13.122 Lecture 2

Shear force and bending moment in floating platform

Ship or freely floating offshore structure is a beam in equilibrium

Overall summation and forces and moments = 0

But shear force and bending moments can and do exist

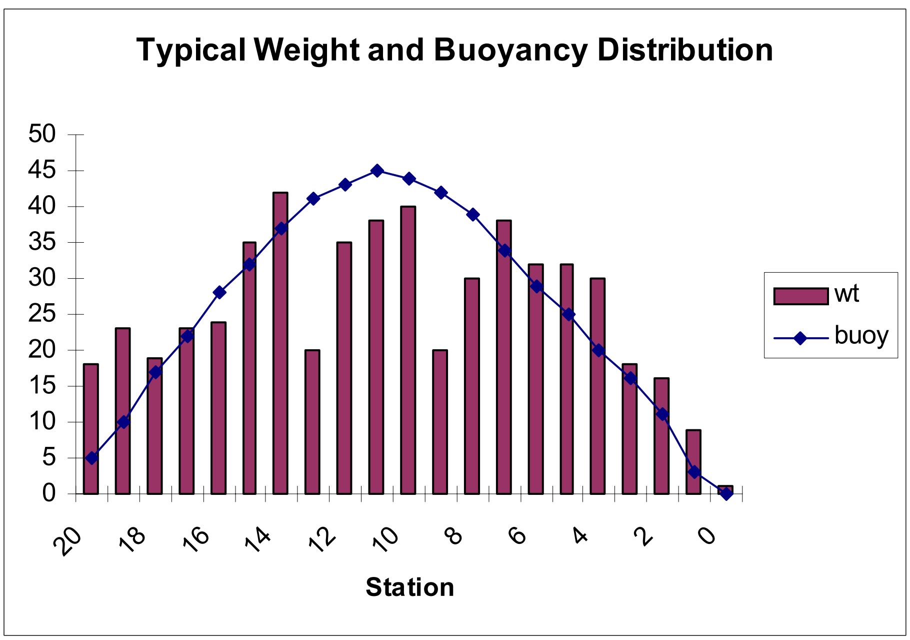
Net force along length f(x) depends on buoyancy - weight at x

Even though these are continuous (not in the mathematical sense) functions of location, typically they are represented by subtotals between regularly spaced locations or stations, normally 10 or 20 between perpendiculars, with segments forward and aft of FP and AP respectively

Weight is dependent on conditions of loading as well as location of equipment (arrangement), structure etc.

Buoyancy is dependent on the immersion of the platform along its length

A typical weight and buoyancy distribution is shown below.

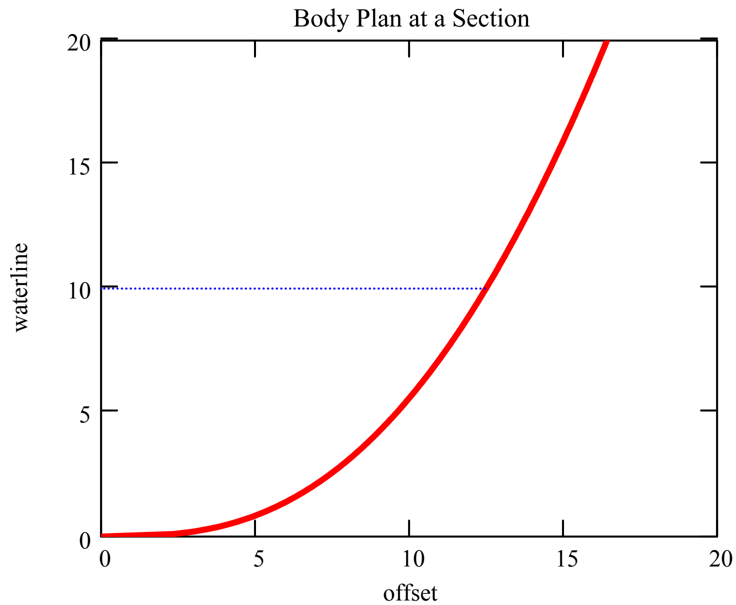


note that station 0 (FP) is to the right.

