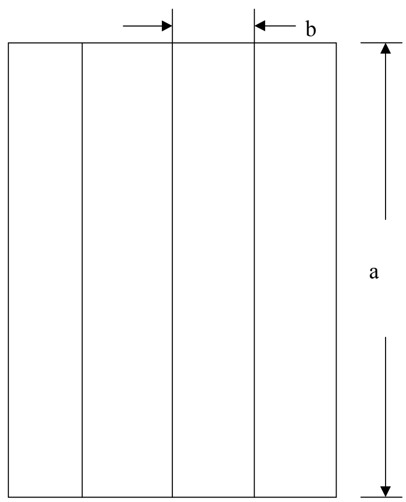
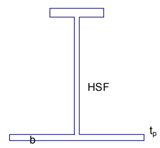
Section 14.2 Ultimate Strength of Stiffened Panels

three failure types

- compression in flange of stiffener (negative bending moment) Mode I
- compression in plate (positive bending moment) Mode II
- tension in flange of stiffener (high positive moment) Mode III





$\gamma_C := 1.5$ $\sigma_C :=$ input

$p :=$ input $\sigma_{ax} := \sigma_C$

j = 5, PS 6, stiffener #6 from catalog

$BSF := 3.94$ $SDEPTH := 7.89$ $TSF := 0.205$ $TSW := .17$ $SCG := 5.35$

plate

$a := 8 \cdot 12$ $b := 23.844$ $t := .375$ $N := 1$

$L := a$

material

$\upsilon := 0.3$ $E := 29.6 \cdot 10^6$ $D := \dfrac{E \cdot t^3}{12 \cdot \left(1 - \upsilon^2\right)}$

allowing for different yield stresses

plate stiffener

$\sigma_{Yp} := 47 \cdot 10^3$ $\sigma_{Ys} := 47 \cdot 10^3$

general parameters:

$HSW := SDEPTH - TSF$ $A_w := (SDEPTH - TSF) \cdot TSW$ $A_f := BSF \cdot TSF$ $A_s := A_w + A_f$

$HSW = 7.685$ $A_w = 1.306$ $A_f = 0.808$ $A_s = 2.114$

$d := SDEPTH - \dfrac{TSF}{2} + \dfrac{t}{2}$ $B := (N + 1) \cdot b$ $A_p := b \cdot t$ $A := A_s + A_p$ $\sigma_{Y\_bar} := \left(\dfrac{A_s \cdot \sigma_{Ys} + \sigma_{Yp} \cdot A_p}{A}\right)$

$d = 7.975$ $B = 47.688$ $A_p = 8.942$ $A = 11.056$ $\sigma_{Y\_bar} = 47000$

For later use and to set the scale on plots, calculate M_p, plastic moment of section

if Ap>Aw+Af; i.e. Ap>At/2 => g is in plate

$$g := \frac{A_p - A_w - A_f}{2 \cdot b} \qquad g = 0.143$$

centroid of upper half

$$y_1 := \frac{\left[\frac{b \cdot g^2}{2} + A_w \cdot \left(\frac{HSW}{2} + g\right)\right] + A_f \left[(g) + HSW + \frac{TSF}{2}\right]}{\frac{A}{2}} \qquad y_1 = 2.145$$

centroid of lower half

$$y_2 := \frac{t - g}{2} \qquad y_2 = 0.116$$

plastic section modulus, if Ap>At/2

$$Z_{P1} := \frac{A}{2} \cdot (y_1 + y_2) \qquad Z_{P1} = 12.498$$

$$TSF = 0.205$$

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if Ap<Aw+Af; i.e. Ap<At/2 => g is is web

$$g := \frac{A_f + A_w - A_p}{2 \cdot TSW} \qquad g = -20.08$$

centroid of upper half

$$y_1 := \frac{A_f \left(HSW - g + \frac{TSF}{2}\right) + \frac{TSW \cdot (HSW - g)^2}{2}}{A_f + A_w - g \cdot TSW}$$

