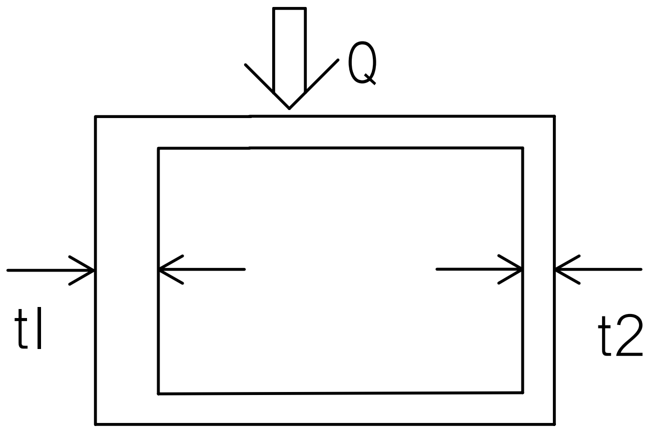
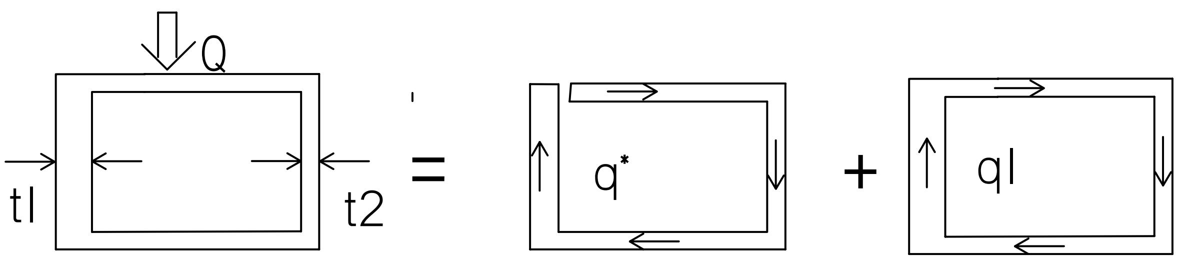
Shear Stress from Shear Load in closed non symmetric section



Shear load is applied such that (pure) bending occurs
can't use symmetry to determine where to start s = 0 arc length parameter

approach: divide into two problems and superpose:



q* is the shear flow we have developed to date opening the section and
q1 is a *constant* shear flow in the closed section

superposition => we add the two flows for the actual shear flow
how do we calculate each. i.e $q = q^* + q1$

We have one condition; the net has to match the applied load

the second comes from the physical situation; the slip at the cut must be 0

$$\text{slip} = \oint \gamma \, ds$$ where integral is circular and γ is the shear strain

$$\oint \gamma \, ds = \oint \frac{\tau}{G} \, ds = \frac{1}{G} \cdot \oint \frac{q}{t} \, ds = 0$$

G the shear modulus = constant =>

$$\oint \frac{q\_star}{t} \, ds + \oint \frac{q_1}{t} \, ds = 0$$

q_1 is constant =>

$$q_1 := \frac{-\int_0^b \frac{q\_star(s)}{t(s)} \, ds}{\oint \frac{1}{t(s)} \, ds}$$

where the numerator is the integral around the cross section and the denominator is as well (circular)

$$q\_star(s) := \frac{Q}{I} \cdot m\_star(s)$$ and $$\int_0^b \frac{q\_star(s)}{t(s)} \, ds = \frac{Q}{I} \cdot \int_0^b \frac{m\_star(s)}{t(s)} \, ds$$

in this case the example is symmetric wrt z axis Iyz = 0 would need to account for assymmetry as above if necessary

