

2.14/2.140 Problem Set 6

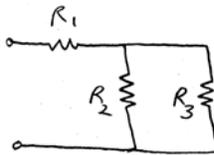
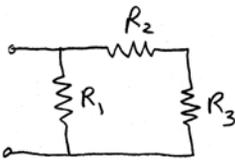
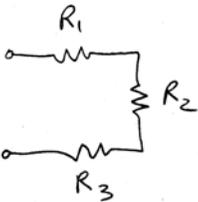
Assigned: Wed. March 12, 2014

Due: Wed. March 19, 2014, in class

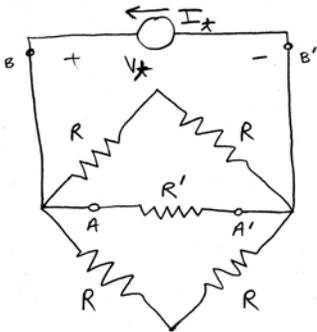
Reading: Lecture notes on Operational Amplifiers

The following problems are assigned to both 2.14 and 2.140 students. The first two problems review some basic circuit calculations. The remainder concern operational amplifier circuits, and studying the stability of these connections.

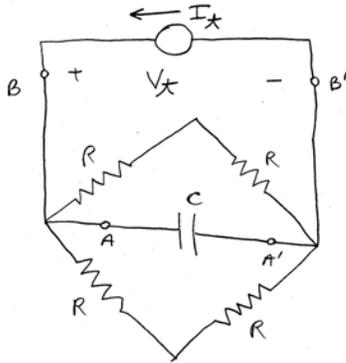
Problem 1 For the circuits shown below, calculate the equivalent resistance seen at the indicated terminals, using the series and parallel resistance formulae.



Problem 2 The circuit shown below is driven by a current source I_t at the terminals B-B'. The resulting voltage across these terminals is defined as V_t .



- a) What is the equivalent resistance R_{eq} seen at the terminals B-B'? Viewed another way, what is the ratio $V_t(t)/I_t(t) = R_{eq}$? The easiest way to solve this problem is to use series and parallel equivalents.
- b) Now we modify the circuit by removing the resistor R' at terminals A-A', and replacing this with the capacitor C shown in the figure below.



What is the equivalent impedance Z_{eq} seen at the terminals B-B'? Viewed another way, what is the ratio $V_t(s)/I_t(s) = Z_{eq}(s)$? Note that here we've switched to the Laplace domain and work in the more general concept of impedances, since the capacitor adds dynamics to the system. The easiest way to solve this problem is to use series and parallel equivalent impedances.

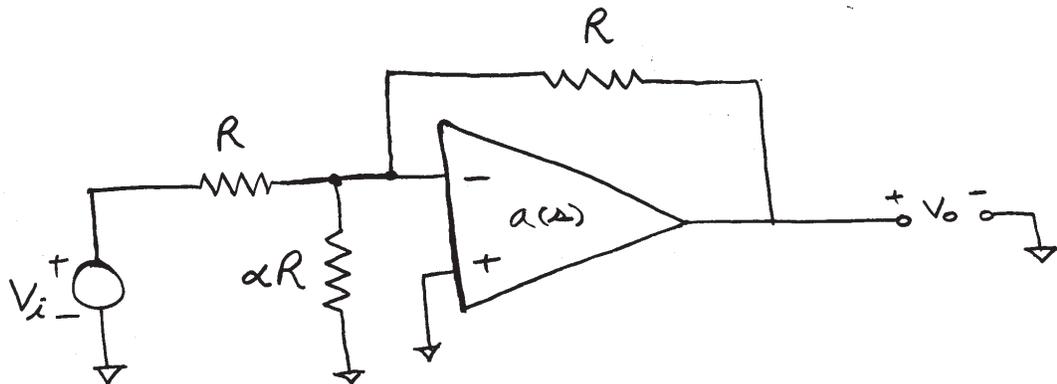
Problem 3 Archive Problem 9.2

For this circuit, make a sketch of the Bode plot for the negative of the loop transmission $L(s)$. What are the loop crossover frequency and phase margin? Also, create a Nyquist plot for the loop, and indicate the loop phase margin and gain margin on your plot. For what value of C will the loop be marginally stable? (Note that there is a small typo: the variables C and C' refer to the same capacitor.)