$$y_1 = 15.926$$

centroid of lower half

$$y_2 := \frac{A_p \cdot \left(g + \frac{t}{2}\right) + \frac{TSW \cdot g^2}{2}}{A_p + g \cdot TSW}$$

$$y_2 = -25.977$$

plastic section modulus, if Ap<At/2

$$Z_{P2} := \frac{A}{2} \cdot (y_1 + y_2) \qquad Z_{P2} = -55.562$$

$$Z_P := \mathrm{if}\left(A_p > \frac{A}{2}, Z_{P1}, Z_{P2}\right) \qquad Z_P = 12.498$$

$$M_P := \sigma_{Y\_bar} \cdot Z_P \qquad M_P = 587397$$

a. Compression failure of stiffener (flange): Mode I (Point E figure 14.2) and curve

- PCSF - Panel Collapse Stiffener Flexure. (Mode I).

geometry of panel

Combination of plate and stiffeners (from p287, equation 8.3.6 in text):

$$C_1 := \frac{A_w \cdot \left(\frac{A}{3} - \frac{A_w}{4}\right) + A_f \cdot A_p}{(A)^2} \qquad I := A \cdot (d)^2 \cdot C_1 \qquad y_f := -d \cdot \frac{\frac{A_w}{2} + A_p}{A} \qquad y_p := d \cdot \left(1 - \frac{\frac{A_w}{2} + A_p}{A}\right)$$

$C_1 = 0.095$ $\quad I = 66.789$ $\quad y_f = -6.921$ $\quad y_p = 1.054$

For maximum moment and center deflection assume simply supported beam:

$$q := p \cdot b \qquad M_o := \frac{q \cdot a^2}{8} \qquad \delta_o := \frac{5 \cdot q \cdot a^4}{384 \cdot E \cdot I} \qquad \delta_o(M_o) := \frac{5 \cdot M_o \cdot a^2}{48 \cdot E \cdot I} \qquad M_o := 1$$ for numbers

Rule of thumb for eccentricity of welded panels: $\Delta := \frac{a}{750}$ $\quad \Delta = 0.128$

$\Delta_I := -\Delta$ applying negative bending moment, apply eccentricity in worse direction

strictly speaking should compare failure stress to torsional buckling limit or yield. Let $\sigma_{aT} := \sigma_{Ys}$

$$\sigma_{Fs} := \min\left(\begin{pmatrix} \sigma_{Ys} \\ \sigma_{aT} \end{pmatrix}\right)$$

$$\rho_I := \sqrt{\frac{I}{A}} \qquad \lambda_I := \frac{a}{\pi \cdot \rho_I} \cdot \sqrt{\frac{\sigma_{Fs}}{E}} \qquad \eta_I(M_o) := \frac{\left(\delta_o(M_o) + \Delta_I\right) \cdot y_f}{(\rho_I)^2} \qquad \mu_I(M_o) := \frac{M_o \cdot y_f}{I \cdot \sigma_{Fs}}$$

$\rho_I = 2.458$ $\quad \lambda_I = 0.495$ $\quad \eta_I(M_o) = 0.147$ $\quad \mu_I(M_o) = -2.205 \times 10^{-6}$

$$\zeta_I(M_o) := 1 - \mu_I(M_o) + \frac{1 + \eta_I(M_o)}{(\lambda_I)^2} \qquad R_I(M_o) := \frac{\zeta_I(M_o)}{2} - \sqrt{\frac{\left(\zeta_I(M_o)\right)^2}{4} - \frac{1 - \mu_I(M_o)}{(\lambda_I)^2}}$$

