Problem ...

A simply supported beam is subject to a distributed load $w = -w_o * \cos\left(2\pi \frac{x}{L}\right)$

Figure shown below

a) Determine reaction forces at the ends
b) Plot shear-force and bending moment diagrams
c) Model the distributed load as a set of four concentrated loads determined by integrating over each quarter length. Locate the load at the mid length of each segment. I.e.

$$F_2 = \int_{\frac{L}{4}}^{\frac{L}{2}} w(x)dx \quad \text{located at } x = \frac{3}{8}L$$

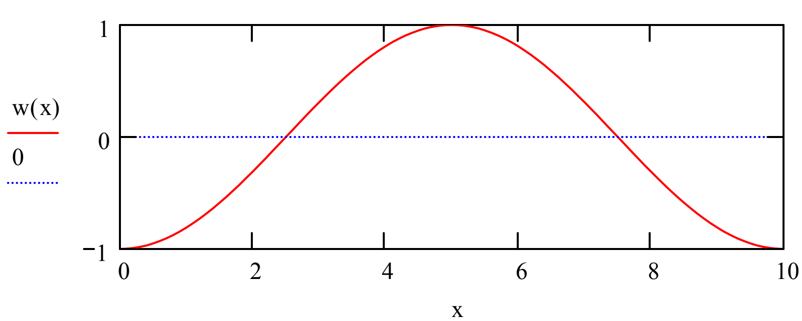
Comment on the static equivalency of this model of the distributed load (is it equivalent in force and moment?)

d) Plot shear-force and bending moment diagram for this loading. Comment on the comparison with b) above.

Solution ....

$w_o := 1$ $\quad L := 10$ $\quad x := 0, 0.1 .. L$ $\quad w(x) := -w_o \cdot \cos\left(2 \cdot \pi \cdot \frac{x}{L}\right)$

as shown, w is positive up but negative in value

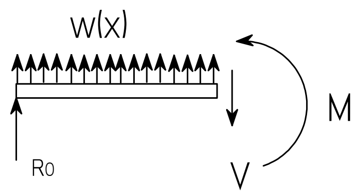


$$\text{sum\_forces} := \int_0^L w(x)\,dx \qquad \text{sum\_forces} = 0 \qquad R_0 + R_L = 0$$

$$\text{moment\_wrt\_R0} := \int_0^L x\cdot w(x)\,dx \qquad \text{moment\_wrt\_R0} = 0$$

a) $R_L\cdot L - \text{moment\_wrt\_R0} = 0 \qquad R_L := 0 \qquad R_0 := 0$

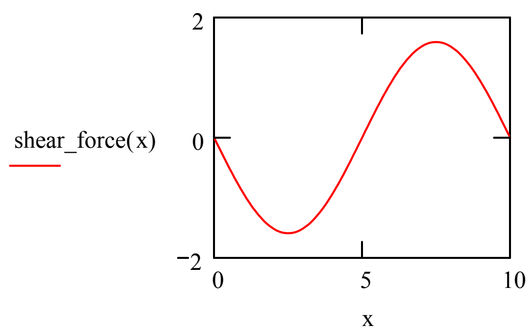
b) equilibrium $R_0 + \int_0^x w(\xi)\,d\xi - V = 0$