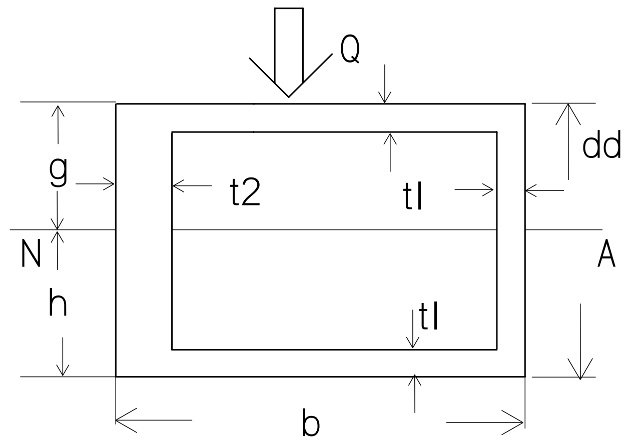
$$q(s) = \frac{Q}{I} \cdot \left(m\_star(s) - \frac{\int_0^b \frac{m\_star(s)}{t(s)} \, ds}{\oint \frac{1}{t(s)} \, ds} \right)$$

let's do an example

Example of closed rectangular cross section non-symmetric subject to shear force

$b := 10 \qquad dd := 8 \qquad Q := 100$

$t_1 = 1 \qquad t_2 = \frac{3}{2}$



$t_d := t_1 \qquad t_s := t_1 \qquad n\_elements := 4$

$t_b := t_1 \qquad t_{ls} := t_2$

$n := 0 .. n\_elements - 1$ index 0 mcd

reference baseline, h is distance between NA and lower segment; g = dd - h is distance to top segment

$$A := \begin{pmatrix} t_d \cdot b \\ t_s \cdot dd \\ t_b \cdot b \\ t_{ls} \cdot dd \end{pmatrix} \qquad A\_total := \sum_n A_n \qquad A\_total = 40 \qquad d := \begin{pmatrix} dd \\ \frac{dd}{2} \\ 0 \\ \frac{dd}{2} \end{pmatrix} \qquad h := \frac{\sum_n (d_n \cdot A_n)}{A\_total}$$

$h = 4$

$$i_0 := \begin{pmatrix} \frac{1}{12} \cdot t_d^3 \cdot b \\ \frac{1}{12} \cdot dd^3 \cdot t_s \\ \frac{1}{12} \cdot t_b^3 \cdot b \\ \frac{1}{12} \cdot dd^3 \cdot t_{ls} \end{pmatrix} \qquad A = \begin{pmatrix} 10 \\ 8 \\ 10 \\ 12 \end{pmatrix} \qquad d = \begin{pmatrix} 8 \\ 4 \\ 0 \\ 4 \end{pmatrix}$$

$g := dd - h$

$g = 4$

$d_g := h$

$$I := \left[\sum_n \left[i_{0_n} + A_n \cdot (d_n)^2 \right] - A\_total \cdot d_g^2 \right] \qquad I = \frac{1285}{3}$$

removed 2 as this is not half section

deck (top)

$$m_{y\_top}(s) := g \cdot t_d \cdot s \cdot (0 < s \le b)$$

$0 < s \le b$

right side

$$m_{y\_side}(s) := \left[g \cdot t_d \cdot b + t_s \cdot \left(g \cdot s - g \cdot b - \frac{1}{2} \cdot s^2 + s \cdot b - \frac{1}{2} \cdot b^2\right)\right] \cdot [\,[b < s \le (b + g + h)]\,]$$

$$[b < s \le (b + g + h)]$$

$$m_{ybgh} := g \cdot t_d \cdot b + t_s \cdot \left[g \cdot (b + g + h) - g \cdot b - \frac{1}{2} \cdot (b + g + h)^2 + (b + g + h) \cdot b - \frac{1}{2} \cdot b^2\right]$$

bottom

$$m_{y\_bottom}(s) := \left[m_{ybgh} - h \cdot t_b \cdot (s - b - g - h)\right] \cdot [b + g + h < s \le (2 \cdot b + g + h)]$$

$$[b + g + h < s \le (2 \cdot b + g + h)]$$

left side

same for as right side with s starting at 2*b +dd, with initial value my(2*b + dd)

$$m_y(s) = \int_0^s y \cdot t \, ds$$

$$m_y(s) = \int_0^{2 \cdot b + dd} y \cdot t \, ds + \int_{2 \cdot b + dd}^{s} y \cdot t \, ds = m_y(2 \cdot b + dd) + t_s \left[\int_{2 \cdot b + dd}^{s} -h + [s - (2 \cdot b + dd)] \, ds\right]$$

