13.122 Lecture 1
Primary Load: Bending Moment and Shear Force

Introduction to course:
- Design process
- Structural design process

General course content:

13.122 Ship Structural Design

A. Loads on ship/offshore platforms
- Calculation of loads
  - buoyancy, shear, bending moment
    - "hand" using excel

B. Review of bending, shear and torsion - open sections

C. Modeling a structure
- Maestro
- checking loads and moments

D. Development of limit states and failure modes
- stress analysis of ship/ocean system structure

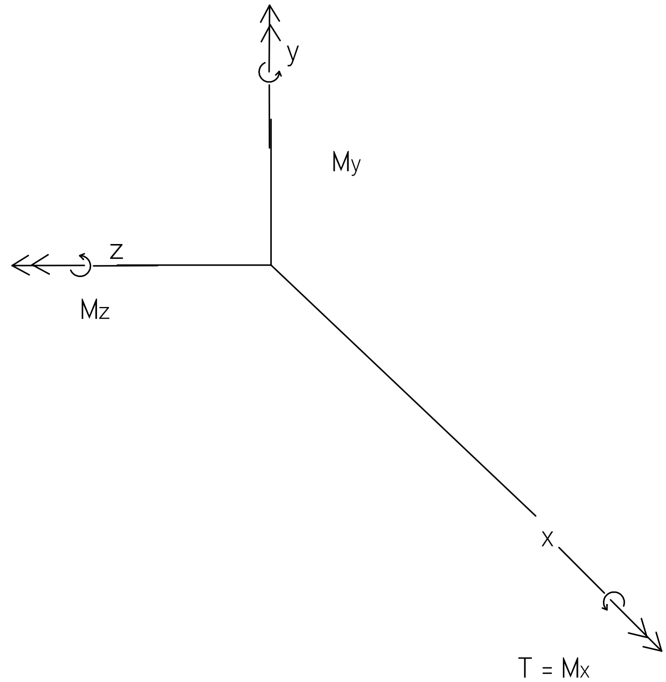
E. Design of section for bending
- project

F. Matrix analysis (Grillage), FEM Introduction

Expected outcome: an ability to effectively use structural design tools with an understanding of the underlying analysis.

Changes from previous years:
- reduced 13.10 review (2002)
- calculation of loads (bending moment and shear force) (2001)
- introduce Maestro earlier for modeling and load analysis (2001)
- introduce open section torsion
- revert to earlier problem sets for mathcad limit state analysis (2001)

Sign Convention



In general, we will use "structural" sign convention described in Shames: 10.2 page 286

An axial force or bending moment acting on a beam cross-section is positive if it acts on a positive face and is directed in a positive coordinate direction. The shear force is positive if it acts in the *negative* direction on a *positive* face.

(positive face defined by outward normal in positive coordinate direction)

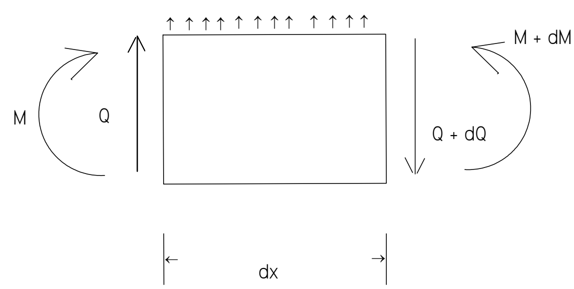
Moment of inertia is defined relative to the axis for measuring distance: $I_z = \int y^2 \, dA$; $I_y = \int z^2 \, dA$

$\sigma_x = \tau_{xx}$ $\tau_{xy} = \tau_{yx}$ etc.

$$\sigma_x = \frac{-M_z \cdot y}{I_z}$$

$$\sigma_x = \frac{M_y \cdot z}{I_y}$$ later .....