$$\text{shear\_force}(x) := \int_0^x w(\xi)\,d\xi$$

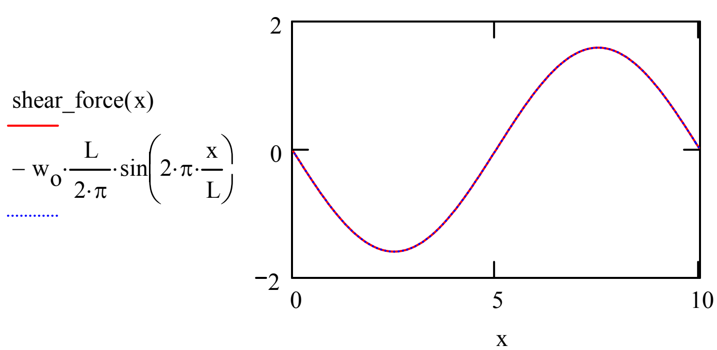
hold this as

$$\text{shear\_force1}(x) := \text{shear\_force}(x)$$



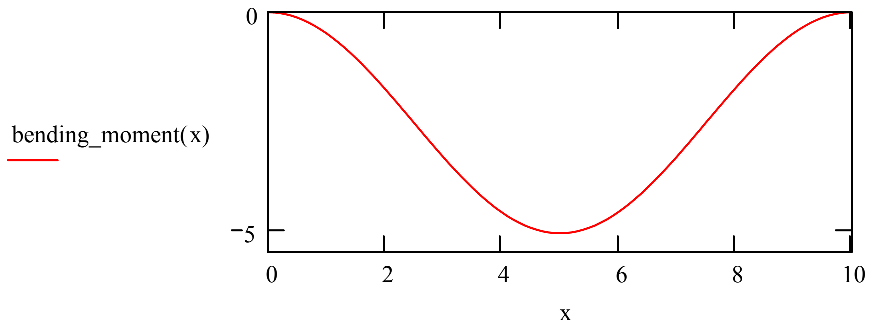
or ... analytically $w1(x) := -w_o\cdot\cos($

$$\text{shear\_force}(x) = \int_0^x w1(\xi)\,d\xi = -w_o\int_0^x \cos\left(2\cdot\pi\cdot\frac{x}{L}\right)d\xi = -w_o\cdot\frac{L}{2\cdot\pi}\cdot\sin\left(2\cdot\pi\cdot\frac{x}{L}\right)$$



curves overlap

equilibrium $R_0 + \int_0^x w(\xi)\,d\xi - V = 0 \qquad \text{bending\_moment}(x) := \int_0^x \text{shear\_force}(\xi)\,d\xi$



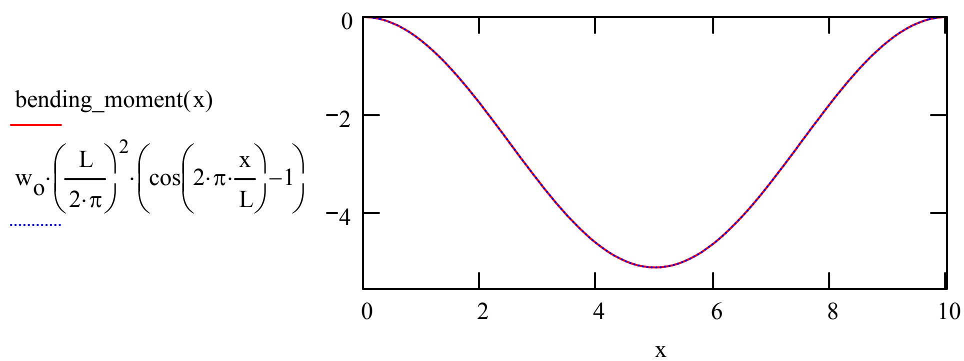
hold this as : $\text{bending\_moment1}(x) := \text{bending\_moment}(x)$

or ... analytically:

$$\text{bending\_moment}(x) = \int_0^x \text{shear\_force}(\xi)\,d\xi = -w_o \cdot \frac{L}{2\cdot\pi} \cdot \left(\int_0^x \sin\left(2\cdot\pi\cdot\frac{x}{L}\right) d\xi \right)$$

$$\text{bending\_moment}(x) = w_o \cdot \left(\frac{L}{2\cdot\pi}\right)^2 \cdot \left(\cos\left(2\cdot\pi\cdot\frac{x}{L}\right) - 1\right)$$

the 1 is from the lower limit



c) model in segments:

following the notation in the notes:

$i := 0..3$ $\quad L = 10$

$$\xi_{i,0} := i\cdot\frac{L}{4} \qquad \xi_{i,1} := (i+1)\cdot\frac{L}{4} \qquad \xi = \begin{pmatrix} 0 & 2.5 \\ 2.5 & 5 \\ 5 & 7.5 \\ 7.5 & 10 \end{pmatrix}$$

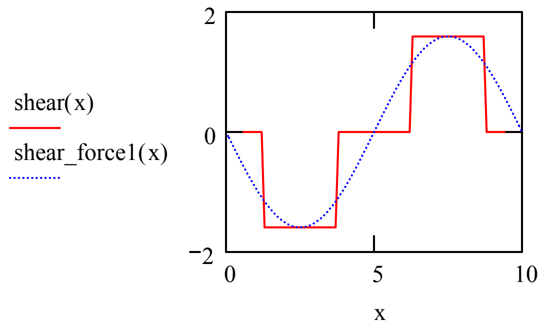
$$f_i := \int_{\xi_{i,0}}^{\xi_{i,1}} w(x)\,dx \qquad f = \begin{pmatrix} -1.592 \\ 1.592 \\ 1.592 \\ -1.592 \end{pmatrix}$$

located at $xx_i := \frac{\xi_{i,1} + \xi_{i,0}}{2}$ $\quad xx = \begin{pmatrix} 1.25 \\ 3.75 \\ 6.25 \\ 8.75 \end{pmatrix}$