Problem 4 Archive Problem 9.5

This circuit can be used as an analog PI controller, for example in a closed-loop current-controlled power amplifier (see Problem G1 below). Calculate the transfer function $V_o(s)/V_i(s)$, under the assumption that the op amp has infinite gain. Use this transfer function to answer the questions posed in this problem.

Problem 5 This problem considers the op amp circuit shown below



Here the resistors take values of R , R , and αR , respectively. The resistors R set the ideal gain of the circuit, which is -1 . The resistor αR can be used to detune the bandwidth of the opamp loop when the amplifier dynamics are too "hot" for the circuit configuration. In particular, like all real systems, op amps have additional high frequency dynamics which can

give bad stability in some circuits. To understand this, assume

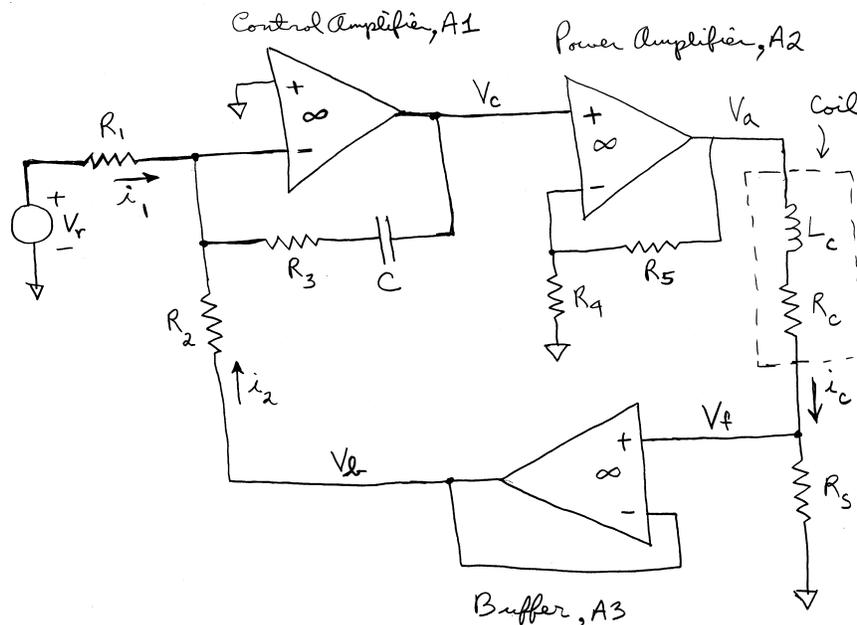
$$a(s) = \frac{5 \times 10^7}{s(10^{-7}s + 1)^2}.$$

The additional pair of poles represent high-frequency dynamics in the internal opamp circuit.