$\zeta_I(M_o) = 5.672$ $\quad R_I(M_o) = 0.844$

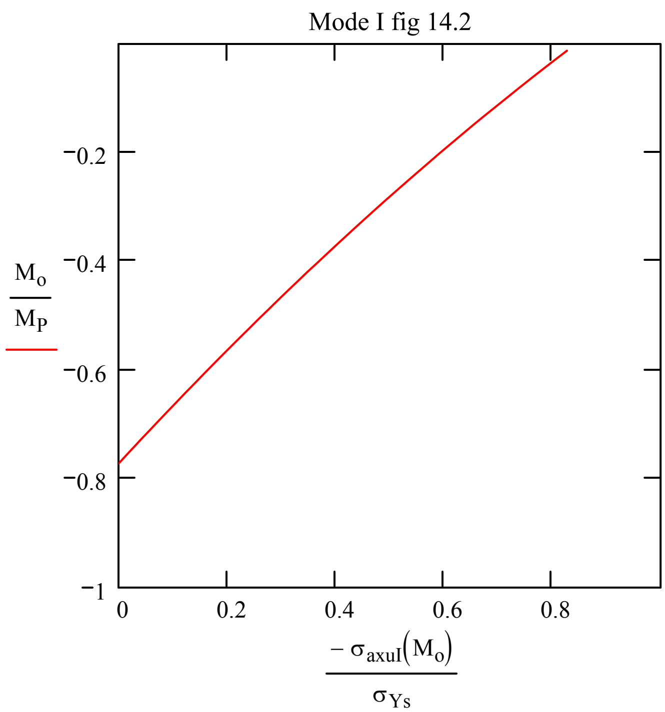
$\sigma_{axuI}(M_o) := -R_I(M_o) \cdot \sigma_{Fs}$ making compression stress negative

$\sigma_{axuI}(M_o) = -39664$

checks with PS 6

$\sigma_{axuI}(0) = -39664$

$M_o := -M_P, (-M_P + 10000) .. 0$ negative Mo $M_P = 587397$



$R_{PCSF1}(M_o) := \frac{\sigma_{ax}}{\sigma_{axuI}(M_o)}$ $\gamma R_{PCSF1}(M_o) := \gamma_C \cdot R_{PCSF1}(M_o)$

Compression failure of plate: Mode II developing $\sigma_{a,ult}$ by first developing $\sigma_{a,tr,ult}$ versus M_o (positive)

as before (Mode I)

$q := p \cdot b$ $\quad M_o := \frac{q \cdot a^2}{8}$ $\quad \delta_o := \frac{5 \cdot q \cdot a^4}{384 \cdot E \cdot I}$ $\quad \delta_o(M_o) := \frac{5 \cdot M_o \cdot a^2}{48 \cdot E \cdot I}$ $\quad M_o := 95370$ for number check

determine failure criteria:

$$\beta := \frac{b}{t} \cdot \sqrt{\frac{\sigma_{Yp}}{E}} \qquad \xi := 1 + \frac{2.75}{(\beta)^2} \qquad T := .25 \cdot \left[2 + \xi - \sqrt{(\xi)^2 - \frac{10.4}{(\beta)^2}} \right]$$

$\tau :=$ input $\quad \sigma_{ay} :=$ input

$\tau := 0$ $\quad \sigma_{ayu} :=$ input

$\sigma_{ay} := 0$

$\beta = 2.534$ $\quad \xi = 1.428$ $\quad T = 0.695$

plate behavior — shear stress — cross axis stress

$$\sigma_{Fp} := \frac{T - .1}{T} \cdot \sigma_{Yp} \cdot \sqrt{1 - 3 \cdot \left(\frac{\tau}{\sigma_{Yp}} \right)^2} \cdot \left(1 - \frac{\sigma_{ay}}{\sigma_{ayu}} \right)$$

$\sigma_{Fp} = 40238$

Set up geometry for transformed plate for combination (from equation 8.3.6 in text):

$$b_{tr} := T \cdot b \qquad A_{ptr} := b_{tr} \cdot t \qquad A_{tr} := A_s + A_{ptr} \qquad C_{1tr} := \frac{A_w \cdot \left(\frac{A_{tr}}{3} - \frac{A_w}{4} \right) + A_f \cdot A_{ptr}}{(A_{tr})^2} \qquad I_{tr} := A_{tr} \cdot (d)^2 \cdot C_{1tr}$$

