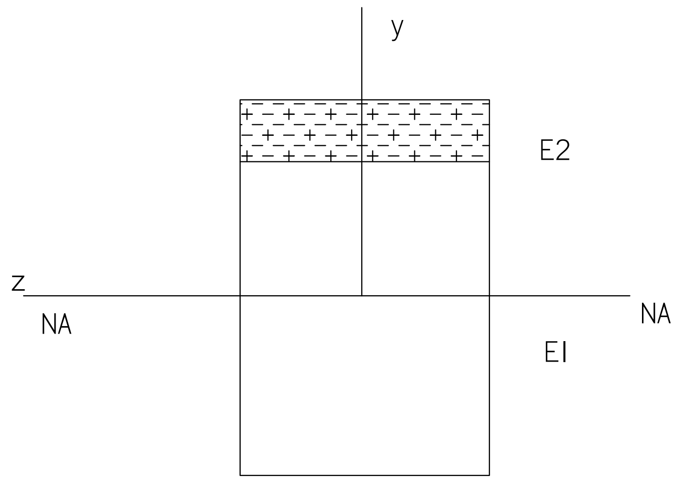
Dissimilar material such as a composite structure:

what if E and I are not constant??

assuming bending only; Mz applied; determine Iz



In this cross section, the upper region has a modulus = E2 where the remainder has modulus E1

as with Euler bending, plane sections remain plane etc....

$$\varepsilon_x = \frac{-y}{R}$$

ε is axial strain

y the distance from the neutral (z) axis

R the radius of curvature

$$\sigma_x = E \cdot \varepsilon_x = \frac{-E \cdot y}{R}$$

Hooke applies (although E is now dependent on y)
signs consistent with Shames 11.2

pure (only) bending => $F_x = \int \sigma_x \, dA = 0 = -\int \frac{E(y) \cdot y}{R} \, dA$ net axial force = 0

suppose we define a parameter Ti such that $T_i = \frac{E_i}{E_1}$ that is, a fraction of a reference modulus E1

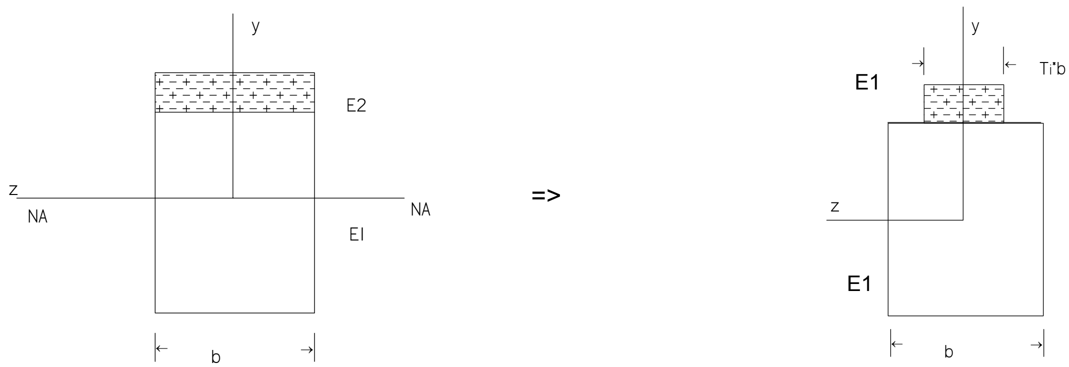
then: $-\int \frac{E(y) \cdot y}{R} \, dA = \frac{-E_1}{R} \int T_i(y) \cdot y \, dA$

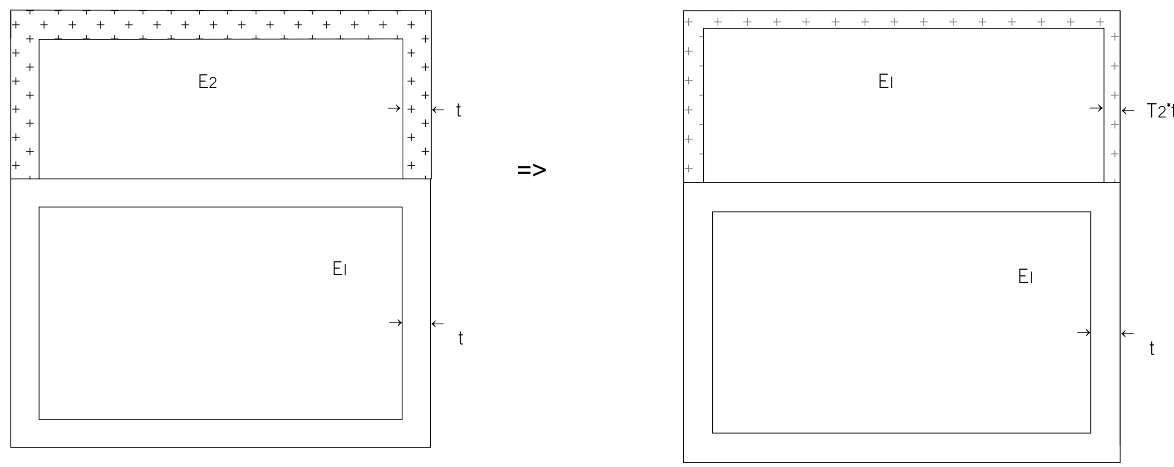
"transfer" Ti to the area, in such a way that y is not affected => $T_i(y) \cdot y \cdot dA = y \cdot (T_i(y) \cdot dA) = y \cdot dy \cdot (T_i(y) \cdot dz)$