define f = load, positive in +y direction,



$$dQ = f \cdot dx$$

$$Q = \int_0^x f(x)\,dx + Q(x = 0) = \int_0^x f(x)\,dx$$

moments around right face =>

$$M + Q \cdot dx + f(x) \cdot dx \cdot \frac{dx}{2} - M + dM = 0 \quad \Rightarrow \quad Q = \frac{d}{dx} M$$

$$M(x) = \int_0^x Q(x)\,dx + M(x = 0) = \int_0^x Q(x)\,dx$$

Part 1: Calculation of loads
buoyancy, shear, bending moment

a) shear and bending moment from distributed force per length
b) shear and bending moment from point force
c) algorithms for calculation

a) shear and bending moment from distributed force per length

suppose we have a distribution of weight per foot w, constant over a distance ξ_0 => ξ_1

mathcad has an expression that handles the < and > relationships:
$a > b$ = 1 if true, 0 if false

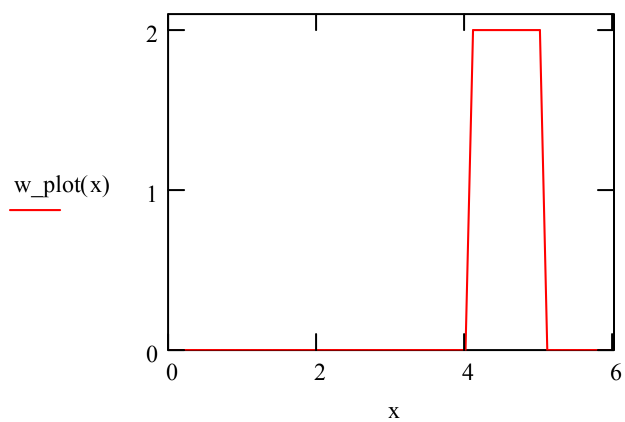
weight per foot   between $\xi_{i,0}$ => $\xi_{i,1}$

$a := 1 \qquad b := 2 \qquad a > b = 0 \qquad b > a = 1$

$$w := \begin{pmatrix} 1 \\ 2 \\ -3 \\ 2 \\ 1 \end{pmatrix} \qquad \xi := \begin{pmatrix} 0 & 1 \\ 1 & 2 \\ 2 & 4 \\ 4 & 5 \\ 5 & 6 \end{pmatrix}$$

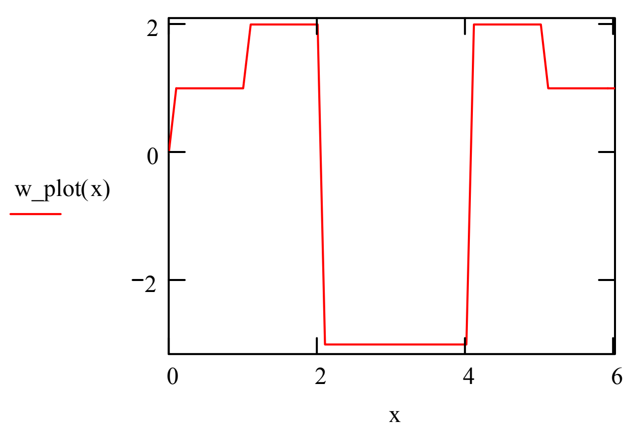
for example: for $i := 3$ and $x := 0, 0.1 .. 6$

$$w\_plot(x) := w_i \cdot (\xi_{i,0} < x \le \xi_{i,1})$$



by superposition: for $i := 0 .. 4$

$$w\_plot(x) := \sum_{i=0}^{4} w_i \cdot (\xi_{i,0} < x \le \xi_{i,1})$$

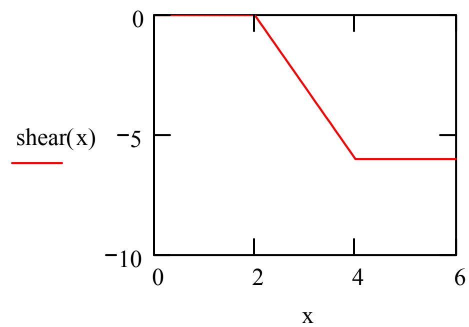


what is shear force from w_i ?