$b_{tr} = 16.572$ $\quad A_{ptr} = 6.215$ $\quad A_{tr} = 8.329$ $\quad C_{1tr} = 0.118$ $\quad I_{tr} = 62.769$

$$y_{ftr} := -d \cdot \frac{\frac{A_w}{2} + b_{tr} \cdot t}{A_{tr}} \qquad y_{ptr} := d \cdot \left(1 - \frac{\frac{A_w}{2} + b_{tr} \cdot t}{A_{tr}} \right) \qquad \rho_{tr} := \sqrt{\frac{I_{tr}}{A_{tr}}} \qquad \lambda := \frac{a}{\pi \cdot \rho_{tr}} \cdot \sqrt{\frac{\sigma_{Fp}}{E}}$$

$y_{ftr} = -6.576$ $\quad y_{ptr} = 1.399$ $\quad \rho_{tr} = 2.745$ $\quad \lambda = 0.41$

Correction for load eccentricity:

$$h := SCG + \frac{t}{2} \qquad \Delta_p := h \cdot A_s \cdot \left(\frac{1}{A_{tr}} - \frac{1}{A} \right) \qquad \eta_p := \Delta_p \cdot \frac{y_{ptr}}{(\rho_{tr})^2}$$

$h = 5.537$ $\quad \Delta_p = 0.347$ $\quad \eta_p = 0.064$

Set up and solve for $R_{II} = \sigma_{a,tr)ult}/\sigma_{Fp}$

$$\eta_{II}(M_o) := \frac{(\delta_o(M_o) + \Delta)\cdot y_{ptr}}{(\rho_{tr})^2} \qquad \mu_{II}(M_o) := \frac{M_o \cdot y_{ptr}}{I_{tr}\cdot \sigma_{Fp}}$$

$$\eta_{II}(M_o) = 0.032 \qquad \mu_{II}(M_o) = 0.053$$

$$\zeta_{II}(M_o) := \frac{1 - \mu_{II}(M_o)}{1 + \eta_p} + \frac{1 + \eta_p + \eta_{II}(M_o)}{(1 + \eta_p)\cdot(\lambda)^2} \qquad R_{II}(M_o) := \frac{\zeta_{II}(M_o)}{2} - \sqrt{\frac{\zeta_{II}(M_o)^2}{4} - \frac{1 - \mu_{II}(M_o)}{(1 + \eta_p)\cdot(\lambda)^2}}$$

$$\zeta_{II}(M_o) = 7.008 \qquad R_{II}(M_o) = 0.859$$

$\sigma_{a,tr})_{ult}$ is now determined from R

$$\sigma_{axtruII}(M_o) := -R_{II}(M_o)\cdot \sigma_{Fp}$$

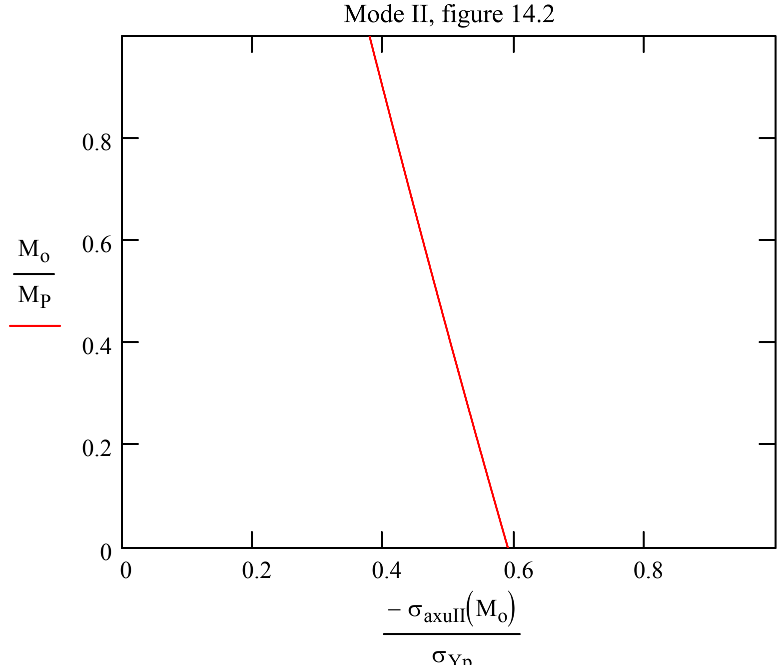
$$\sigma_{axtruII}(M_o) = -34579$$

Convert back to untransformed geometry and corresponding stress

$$\sigma_{axuII}(M_o) := \sigma_{axtruII}(M_o)\cdot \frac{A_{tr}}{A} \qquad \sigma_{axuII}(M_o) = -26050$$