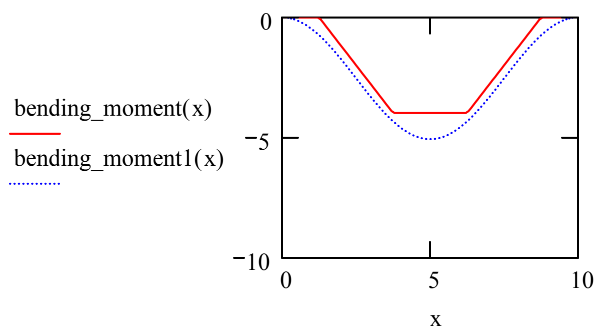
$x := 0, 0.1 .. L \qquad ll := 0 \qquad ul := 3$

$$shear(x) := \sum_{i=ll}^{ul} f_i \cdot (x \geq xx_i)$$

plotted with value from distributed



$$bending\_moment(x) := \sum_{i=ll}^{ul} f_i \cdot (x - xx_i) \cdot (x \geq xx_i)$$



comment ... shear at the quarter points right on! bending moment a bit underestimated

$$\frac{bending\_moment\left(\frac{L}{2}\right)}{bending\_moment1\left(\frac{L}{2}\right)} = 0.785$$

about 20 % low .. but with just 4 segments

do the same but with 8 segments: (not expected)

$i := 0 .. 7 \qquad L = 10$

$$\xi_{i,0} := i \cdot \frac{L}{8} \qquad \xi_{i,1} := (i+1) \cdot \frac{L}{8} \qquad \xi = \begin{pmatrix} 0 & 1.25 \\ 1.25 & 2.5 \\ 2.5 & 3.75 \\ 3.75 & 5 \\ 5 & 6.25 \\ 6.25 & 7.5 \\ 7.5 & 8.75 \\ 8.75 & 10 \end{pmatrix}$$

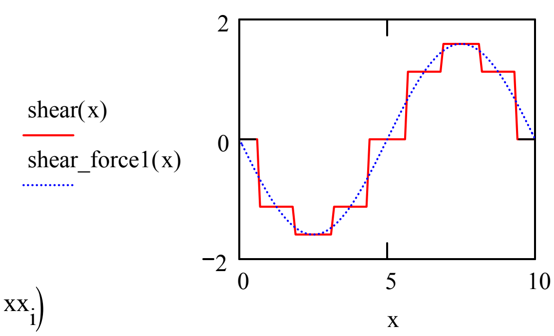
$$f_i := \int_{\xi_{i,0}}^{\xi_{i,1}} w(x)\,dx \qquad f = \begin{pmatrix} -1.125 \\ -0.466 \\ 0.466 \\ 1.125 \\ 1.125 \\ 0.466 \\ -0.466 \\ -1.125 \end{pmatrix}$$

located at $xx_i := \dfrac{\xi_{i,1} + \xi_{i,0}}{2}$ $\qquad xx = \begin{pmatrix} 0.625 \\ 1.875 \\ 3.125 \\ 4.375 \\ 5.625 \\ 6.875 \\ 8.125 \\ 9.375 \end{pmatrix}$

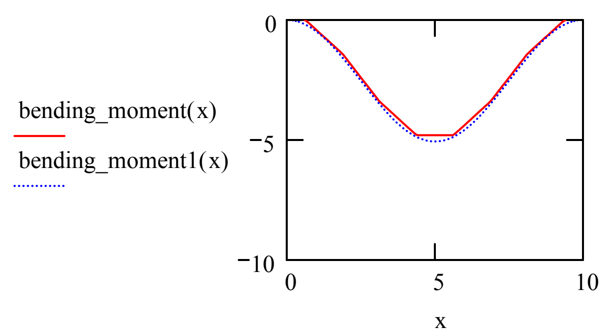
$x := 0, 0.1 .. L \qquad ll := 0 \qquad ul := 7$

$$shear(x) := \sum_{i = ll}^{ul} f_i \cdot \left(x \ge xx_i\right)$$

plotted with value from distributed



$$bending\_moment(x) := \sum_{i = ll}^{ul} f_i \cdot \left(x - xx_i\right) \cdot \left(x \ge xx_i\right)$$



comment ... shear at the quarter points right on! bending moment a bit underestimated

$$\frac{bending\_moment\left(\frac{L}{2}\right)}{bending\_moment1\left(\frac{L}{2}\right)} = 0.948$$

about 5 % low .. with 8 segments