$$\text{shear}(x) := \int_0^x w_i(\xi)\,d\xi \quad = \quad \begin{cases} 0 & \xi_{i,0} > x \\ w_i \cdot (x - \xi_{i,0}) & \xi_{i,0} < x \le \xi_{i,1} \\ w_i \cdot (\xi_{i,1} - \xi_{i,0}) & x > \xi_{i,1} \end{cases}$$

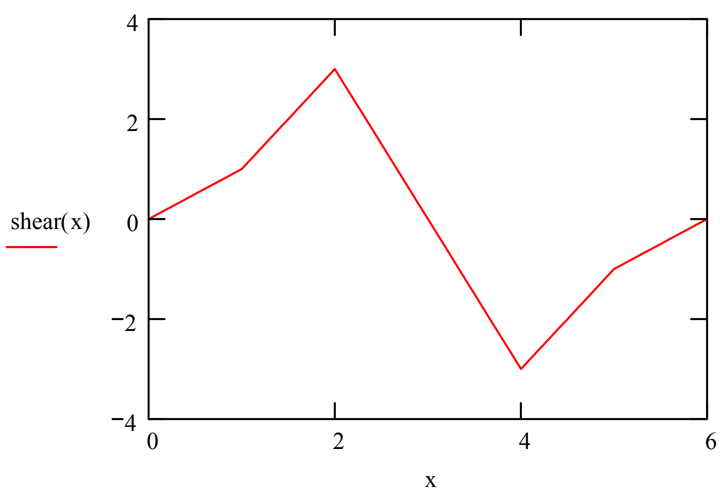
for example: $i := 2$

$$\text{shear}(x) := w_i \cdot (x - \xi_{i,0}) \cdot (\xi_{i,0} < x \le \xi_{i,1}) + w_i \cdot (\xi_{i,1} - \xi_{i,0}) \cdot (x > \xi_{i,1})$$



again by superposition: $i := 0..4$ using
lower limit = $ll := 0$ and upper limit = $ul := 4$

$$\text{shear}(x) := \sum_{i = ll}^{ul} \left[w_i \cdot (x - \xi_{i,0}) \cdot (\xi_{i,0} < x \le \xi_{i,1}) + w_i \cdot (\xi_{i,1} - \xi_{i,0}) \cdot (x > \xi_{i,1}) \right]$$



for later comparison, let's rename this $\text{shear}_1(x) := \text{shear}(x)$
note that shear curve starts and ends at 0; why??

$$\sum_i w_i \cdot (\xi_{i,1} - \xi_{i,0}) = 0$$

now what is bending moment from weight per foot as above?

$$\text{bending\_moment}(x) := \int_0^x \text{shear}(\xi)\, d\xi =$$

$$\int_0^x \left[w_i\cdot(x-\xi_{i,0})\cdot(\xi_{i,0} < x \le \xi_{i,1}) + w_i\cdot(\xi_{i,1}-\xi_{i,0})\cdot(x > \xi_{i,1})\right] d\xi$$

$= \;\; 0$ when ; $x < \xi_{i,0}$

$= w_i\cdot\left[\left(\frac{x^2}{2} - \xi_{i,0}\cdot x\right) - \left[\frac{(\xi_{i,0})^2}{2} - (\xi_{i,0})^2\right]\right]$ when $\xi_{i,0} < x < \xi_{i,1}$ or : $= \frac{w_i}{2}\cdot(x-\xi_{i,0})^2$ after simplifying