$M_o := 0, 1000 .. M_P$ positive Mo $M_P = 587396.758$

Mode II, figure 14.2



$\sigma_{axuII}(95370) = -26050$

checks with PS 6

appropriate partial safety factor, PCSF2:

$$R_{PCSF2}(M_o) := \frac{\sigma_{ax}}{\sigma_{axuII}(M_o)} \qquad \gamma R_{PCSF2}(M_o) := \gamma_C \cdot R_{PCSF2}(M_o)$$

Tensile yield in flange leading to total plate plus stiffener failure; Mode III: getting relationship for intersection with plate compression failure (line GH in figure 14.2):

$M_{oG} := 1$ for number check

For line GH at point G:

$$\lambda_{GH} := \frac{a}{\pi \cdot \rho_{tr}} \cdot \sqrt{\frac{\sigma_{Ys}}{E}} \qquad \delta_{oG}(M_{oG}) := \frac{5 \cdot M_{oG} \cdot a^2}{48 \cdot E \cdot I} \qquad \eta_{pGH} := \Delta_p \cdot \frac{y_{ftr}}{(\rho_{tr})^2} \qquad \eta_{GH}(M_{oG}) := \frac{(\delta_{oG}(M_{oG}) + \Delta) \cdot y_{ftr}}{(\rho_{tr})^2}$$

$\lambda_{GH} = 0.444$ $\qquad \delta_{oG}(M_{oG}) = 4.856 \times 10^{-7}$ $\qquad \eta_{pGH} = -0.303$ $\qquad \eta_{GH}(M_{oG}) = -0.112$

$$\mu_{GH}(M_{oG}) := \frac{-M_{oG} \cdot y_{ftr}}{I_{tr} \cdot \sigma_{Ys}} \qquad \zeta_{GH}(M_{oG}) := \frac{1 - \mu_{GH}(M_{oG})}{1 + \eta_{pGH}} - \frac{1 + \eta_{pGH} + \eta_{GH}(M_{oG})}{(1 + \eta_{pGH}) \cdot (\lambda_{GH})^2} \qquad \zeta_{GH}(M_{oG}) = -2.835$$

$\mu_{GH}(M_{oG}) = 2.229 \times 10^{-6}$

changed sign

either root may play in result

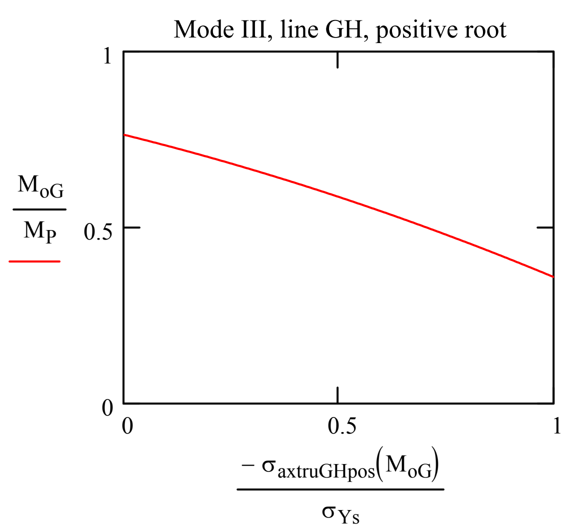
$$R_{GHneg}(M_{oG}) := \frac{\zeta_{GH}(M_{oG})}{2} - \sqrt{\frac{(\zeta_{GH}(M_{oG}))^2}{4} + \frac{1 - \mu_{GH}(M_{oG})}{(1 + \eta_{pGH}) \cdot (\lambda_{GH})^2}} \qquad R_{GHneg}(M_{oG}) = -4.467$$