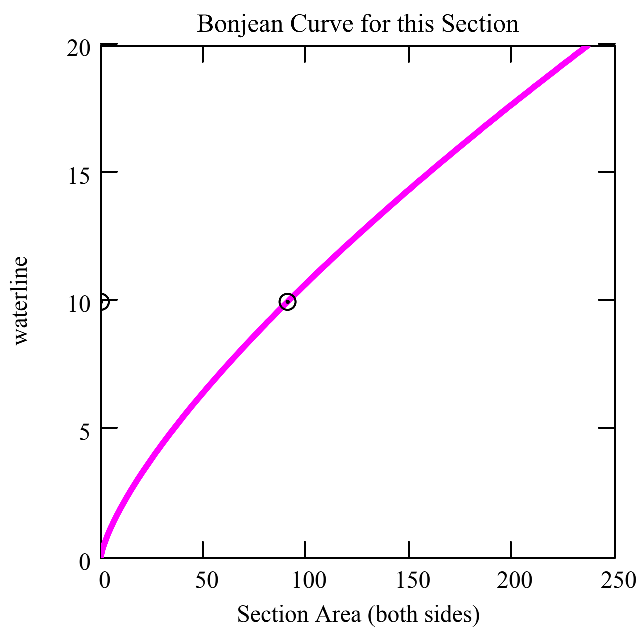
To calculate local buoyancy, it is useful to have a set of curves at each station representing the section area. The local

buoyancy per unit length is = area of the immersed section * density of sea water.   Ref: Archimedes

consider a section:

