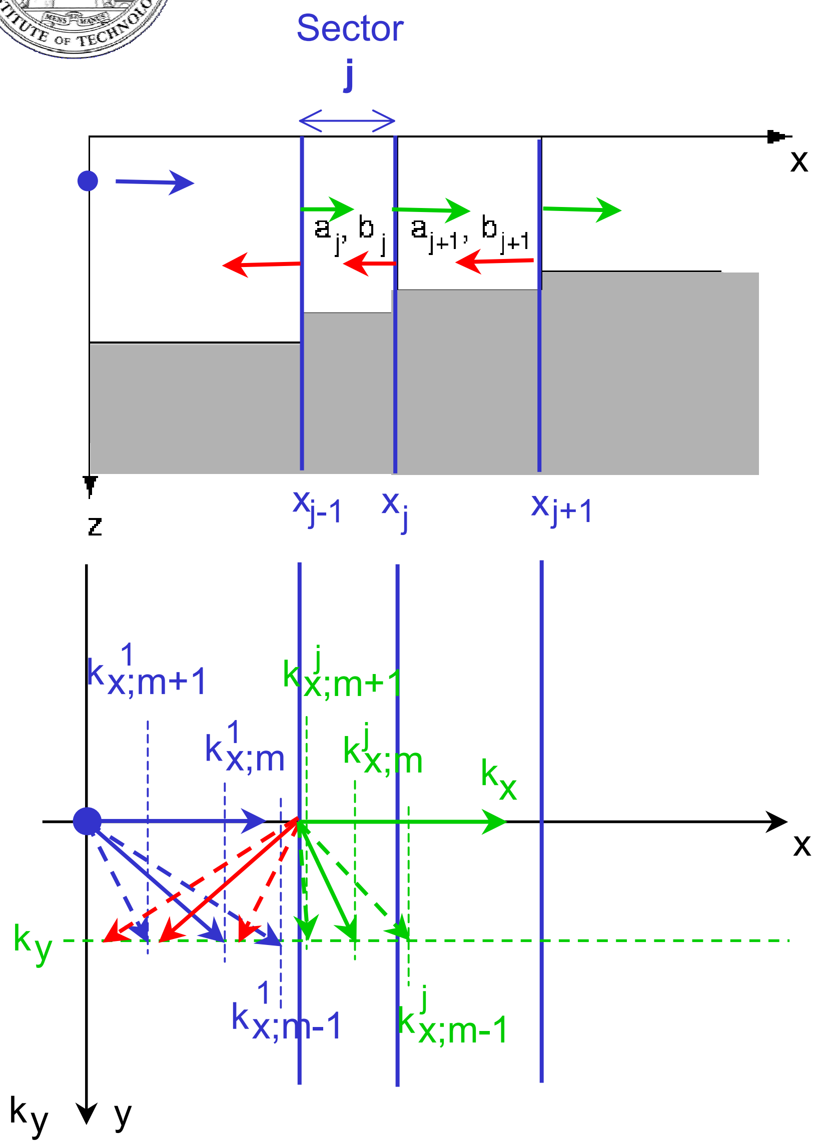
$$\frac{1}{\rho^{j+1}(z)}\frac{\partial p^{j+1}(x^j;k_y)}{\partial x} = \frac{1}{\rho^j(z)}\frac{\partial p^j(x^j;k_y)}{\partial x}$$

Matrix Notation

$$\mathbf{a}^{j+1} - \mathbf{E}_2^{j+1}\,\mathbf{b}^{j+1} = \widehat{\mathbf{C}}^j(k_y)\left(\mathbf{E}_1^j\,\mathbf{a}^j - \mathbf{b}^j\right) .$$

Only difference for 3D

$$\widehat{\mathbf{C}}_{lm}(k_y) = \boxed{\frac{k_{xm}^j(k_y)}{k_{xl}^{j+1}(k_y)}}\int \frac{\Psi_l^{j+1}(z)\,\Psi_m^j(z)}{\rho_j(z)}\,dz\,.$$