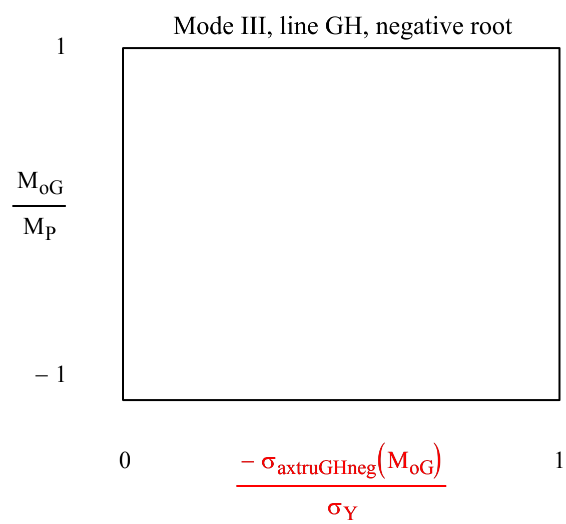
$$R_{GHpos}(M_{oG}) := \frac{\zeta_{GH}(M_{oG})}{2} + \sqrt{\frac{(\zeta_{GH}(M_{oG}))^2}{4} + \frac{1 - \mu_{GH}(M_{oG})}{(1 + \eta_{pGH}) \cdot (\lambda_{GH})^2}} \qquad R_{GHpos}(M_{oG}) = 1.631$$

$\sigma_{axtruGHneg}(M_{oG}) := R_{GHneg}(M_{oG}) \cdot (-\sigma_{Ys})$ $\qquad \sigma_{axtruGHneg}(0) = 209938$

$\sigma_{axtruGHpos}(M_{oG}) := R_{GHpos}(M_{oG}) \cdot (-\sigma_{Ys})$ $\qquad \sigma_{axtruGHpos}(0) = -76681$

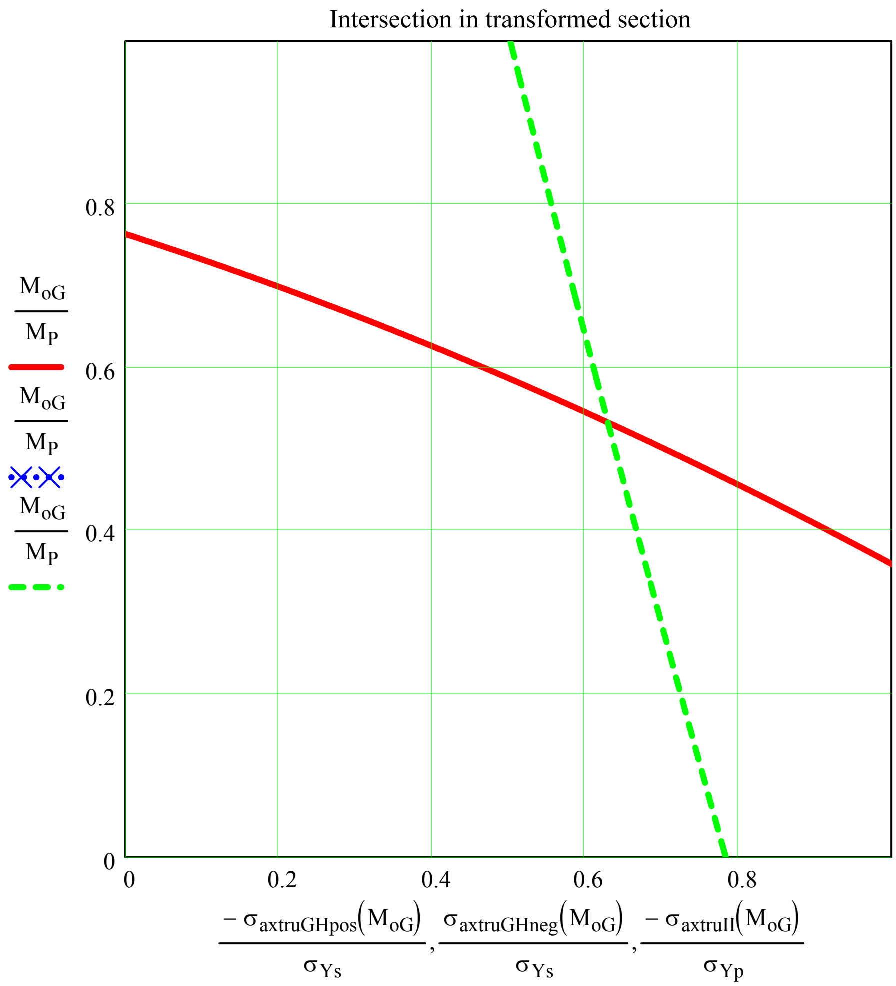
$M_{oG} := 0, 1000 .. M_P$ positive Mo $\qquad M_P = 587396.758$ $\qquad$ should be converted to not tr