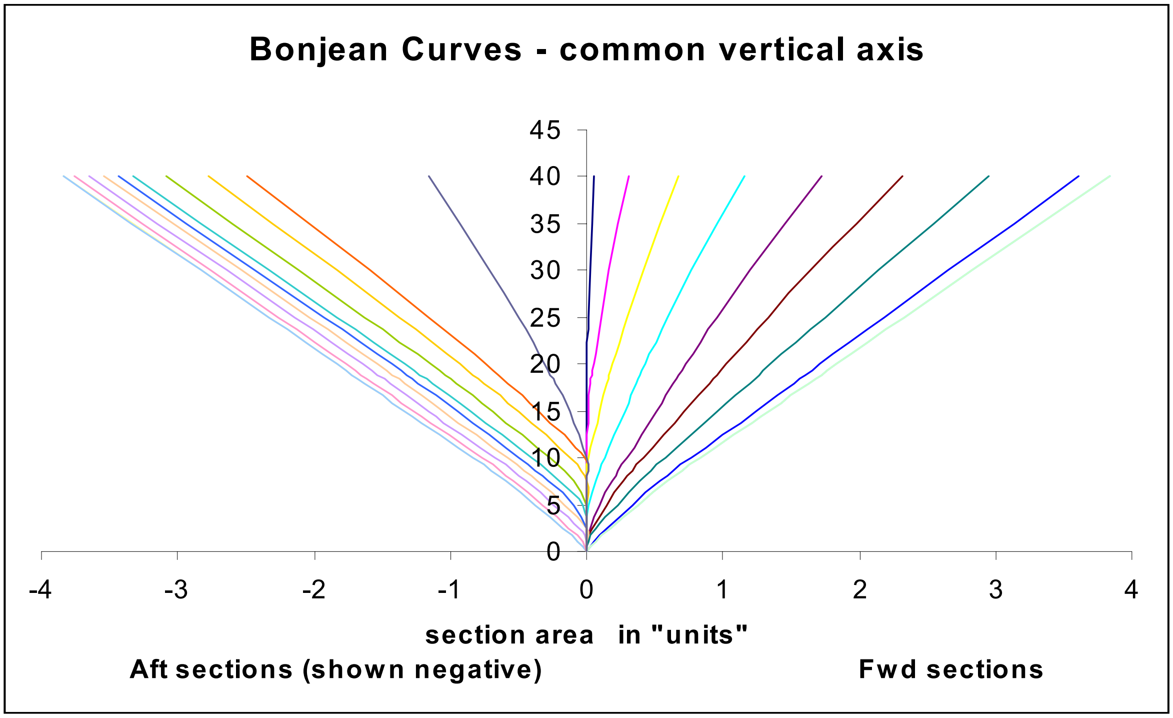


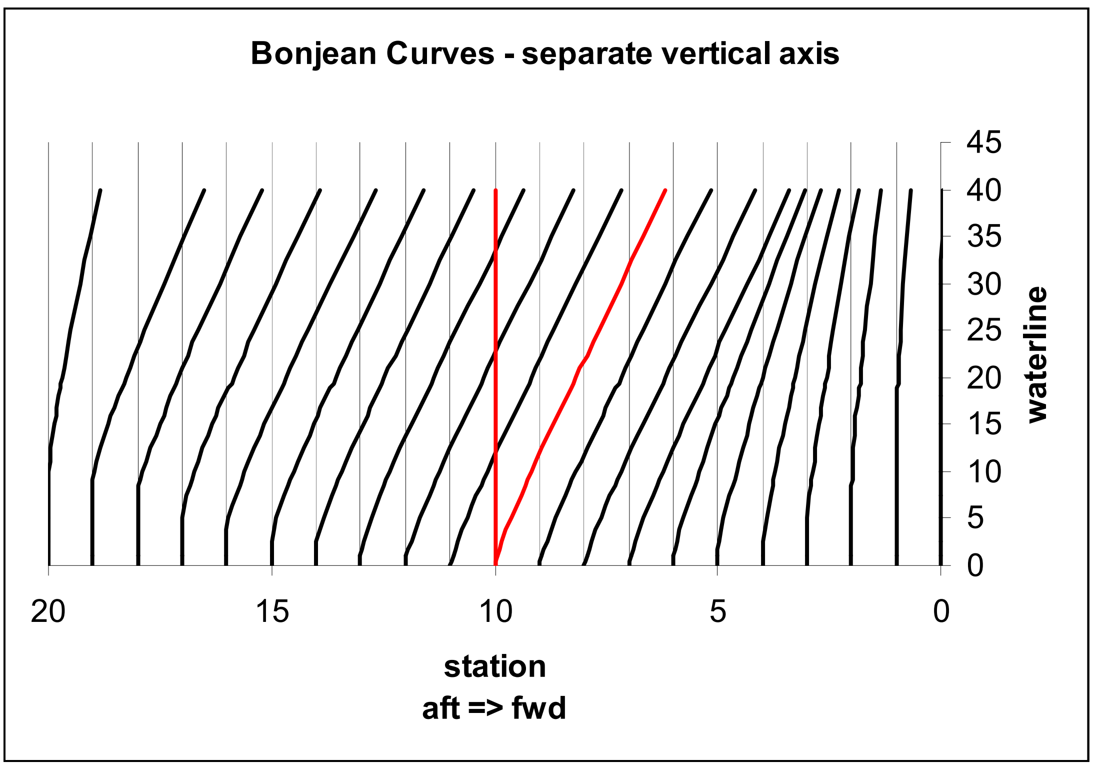
The section area is the area below each waterline. The curve of area below the respective waterline is the section area or Bonjean curve.



shown is the section area at the 10 waterline $sa(10) = 90.833$

note that the section area is typically a large number so it is typical to represent the area in "inches" on a plot or some scaled units. We'll see why for some plots below.





Ref: PNA figures 33 and 34.
It can be a little confusing. The curve for station 10 is emphasized with its axis.

In general the approach to computing the shear and bending moment is first to "balance" the platform on the selected wave. (This might be stillwater i.e. 0 wave height.) US Navy practice has been to use a trochoidal wave with height = 1.1*sqrt(length). Note that this relationship is dependent on the units selected and only applicable with length in feet. This may change in the near future. ABS uses an increment to be added to the stillwater bending moment based on at sea test data. In any case some calculation of a moment in this manner is the norm. Several such calculations will be required to evaluate the spectrum of loading conditions.

<u>What do we mean by "balance"?</u>

If we assume the platform is immersed to a mean draft, say at midships, with an angle of trim, this will determine the immersion of each station along the length. The intersection of the waterline with each station (the vertical axis - not the section area curve on the plot above) determines the buoyancy per unit length at that station from:

$$\text{buoyancy\_per\_length}(x) := \text{section\_area\_immersed}(x)\cdot\text{density\_sea\_water}$$ e.g.tons per foot

total buoyancy: $B := \int_0^L \text{buoyancy\_per\_length}(x)\,dx$

longitudinal center of buoyancy: $\text{lcb} := \dfrac{\int_0^L \text{buoyancy\_per\_length}(x)\cdot x\,dx}{B}$