using ll for 2*b + dd

$$[2 \cdot b + dd < s \le (2 \cdot b + 2dd)]$$

$$\int_{ll}^{s} -h + (\sigma - ll) \, d\sigma \rightarrow$$

$$m_y(2 \cdot b + dd) = m_{y\_bottom}(2 \cdot b + dd) \qquad m_{y\_bottom}(2 \cdot b + dd) = 0$$

$$ll := 2 \cdot b + dd$$

$$m_{y\_left\_side}(s) := \left[m_{y\_bottom}(2 \cdot b + dd) + t_s \cdot \left(-4 \cdot s + \frac{1}{2} \cdot s^2 - ll \cdot s + 4 \cdot ll + \frac{1}{2} \cdot ll^2\right)\right] \cdot [\,[2 \cdot b + dd < s \le (2 \cdot b + 2dd)]\,]$$

$s := 0, 0.1 .. b + dd + b + dd$

$m_y(s) := m_{y\_top}(s) + m_{y\_side}(s) + m_{y\_bottom}(s) + m_{y\_left\_side}(s)$

need also t(s)

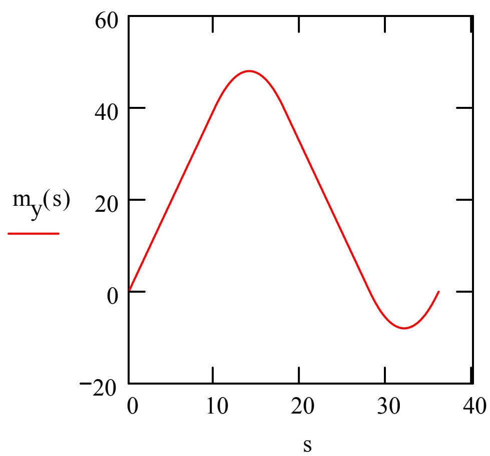
$t\_top(s) := t_d \cdot (0 \le s \le b)$ $\quad$ $t\_side(s) := t_s \cdot [b < s \le (b + g + h)]$

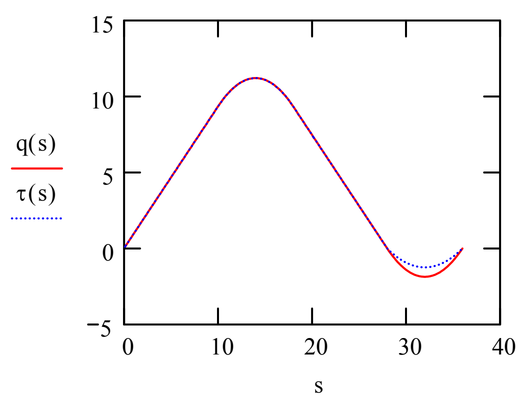
$t\_bottom(s) := t_b \cdot (b + g + h < s < 2 \cdot b + g + h)$ $\quad$ $t\_left\_side(s) := t_{ls} \cdot [2 \cdot b + dd \le s \le (2 \cdot b + 2dd)]$

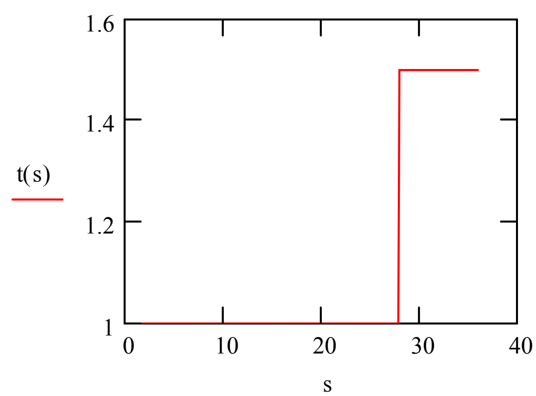
$t(s) := t\_top(s) + t\_side(s) + t\_bottom(s) + t\_left\_side(s)$