examining intersection: working with $\sigma_{a,tr,ult}$ (14.2.29; $R_{II}*\sigma_{Fp} = -R_{GH}*\sigma_{Ys}$):

$M_{oG} := 0, 1000 .. M_P$ positive Mo



Assume value for M_{oG} and iterate
(M_{oG}>Mo; fails in Mode I or II)

$M_{ratio} := 0.532$ trial value

$M_{oG} := M_{ratio} \cdot M_P$ $\sigma_{axtruII}(M_{oG}) = -29613$

$M_{oG} = 312495$ $\sigma_{axtruGHpos}(M_{oG}) = -29688$

$M_P = 587397$

$\sigma_{axtruGHneg}(M_{oG}) = 164531$

if stress levels match, M_{oG} is fixed for Mode III plot:

limit state

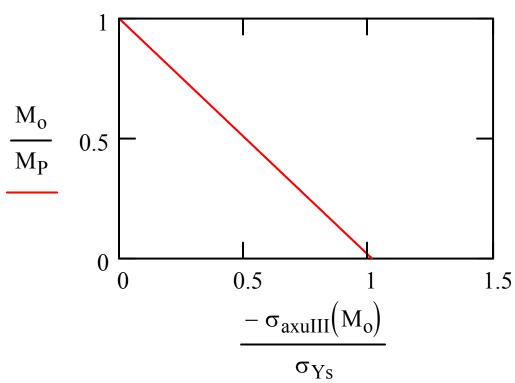
$$\sigma_{auG} := \sigma_Y \cdot \frac{A_{tr}}{A} \cdot R_{GH} \qquad \sigma_{au} := \frac{M_P - M_o}{M_P - M_{oG}} \cdot \sigma_{auG} \qquad \gamma R_{PCSF3} := \gamma_C \cdot \frac{\sigma_C}{\sigma_{au}} \qquad (6\text{-}32)$$

Mode III plot, using M_{oG} and stress level in non-transformed section

$\sigma_{axuII}(M_{oG}) = -22309$

$$\sigma_{axuIII}(M_o) := \frac{M_P - M_o}{M_P - M_{oG}} \cdot \sigma_{axuII}(M_{oG})$$