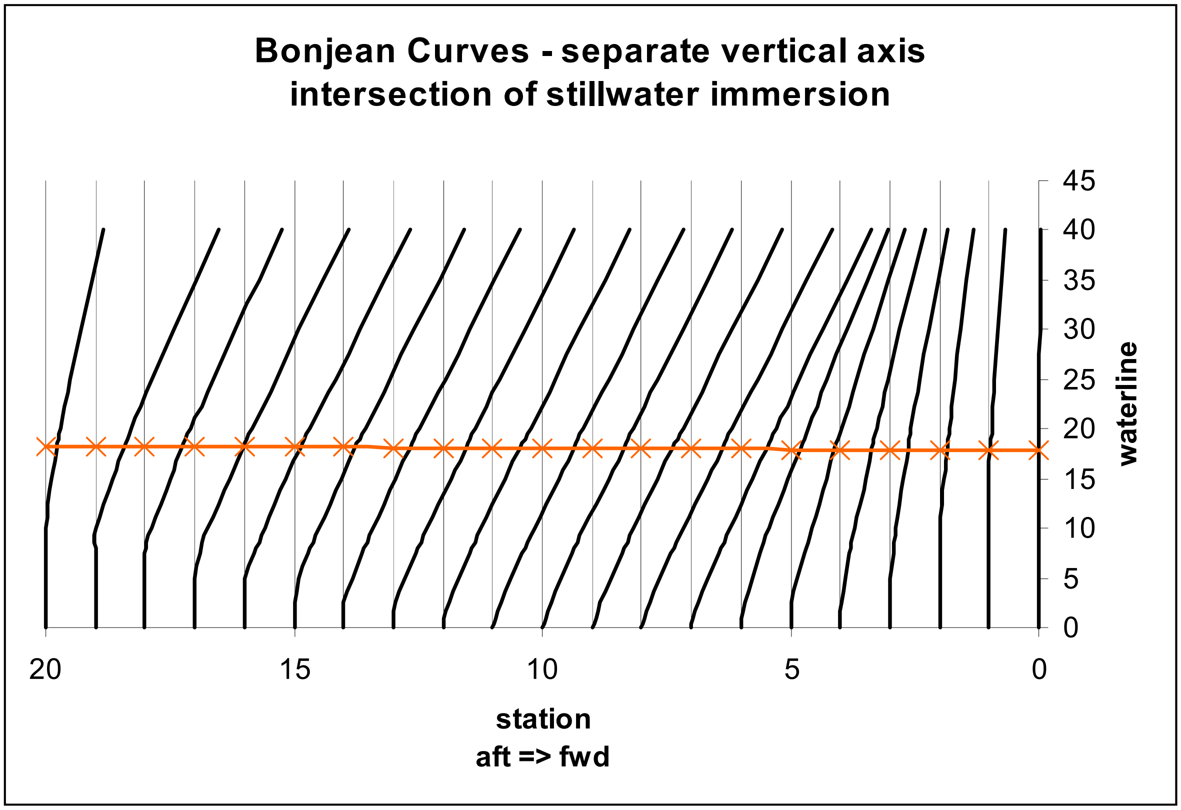
To satisfy the two rules of naval architecture: total buoyancy must = total weight