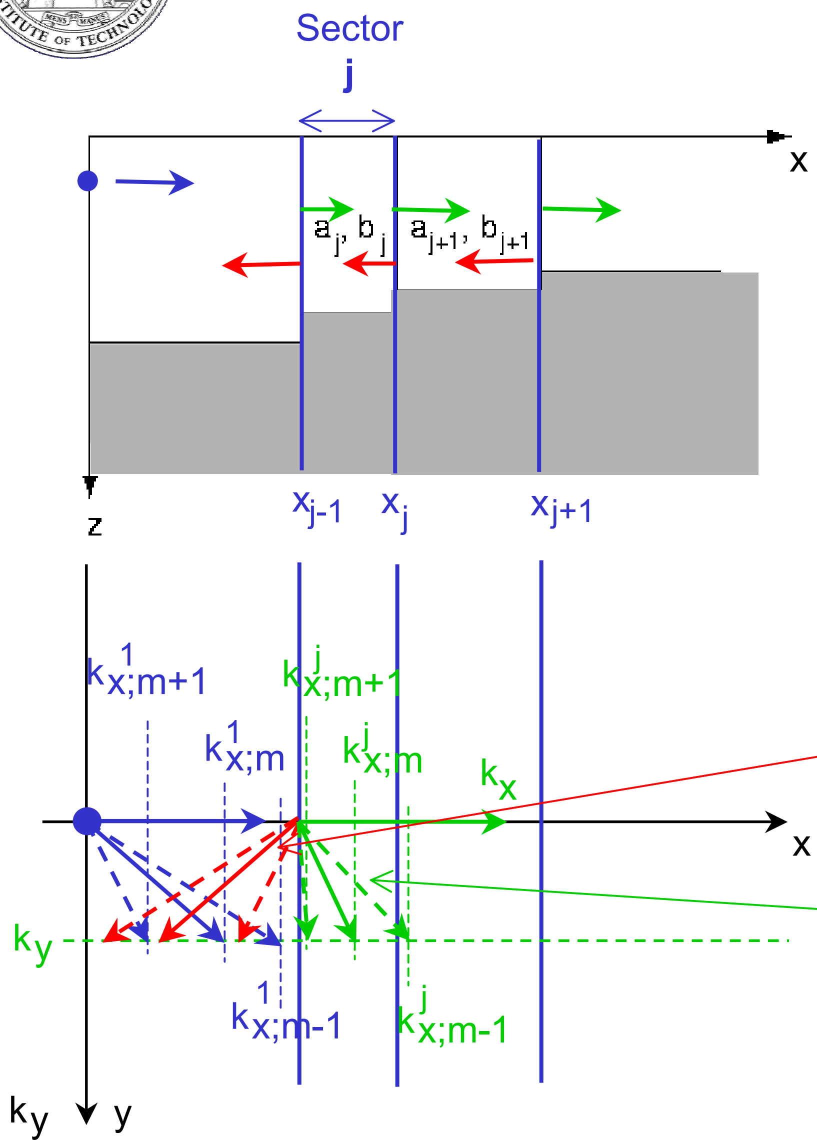


Combined Coupling Equations

$$\begin{bmatrix} \mathbf{a}^{j+1} \\ \mathbf{b}^{j+1} \end{bmatrix} = \begin{bmatrix} \mathbf{R}_1^j & \mathbf{R}_2^j \\ \mathbf{R}_3^j & \mathbf{R}_4^j \end{bmatrix} \begin{bmatrix} \mathbf{a}^j \\ \mathbf{b}^j \end{bmatrix},$$

$$\begin{aligned}
\mathbf{R}_1^j &= \frac{1}{2}\left(\widetilde{\mathbf{C}}^j + \widehat{\mathbf{C}}^j\right)\mathbf{E}_1^j\,, \\
\mathbf{R}_2^j &= \frac{1}{2}\left(\widetilde{\mathbf{C}}^j - \widehat{\mathbf{C}}^j\right)\,, \\
\mathbf{R}_3^j &= \frac{1}{2}\left(\widetilde{\mathbf{C}}^j - \widehat{\mathbf{C}}^j\right)\left(\mathbf{E}_2^{j+1}\right)^{-1}\mathbf{E}_1^j\,, \\
\mathbf{R}_4^j &= \frac{1}{2}\left(\widetilde{\mathbf{C}}^j + \widehat{\mathbf{C}}^j\right)\left(\mathbf{E}_2^{j+1}\right)^{-1}\,.
\end{aligned}$$

One-Way Coupled Modes



Coupling Equations Interface j

$$\begin{bmatrix} \mathbf{a}^{j+1} \\ \mathbf{b}^{j+1} \end{bmatrix} = \begin{bmatrix} \mathbf{R}_1 & \mathbf{R}_2 \\ \mathbf{R}_3 & \mathbf{R}_4 \end{bmatrix} \begin{bmatrix} \mathbf{a}^{j} \\ \mathbf{b}^{j} \end{bmatrix} .$$

Ignore Backscatter from next interface:

$$\mathbf{b}^{j+1} = \mathbf{0}$$

Back-Scattered Amplitudes

$$\mathbf{b}^{j} = -\mathbf{R}_4^{-1}\mathbf{R}_3\,\mathbf{a}^{j} .$$

Forward-Scattered Amplitudes

$$\mathbf{a}^{j+1} = \left(\mathbf{R}_1 - \mathbf{R}_2\,\mathbf{R}_4^{-1}\mathbf{R}_3\right)\mathbf{a}^{j} ,$$

Approximate Single-Scatter Solution

$$\mathbf{a}^{j+1} = \mathbf{R}_1\,\mathbf{a}^{j} .$$

3-D Modal Modeling Framework

3-D Ocean Environment

[See Jensen Fig 5.19a]

Range-Independent Sectors

[See Jensen Fig 5.19b]

Computational Procedure

1. Pre-compute modes for all sectors
2. Each source-receiver combination
   - Horizontal ray tracing, all mode combinations
   - Local single-scattering approximation in plane geometry
   - Approximate accounting for geometric spreading $r^{-1/2}$