$Q = 100$

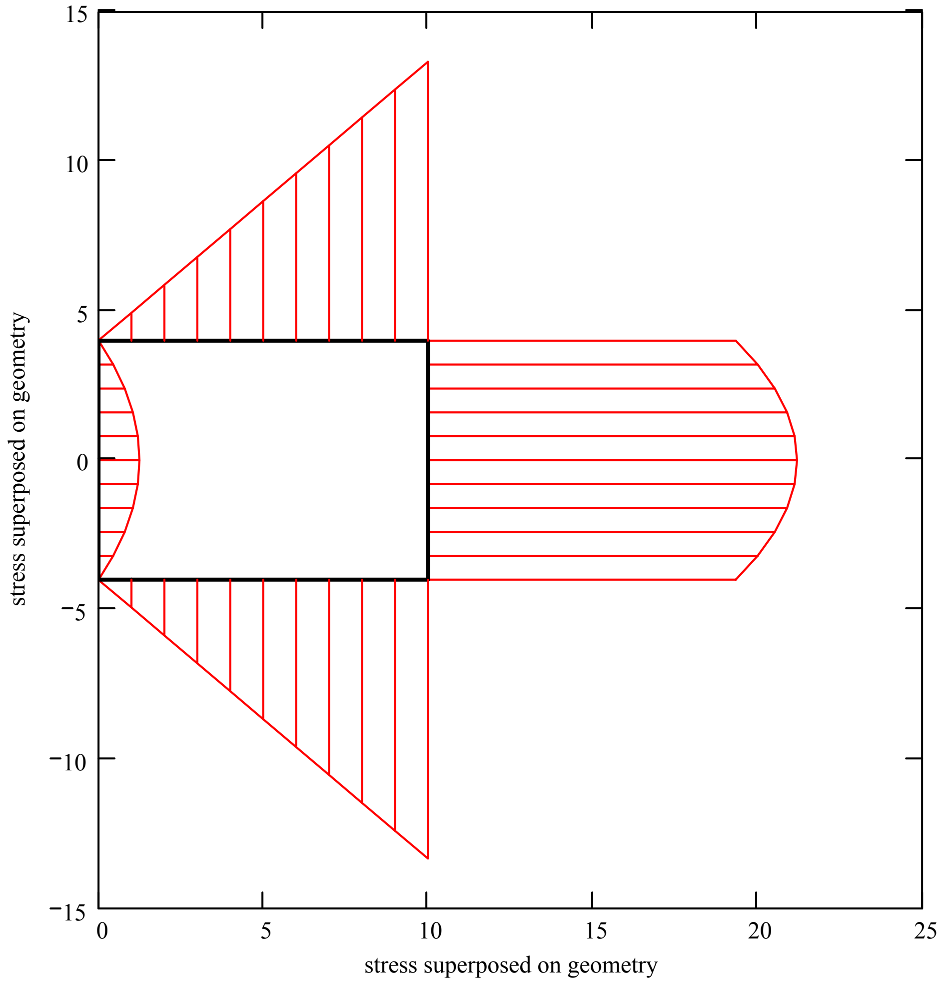
$$q(s) := \frac{Q \cdot m_y(s)}{I} \qquad \tau(s) := \frac{Q \cdot m_y(s)}{I \cdot t(s)}$$







plot of stress distribution around cross section for q_star
magnitudes shown positive is out of section
this is shear stress

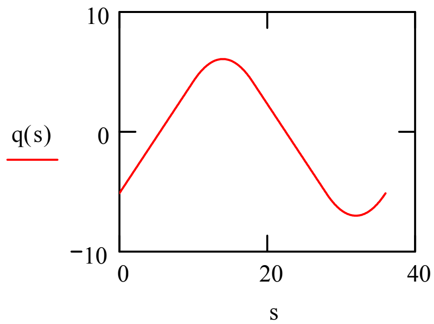


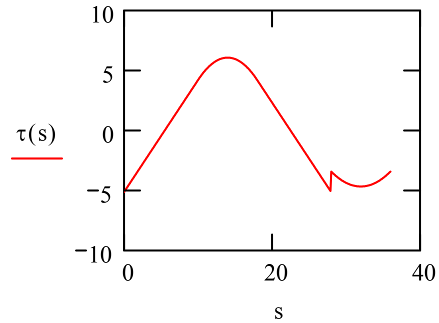
$$m\_star(s) := m_y(s)$$

$$q_1 := \frac{\displaystyle\int_0^{2\cdot b+2\cdot dd} \frac{m\_star(s)}{t(s)}\,ds}{\displaystyle\int_0^{2\cdot b+2\cdot dd} \frac{1}{t(s)}\,ds} \qquad q_1 = \frac{55613827}{2524920}$$