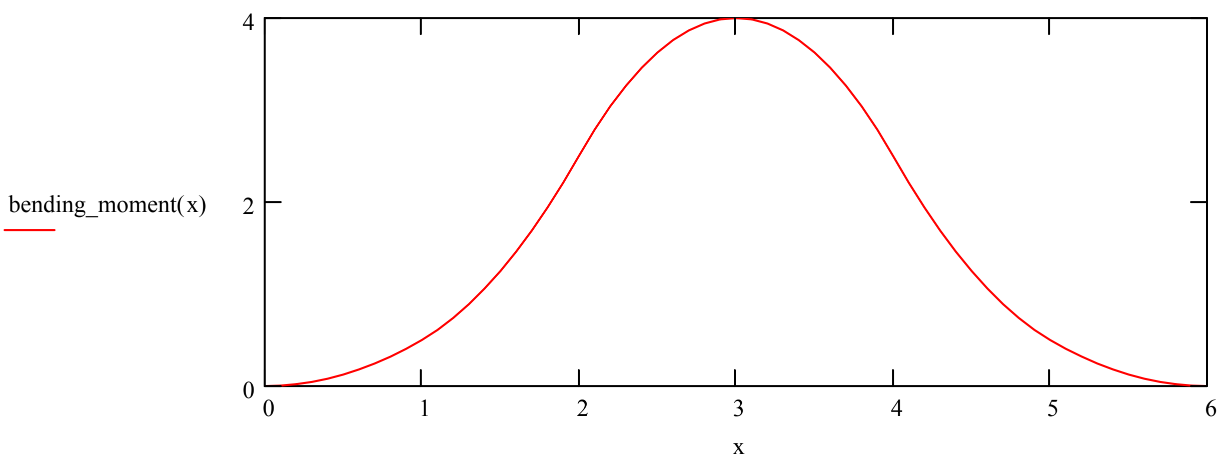
$= \left[\frac{w_i}{2}\cdot(x-\xi_{i,0})^2\right]\cdot(\xi_{i,0} < x \le \xi_{i,1})$ using the notation above.

and $= w_i\cdot\left[(\xi_{i,1}-\xi_{i,0})\cdot(x-\xi_{i,1}) + \frac{1}{2}\cdot(\xi_{i,1}-\xi_{i,0})\cdot(\xi_{i,1}-\xi_{i,0})\right]\cdot(x > \xi_{i,1})$ when $x > \xi_{i,1}$

simplifying => $\left(\frac{x^2}{2} - \xi_{i,0}\cdot x\right) - \left[\frac{(\xi_{i,0})^2}{2} - (\xi_{i,0})^2\right] \quad = \quad \frac{x^2}{2} - \xi_{i,0}\cdot x + \frac{(\xi_{i,0})^2}{2} \quad = \quad \frac{1}{2}\cdot(x-\xi_{i,0})^2$

therefore; bending moment (x) = for lower limit: $ll := 0$ and upper limit = $ul := 4$

$$\text{bending\_moment}(x) := \sum_{i=ll}^{ul} \left[w_i\cdot\left[\frac{1}{2}\cdot(x-\xi_{i,0})^2\right]\cdot(\xi_{i,0} < x \le \xi_{i,1}) \ldots \atop + w_i\cdot\left[\left[(\xi_{i,1}-\xi_{i,0})\cdot(x-\xi_{i,1}) + \frac{1}{2}\cdot(\xi_{i,1}-\xi_{i,0})\cdot(\xi_{i,1}-\xi_{i,0})\right]\cdot(x > \xi_{i,1})\right] \right]$$



for later comparison we will save this as $\text{bending\_moment}_1(x) := \text{bending\_moment}(x)$

b) shear and bending moment from point force

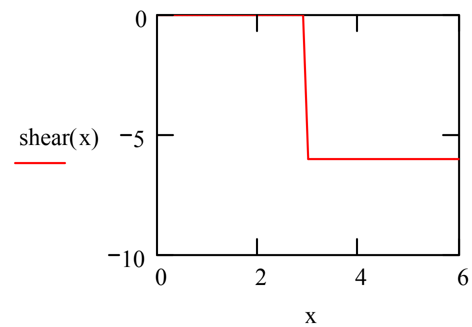
now consider if load is concentrated at a point between $\xi_{i,0}$ and $\xi_{i,1}$;

define $f_i := w_i \cdot (\xi_{i,1} - \xi_{i,0})$ when x = xx_i i. e. and we will define $xx_i := \frac{(\xi_{i,1} + \xi_{i,0})}{2}$

this is not necessary as xx_i can be at any position (but usually between the end points).