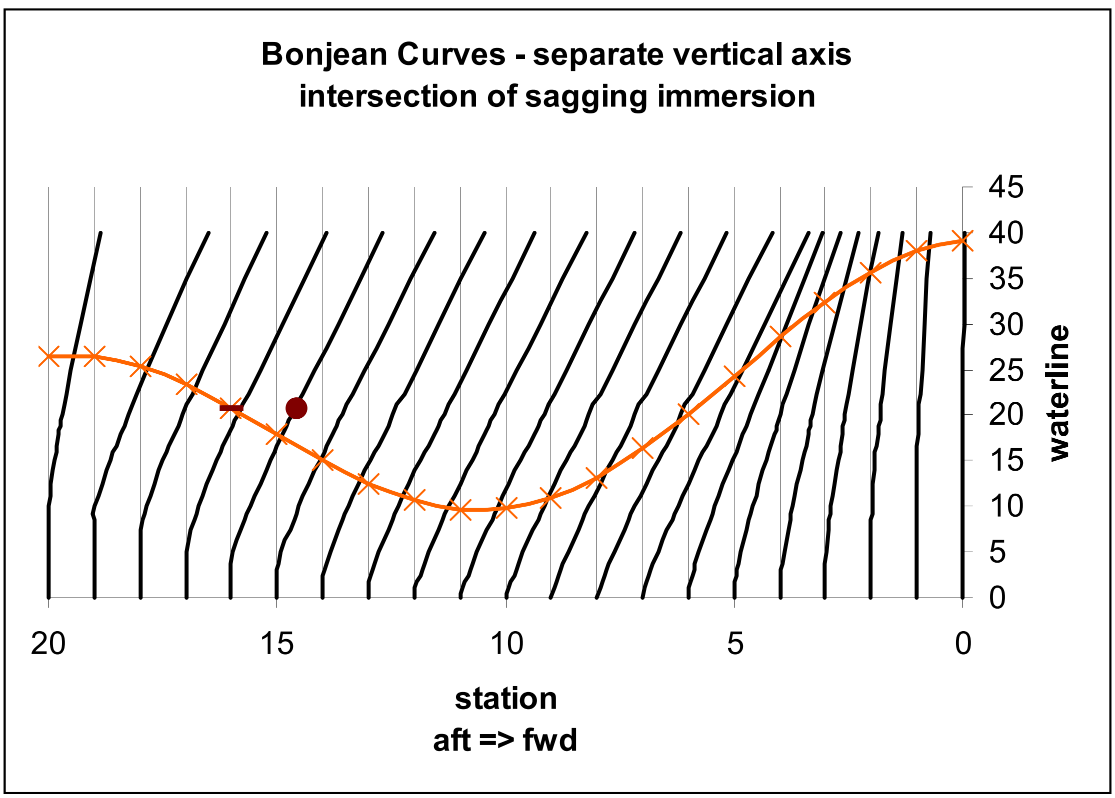
and

lcb = lcg

(horizontal cgs must match but not the vertical)

An example of the intersection of a stillwater and a wave profile along the length are shown below.


Bonjean Curves - separate vertical axis
intersection of stillwater immersion
45
40
35
30
25
20
15
10
5
0
waterline
20
15
10
5
0
station
aft => fwd


Bonjean Curves - separate vertical axis
intersection of sagging immersion
45
40
35
30
25
20
15
10
5
0
waterline
20
15
10
5
0
station
aft => fwd

remember that the local waterline intersects the vertical axis (ordinate) not the section area (abscissa). Given the intersection with the ordinate, the section area is determined by reading from the appropriate curve at that height. On the above plot, the section area for station 16 is emphasized. The wave intersects the vertical axis at about 20 and the value of section area is ~ 1.5 units.

Computational routines vary, but one approach might be to

1) average the buoyancy between two stations (to be consistent with the location of the weight)
2) the increment for buoyancy is then:

for example:

$\text{number\_of\_stations} := 20$ numbered 0 - 20 (might call this 20 sections)

$\text{station\_spacing} := 1$

$i := 0 .. \text{number\_of\_stations} - 1$ this is one fewer than the number of stations (index starts from 0) and/or in between two stations only provides one average force.

$\text{nsta} := \text{number\_of\_stations}$

$$\text{buoyancy}_i := \left(\frac{\text{section\_area}_i + \text{section\_area}_{i+1}}{2} \right) \cdot \text{density\_sea\_water} \cdot \text{station\_spacing}$$

the x location is: $xf_i := i + 0.5$

$$B := \sum_{i=0}^{\text{nsta}-1} \text{buoyancy}_i$$ and: $$\text{lcb} := \frac{\left(\sum_{i=0}^{\text{nsta}-1} \text{buoyancy}_i \cdot xf_i \right)}{B}$$

The assumed value of mean draft and trim angle is then adjusted until B = W and lcb = lcg.

for example: after some iterations a draft and trim is determined that has the following buoyancy per length distribution:

$xf_i := i + 0.5$ is the midpoint between station i and i+1

$xf_i =$

	0
0	0.5
1	1.5
2	2.5
3	3.5
4	4.5
5	5.5
6	6.5
7	7.5
8	8.5
9	9.5
10	10.5
11	11.5
12	12.5
13	13.5
14	14.5
15	15.5
16	16.5
17	17.5
18	18.5
19	19.5

buoyancy at xf when balanced.

$$buoyancy := \begin{pmatrix} 6 \\ 34 \\ 98 \\ 192 \\ 293 \\ 400 \\ 521 \\ 606 \\ 628 \\ 629 \\ 630 \\ 617 \\ 582 \\ 540 \\ 500 \\ 461 \\ 409 \\ 336 \\ 262 \\ 160 \end{pmatrix}$$

weight at xf

$$weight := \begin{pmatrix} 20 \\ 40 \\ 70 \\ 111 \\ 211 \\ 332 \\ 453 \\ 544 \\ 735 \\ 775 \\ 765 \\ 755 \\ 655 \\ 551 \\ 493 \\ 433 \\ 373 \\ 312 \\ 181 \\ 91 \end{pmatrix}$$