$$q(s) := \frac{Q}{I}\cdot\left(m\_star(s) - q_1\right)$$

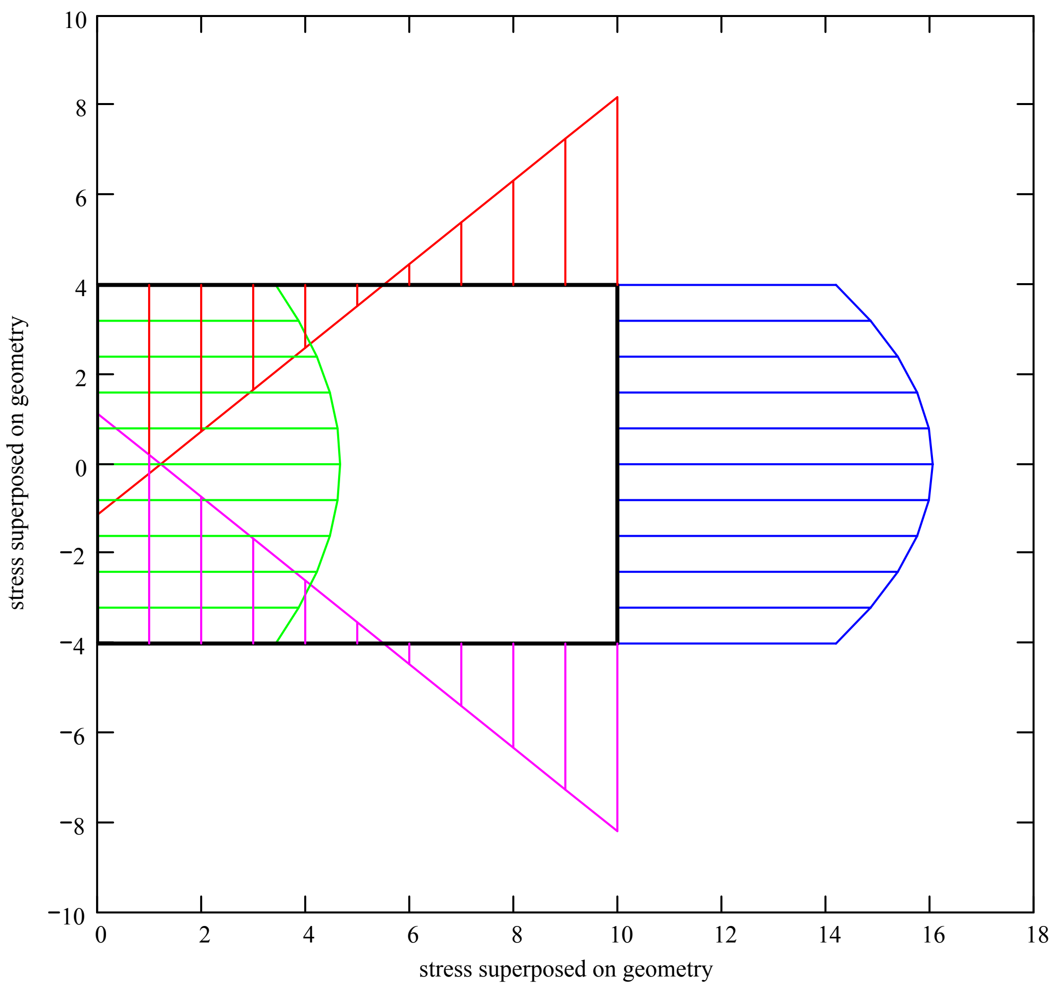
$$\tau(s) := \frac{q(s)}{t(s)}$$





$t_1 \equiv 1.0 \qquad t_2 \equiv 1.5$

plot of stress distribution around cross section
magnitudes shown positive outward
this is shear stress



now that we have considered on such cross section, what if there are one or more adjacent?