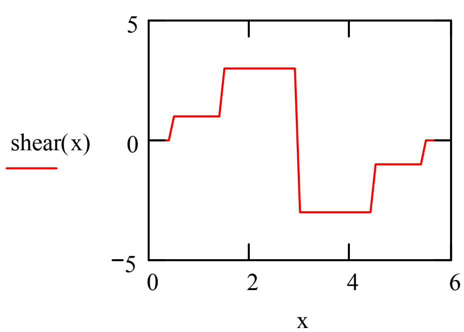
In this case $shear(x) := f_i \cdot (x \geq xx_i)$

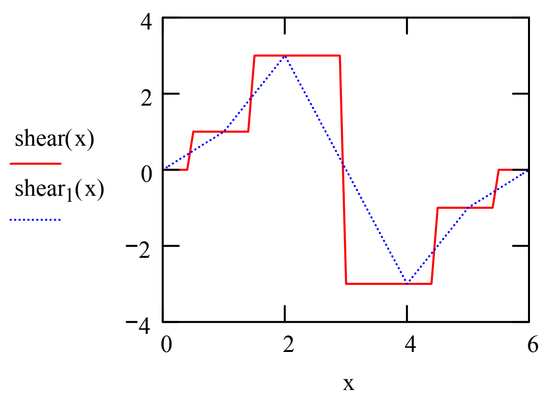
for example: $i := 2$ $xx_i = 3$ $shear(x) := f_i \cdot (x \geq xx_i)$



as above total shear along beam with $ll := 0$ and $ul := 4$ => $shear(x) := \sum_{i=ll}^{ul} f_i \cdot (x \geq xx_i)$

let's compare this version with the earlier weight per foot $shear_1(x)$





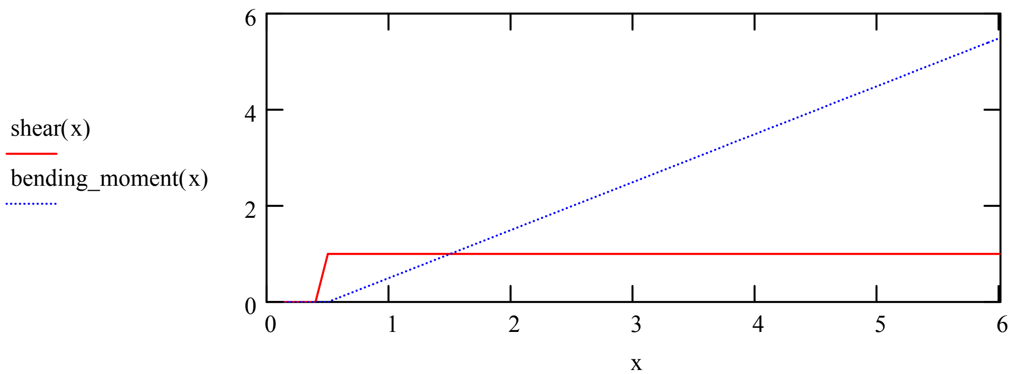
now for bending moment: $\text{bending\_moment}(x) := \int_0^x \text{shear}(\xi)\,d\xi$; as above let's look first at bending moment

from each component: $\text{shear}(x) := f_i \cdot (x \ge xx_i)$

$$\text{bending\_moment}(x) := \int_0^x f_i \cdot (x \ge xx_i)\,d\xi \;=\; \text{bending\_moment}(x) := f_i \cdot (x - xx_i) \cdot (x \ge xx_i)$$