"balance" is defined by total buoyancy B = total weight W and lcb = lcg:

$$B := \sum_{i=0}^{nsta-1} buoyancy_i \qquad W := \sum_{i=0}^{nsta-1} weight_i$$

$$B = 7904 \qquad W = 7900$$

$$lcb := \frac{\left(\sum_{i=0}^{nsta-1} buoyancy_i \cdot xf_i\right)}{B} \qquad lcg := \frac{\left(\sum_{i=0}^{nsta-1} weight_i \cdot xf_i\right)}{W}$$

$$lcb = 11.007 \qquad lcg = 11.01$$

close enough!!

At each station (usually in between - at midpoint) there is a net UP or DOWN force due to buoyancy -weight =>

$f_i := buoyancy_i - weight_i$ in this sense we are defing buoyancy (UP) as positive.

Shear can be calculated starting from one end (zero shear: free) .

$shear_0 := 0$ and $shear_{i+1} := shear_i + f_i$ where i = 0;...number_of_stations-1 => nsat values, 0 and 0 -> nsta-1

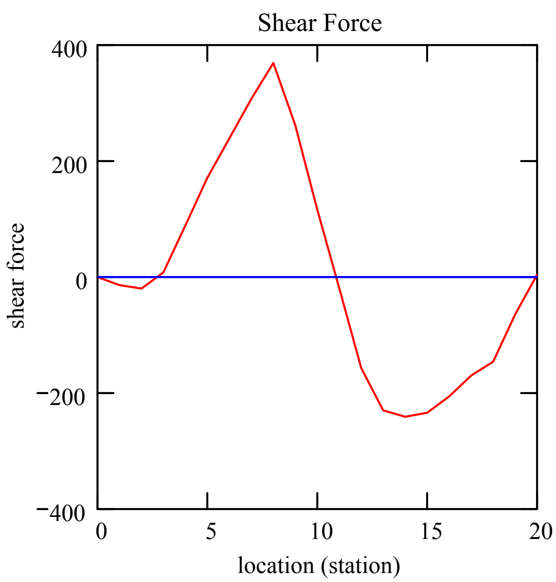
(neglecting net forces forward of FP and aft of AP - include those in similar manner)

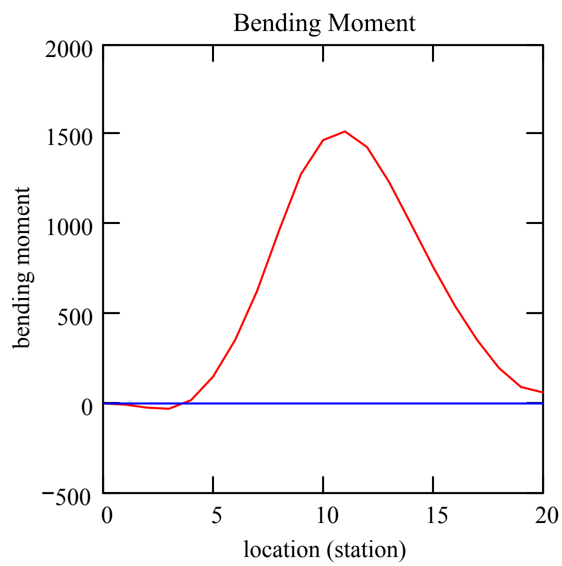
Then bending moment is:

$$bending\_moment_0 := 0 \text{ and } bending\_moment_{i+1} := bending\_moment_i + \frac{(shear_i + shear_{i+1})}{2} \cdot station\_spacing$$

where $station\_spacing$ is the station spacing.

station location is defined as follows: $i := 0 .. nsta$ $\quad xs_i := i$





as a check on achieving a true "balance", the shear force and bending moment should be zero at the end (x_s = 20 station). The above data is close but there is a relatively small remainder.

This same calculation is done for various conditions of loading, wave immersion etc. for a static determination of shear force and bending moment.

This is the calculation underlying DDS 100-6 Longitudinal Strength Calculation, Ship's Hull Characteristics Program (SHCP) and Maestro load calculations... at least.