$\sigma_{axuIII}(M_o)$



$M_o :=$ input if Mode III is relevant failure mode

$$\gamma R_{PCSF3} := \gamma_C \cdot \frac{\sigma_C}{\sigma_{axuIII}(M_o)}$$

$\sigma_{axuIII}(M_o) = -47668.439$

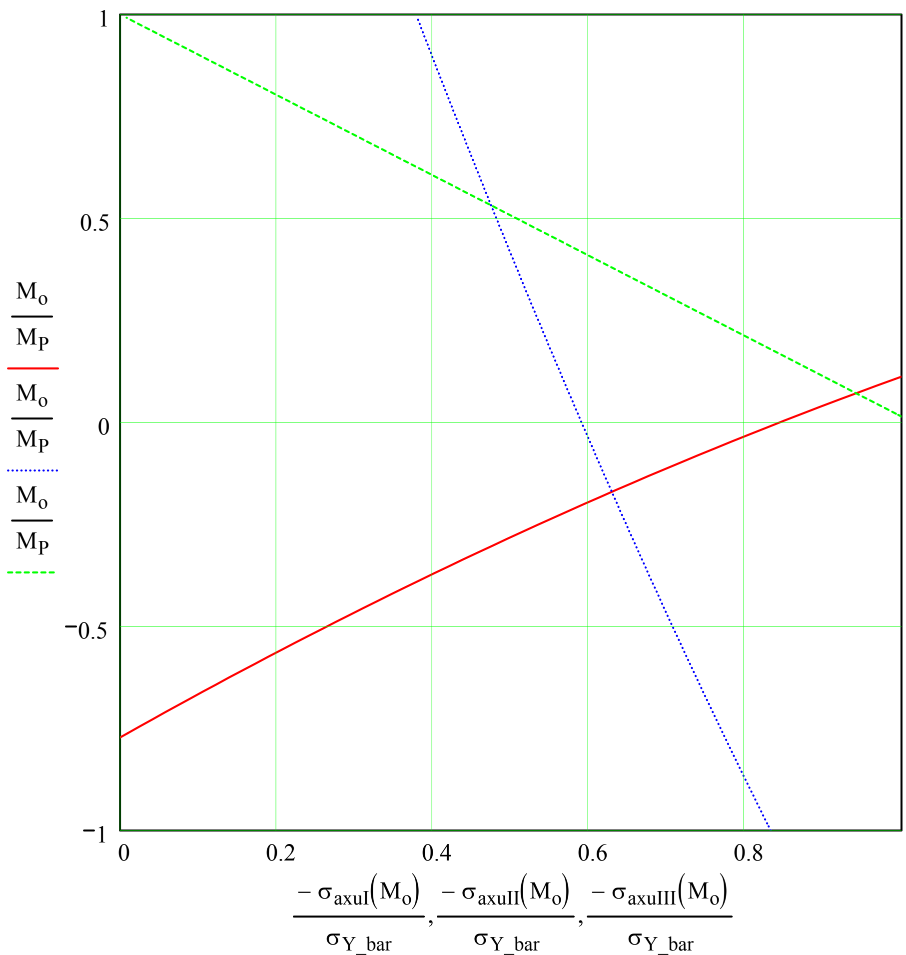
$$\frac{\sigma_{axuI}(M_o)}{\sigma_{Y\_bar}}, \frac{\sigma_{axuII}(M_o)}{\sigma_{Y\_bar}}, \frac{\sigma_{axuIII}(M_o)}{\sigma_{Y\_bar}}$$

$\sigma_{axuII}(M_o) = -27748.195$

$\sigma_{axuI}(M_o) = -39664.127$

attempting to plot all three modes:

$$M_o := -M_P, (-M_P + 10000) .. M_P$$ positive and negative Mo



trying to be a little fancier

$M_{oG} = 312495$

$$\frac{\sigma_{axuI}(M_o)}{\sigma_{Y\_bar}}, \frac{\sigma_{axuII}(M_o)}{\sigma_{Y\_bar}}, \frac{\sigma_{axuIII}(M_o)}{\sigma_{Y\_bar}}$$

$$modeI(M_o) := if\left(M_o < 0, \frac{-\sigma_{axuI}(M_o)}{\sigma_{Y\_bar}}, 0\right)$$

$$modeII(M_o) := if\left(M_o > M_{oG}, 0, if\left(M_o < 0, 0, \frac{-\sigma_{axuII}(M_o)}{\sigma_{Y\_bar}}\right)\right)$$

$$modeIII(M_o) := if\left(M_o > M_{oG}, \frac{-\sigma_{axuIII}(M_o)}{\sigma_{Y\_bar}}, 0\right)$$

$$M_o := -M_P, (-M_P + 1000) .. M_P$$