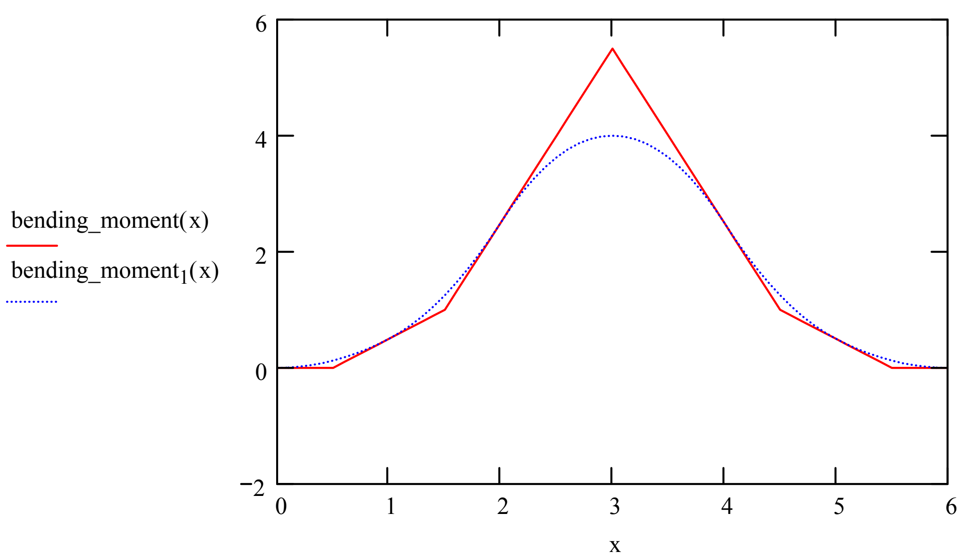
plotting shear and bending moment for $i := 0$; $\text{shear}(x) := f_i \cdot (x \ge xx_i)$ and

$\text{bending\_moment}(x) := f_i \cdot (x - xx_i) \cdot (x \ge xx_i)$



and as above total bending moment is the superposition of all components:

$\text{bending\_moment}(x) := \sum_{i=ll}^{ul} f_i \cdot (x - xx_i) \cdot (x \ge xx_i)$ again comparing with the weight per foot from above



x

this doesn't look all that great, but we let a constant 3 span two intervals, what if we split it into two parts?