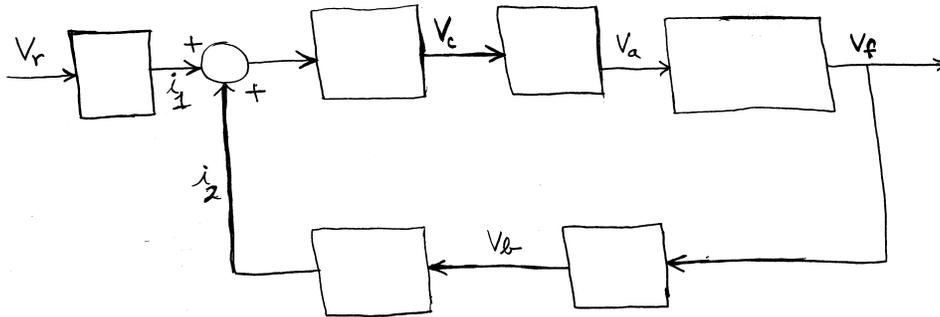
- a) First, we remove the resistor αR by letting $\alpha \rightarrow \infty$ (infinite resistance is an open circuit). For this configuration, calculate the loop crossover frequency and phase margin. Use Matlab to plot the unit step response of the circuit. You should find that this system is unstable. Compare the ring frequency of the unstable step response signal with the natural frequency of the closed loop dominant pole pair and with the loop crossover frequency. You should see that $\omega_n \approx \omega_c$. (When making ring frequency measurements on the step response, be sure to convert from Hz to rad/sec.)
- b) Conduct a Nyquist analysis for the loop in part a), which should indicate the loop instability. Be sure to clearly show the number of encirclements, and resulting number of right half plane closed-loop poles. Indicate the loop phase margin (negative) and gain margin (less than one) on the Nyquist plot.
- c) Draw a root locus plot for the loop of part a). Indicate the closed-loop root locations for the nominal design with $\alpha \rightarrow \infty$.
- d) Now pick a value of α which results in a loop phase margin $\phi_m = 45^\circ$. For this value of α , what is the loop crossover frequency ω_c ? Use Matlab to plot the unit step response of this detuned circuit. Compare the natural frequency of the closed loop dominant pole pair with the loop crossover frequency.
- e) Conduct a Nyquist analysis for the loop in part d), which should indicate the loop stability. Indicate the loop phase margin and gain margin on the Nyquist plot.
- f) Draw a root locus plot indicating where the closed-loop poles are located for the value of α from part d).

The following problems are assigned to only 2.140 students. Students in 2.14 are welcome to work these, but no extra credit will be given.

Problem G1 In the lab you will see power amplifiers used to control the *voltage* on a load such as a voice coil or motor. In practice, it is also common to use power amplifiers in a feedback loop to control the *current* through a load. This problem investigates the design of such current-controlled amplifier configurations. In the circuit shown below, the load is represented by a coil with inductance L_c and resistance R_c . The load current i_c is supplied by the power amplifier A2. The current is measured by passing it through the sense resistor R_s ; the voltage V_f is then proportional to the load current. Thus if V_f is kept at a desired level, the load current can be regulated. The buffer amplifier A3 is used to avoid loading the main coil circuit; that is, current i_2 is supplied by amplifier A3. The control amplifier A1 is used to compare the reference voltage V_r with the feedback voltage V_b , and then to add dynamics to the loop via R_3 and C . That is, amplifier A1 is the compensation for the current feedback loop. Note that all the op amps are modeled as having infinite gain. All voltages labeled on the circuit are with respect to circuit ground.



- a) The current feedback loop may be represented in the form of the block diagram shown below.



Develop expressions for each of the transfer functions in the blocks of this block diagram, and fill them in on your own version of the block diagram. Carefully note the system variables already shown on the block diagram, and fill in the transfer functions appropriately.

- b) Suppose the input V_r has the constant value $V_r = V_0$. What is the steady-state load current i_C for this constant reference input? Explain your results.
- c) Now let the parameters take the values $R_1 = R_2 = R_4 = 10 \text{ k}\Omega$, $R_5 = 50 \text{ k}\Omega$, $R_s = 1 \text{ }\Omega$, $L_c = 100 \text{ mH}$, and $R_c = 10 \text{ }\Omega$. Chose the values of R_3 and C such that the closed-loop system poles have a natural frequency of $\omega_n = 1000 \text{ rad/sec}$ and a damping ratio of $\zeta = 0.4$.
- d) Make a sketch of the Bode plot for the negative of the loop transmission $L(s)$. What are the loop crossover frequency and phase margin? How do these relate to the closed-loop natural frequency and damping ratio from part c)?
- e) For these parameter values, let the input be a negative unit step $V_r = -u_s(t)$. Make a graph of the response in current $i_C(t)$.

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2.14 / 2.140 Analysis and Design of Feedback Control Systems
Spring 2014

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