which means "transfer" to dz, and in rectangular shape, equivalent to applying to z dimension, for thin walled vertical sections that's the thickness. that is over the different moduli:

$$\int T_i(y) \cdot y \, dA = T_i \cdot b \cdot \int y \, dy$$

in a horizontal thin walled section, y ~ constant => can still apply to thickness:





$$\int T_i(y)\cdot y\,dA = 0$$ => NA is at the cg of the transformed section

now continue looking at the bending moment:

$$M_z = -\int \sigma_x \cdot y\,dA = \frac{-1}{R}\cdot\int E(y)\cdot y^2\,dA = \frac{-E_1}{R}\cdot\int T_i\cdot y^2\,dA$$ using the relation defined above

define $$I_{z\_tr} = \int T_i\cdot y^2\,dA \quad => \quad \frac{1}{R} = \frac{M_z}{E_1\cdot I_{z\_tr}}$$

and again we can apply Ti to the z dimension in vertical sections and to the y in horizontal

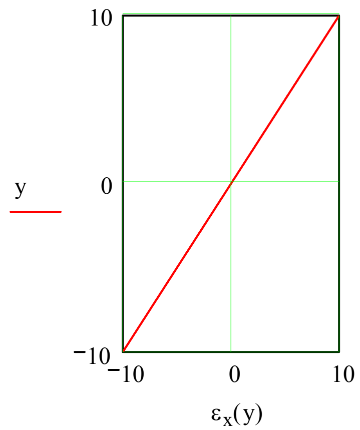
$$\varepsilon_x = \frac{-y}{R} \qquad \sigma_x = E\cdot\varepsilon_x = \frac{-E(y)\cdot y}{R} = -E(y)\cdot\frac{M_z}{E_1\cdot I_{z\_tr}}\cdot y$$

where we have written E(y) as it varies

$$T_i = \frac{E_i}{E_1} \qquad \sigma_x = -E\cdot(y)\cdot\frac{M_z}{E_1\cdot I_{z\_tr}}\cdot y = -T_i\cdot(y)\cdot E_1\cdot\frac{M_z}{E_1\cdot I_{z\_tr}}\cdot y = -T_i\cdot(y)\cdot\frac{M_z}{I_{z\_tr}}\cdot y$$

assume for plot like text $R := -1$

$$\varepsilon_x(y) := \frac{-y}{R}$$



(plane sections remain plane)

to evaluate stress
at yi (where modulus changes, substitute E(y) = Ti*E1 =>

$I_{z\_tr} := 1$ $\quad M_z := -1$ $\quad yy := \begin{pmatrix} 0 \\ 0.7 \\ 1.0 \end{pmatrix}$ $\quad TT := \begin{pmatrix} 1 \\ 0.5 \end{pmatrix}$

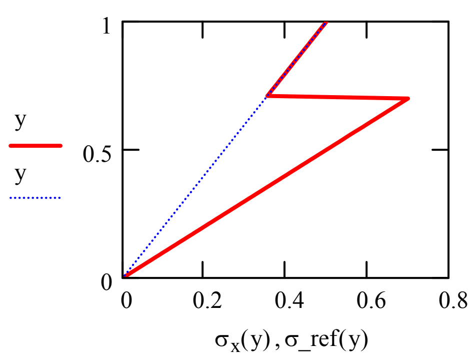
$i := 0..1$ $\quad y := 0, 0.01..1$

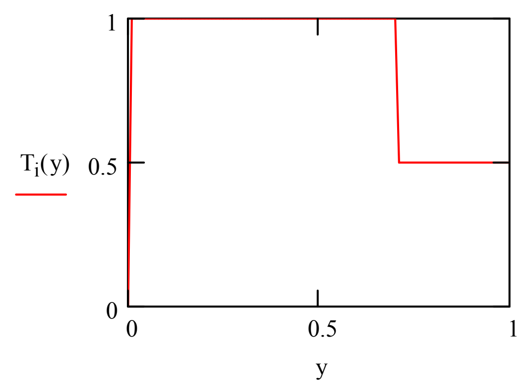
$$T_i(y) := \sum_i TT_i \cdot \left(yy_i < y \le yy_{i+1}\right)$$

summation is to superpose the values appropriate to the (y) dependence

$$\sigma_x(y) := -T_i(y) \cdot \frac{M_z}{I_{z\_tr}} \cdot y \qquad \sigma\_ref(y) := -TT_1 \cdot \frac{M_z}{I_{z\_tr}} \cdot y$$

hence the stress variation is





$x := 0..10$ $v(x) := 1$ $E := 1$ $I := 1$ $constant := 0$ $z := 0..10$

$I_n := 1$ $n := 1..2$ $d_g := 1$ $A_n := 1$ $i0_n := 1$

$d_n := 1$

$n := 1..2$ $t := 1$

$V(x) := 1$ $N_0 := 0$

$\sigma_z := 6$ $\tau_{xz} := 1$

$\tau_{yz} := 4$

$\sigma_1 := 1$ $\tau_{xy} := 1$ $\sigma_2 := 2$ $\sigma_3 := 3$ $\sigma_y := 5$ $\sigma_Y := 4$ $\sigma_x := 2$