weight per foot   between $\xi_{i,0}$ => $\xi_{i,1}$

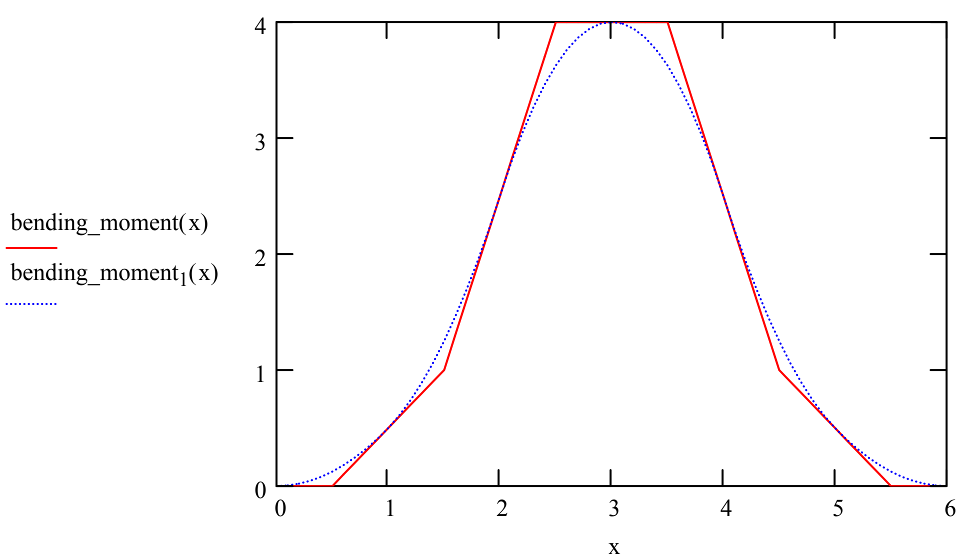
$i := 0..5$

$ll := 0$

$ul := 5$

$$w := \begin{pmatrix} 1 \\ 2 \\ -3 \\ -3 \\ 2 \\ 1 \end{pmatrix} \qquad \xi := \begin{pmatrix} 0 & 1 \\ 1 & 2 \\ 2 & 3 \\ 3 & 4 \\ 4 & 5 \\ 5 & 6 \end{pmatrix} \qquad xx_i := \frac{\left(\xi_{i,1} + \xi_{i,0}\right)}{2} \qquad xx = \begin{pmatrix} 0.5 \\ 1.5 \\ 2.5 \\ 3.5 \\ 4.5 \\ 5.5 \end{pmatrix}$$

$$\text{bending\_moment}(x) := \sum_{i=ll}^{ul} f_i \cdot \left(x - xx_i\right) \cdot \left(x \geq xx_i\right) \qquad f_i := w_i \cdot \left(\xi_{i,1} - \xi_{i,0}\right)$$



c) algorithms for calculation

problem: distribution of point loads along length, located at ~midpoint between xi and xi+1

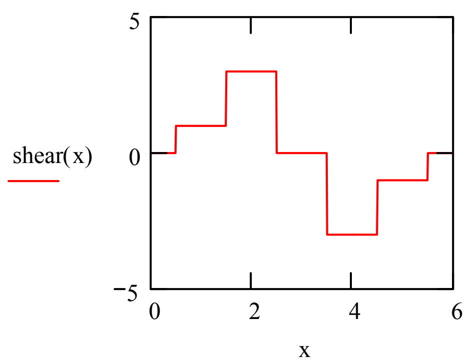
$nsta := 7$

$ul := nsta - 1 \qquad ll := 0 \qquad\qquad j := 0 .. ul - 1$

$$f := \begin{pmatrix} 1 \\ 2 \\ -3 \\ -3 \\ 2 \\ 1 \end{pmatrix} \qquad xs := \begin{pmatrix} 0 \\ 1 \\ 2 \\ 3 \\ 4 \\ 5 \\ 6 \end{pmatrix} \qquad xx_j := \frac{xs_j + xs_{j+1}}{2} \qquad xx = \begin{pmatrix} 0.5 \\ 1.5 \\ 2.5 \\ 3.5 \\ 4.5 \\ 5.5 \end{pmatrix}$$

$x := xs_0, xs_0 + 0.01 .. xs_{ul}$

$$shear(x) := \sum_{i = ll}^{ul-1} f_i \cdot (x \ge xx_i)$$

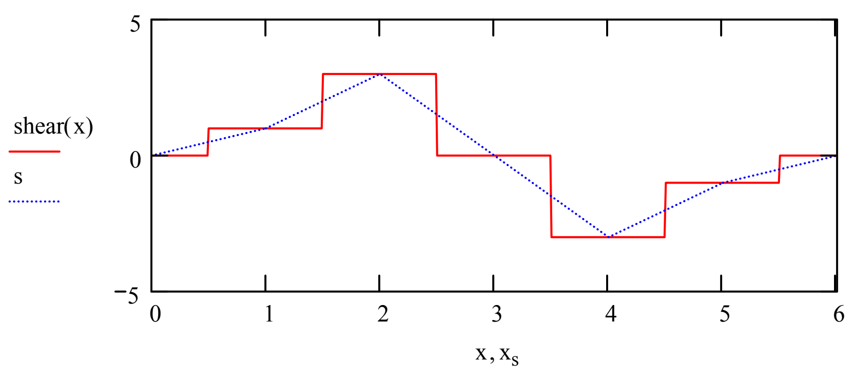


can represent by values at endpoint of stations: eliminates (x>xs) term

location

$s_0 := 0 \qquad i := 0 .. ul - 1 \qquad s_{i+1} := s_i + f_i$

$$s = \begin{pmatrix} 0 \\ 1 \\ 3 \\ 0 \\ -3 \\ -1 \\ 0 \end{pmatrix} \qquad xs = \begin{pmatrix} 0 \\ 1 \\ 2 \\ 3 \\ 4 \\ 5 \\ 6 \end{pmatrix}$$



bending moment algorithm: as above

$$bm = \int_0^x shear(x)\,dx$$

$$bm\left(x = xs_i\right) = \int_0^x shear(x)\,dx = \int_0^{xs_{i-1}} shear(x)\,dx + \int_{xs_{i-1}}^{xs_i} shear(x)\,dx = bm\left(x = xs_{i-1}\right) + \int_{xs_{i-1}}^{xs_i} shear(x)\,dx$$

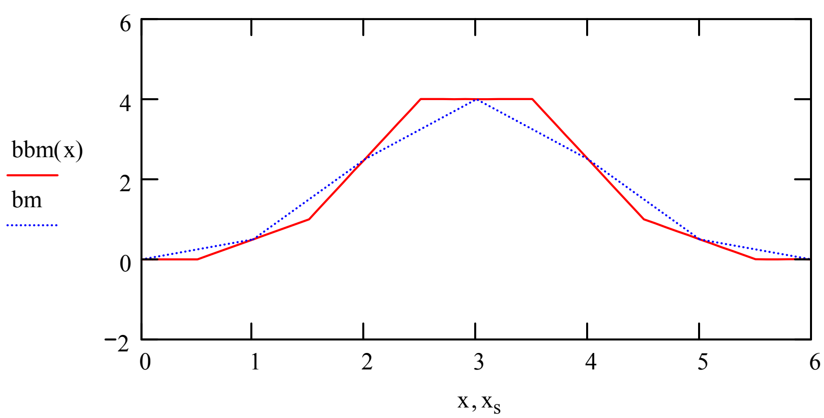
either model for shear =>

$$\int_{xs_{i-1}}^{xs_i} shear(x)\,dx = \left(shear\left(xs_i\right) + shear\left(xs_{i-1}\right)\right)\cdot\frac{\left(xs_i - xs_{i-1}\right)}{2} = \left(\frac{s_{i-1} + s_i}{2}\right)\cdot\left(xs_i - xs_{i-1}\right)$$

=> in index form:

$$bm_0 := 0 \qquad i := 1 .. 6 \qquad bm_i := bm_{i-1} + \left(\frac{s_{i-1} + s_i}{2}\right)\cdot\left(xs_i - xs_{i-1}\right)$$

$$x := xs_0, xs_0 + 0.1 .. xs_{ul} \qquad bbm(x) := \int_0^x shear(x)\,dx$$ for comparison



this algorithm is amenable to iterative schemes such as could be done in excel:

load	station	shear	bending	
	location	force	moment	
	0	0	0	
1				
	1	1	0.5	
2				
	2	3	2.5	
-3				
	3	0	4	
-3				
	4	-3	2.5	
2				
	5	-1	0.5	
1				
	6	0	0	

$$s = \begin{pmatrix} 0 \\ 1 \\ 3 \\ 0 \\ -3 \\ -1 \\ 0 \end{pmatrix} \qquad bm = \begin{pmatrix} 0 \\ 0.5 \\ 2.5 \\ 4 \\ 2.5 \\ 0.5 \\ 0 \end{pmatrix}$$

next we will develop the load from weight-buoyancy