

2.171 Problem Set 3

Assigned: Mon. Sept 25, 2006

Due: Wed. Oct 4, 2006, in class

Reading: F,P,&W Ch. 4, starred sections optional; F,P,&W 5.1, 5.2, 5.3
(Sections 5.4 and 5.5 are only of historical interest at this point in time;
these techniques are largely obsolete in the present era.)

Problem 1 FPW 4.19 Use Matlab to plot the discrete-time Bode plot for this plant assuming a DC gain of unity. Also, use Matlab to plot the system unit step response.

Problem 2 FPW 4.28

Problem 3 FPW 4.29

Problem 4 Consider a continuous-time signal $y_a(t) = \sin \omega t$ which is sampled at a rate of T seconds/sample to yield $y(k) = y_a(kT)$.

a) Show that $y(k) = \sin \Omega k$ where $\Omega = \omega T$.

b) One can notice that although the sampled sinusoid is a pure harmonic at frequency Ω , the time waveform appears to consist of beats as Ω approaches π . Some insight into this appearance may be gained as follows. Specifically, show that a discrete-time signal $y(k) = \sin \Omega k$ may be written as $y(k) = -\sin \Delta k \cos \pi k$, and thus as a beat tone between frequencies Δ and π , where $\Delta \equiv \pi - \Omega$.

Hint: First use the periodicity of discrete-time frequencies to write

$$\sin \Omega k = \frac{1}{2} [\sin \Omega k + \sin(\Omega - 2\pi)k].$$

The following trig identities may be helpful:

$$\sin a + \sin b = \sin(A + B) + \sin(A - B) = 2 \sin A \cos B,$$

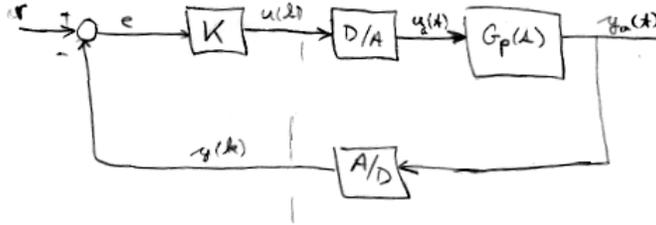
where $A = (a + b)/2$, and $B = (a - b)/2$.

c) Make a sketch of one period of the waveform $y(k) = \sin(5\pi/6)k$. Indicate graphically how the result of part b) applies to this waveform.

d) Assume $T = 1$ and that $y_a(t) = \sin(5\pi/6)t$, so that the waveform in part c) could result from sampling this sinusoid without aliasing. Sketch the relationship between the waveform $y_a(t)$ and its samples $y(k)$.

e) For the example in part c) we have chosen Ω only slightly less than π so that the beat tone Δ is at a relatively low frequency. However, the result from b) is general. Explain what happens to the apparent beating phenomena as Ω is small relative to π , so that the sampled signal looks very much like a sinusoid, with little apparent beating. You may want to illustrate the point with a specific numerical example.

Problem 5 Consider the loop shown below in a figure taken from the notes handed out in class last week.



Assume that the plant transfer function $G_p(s)$ is as given in F,P,&W Problem 4.28, and that we sample with $T=10$ msec as in that problem. Now assume that the digital controller is a pure proportional gain K as shown in the figure. The reference r is assumed to be a discrete-time signal.

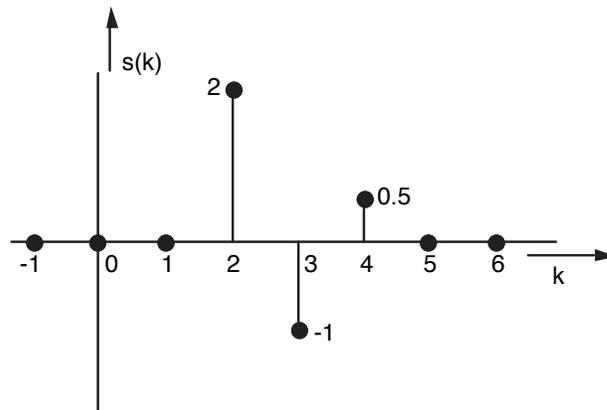
- Calculate the closed-loop transfer function $Y(z)/R(z)$
- Calculate the closed-loop poles and zeros of the system as a function of K .
- For what range of K is the closed loop system stable?
- Use Matlab to simulate the closed-loop step response for several interesting values of K . Explain these responses in light of the closed loop pole and zero locations.

Problem 6 Consider a system with a transfer function

$$H(z) = \frac{0.5z^5 - 0.5}{z^6 + 0.5z^5} \quad (1)$$

- Write the difference equation corresponding to (1). What is the DC gain of this filter?
- Draw a minimal block diagram representation of this system in terms of blocks consisting of unit delays, summing junctions, and gains. What is the order of this system?
- Sketch the response of this system to a unit pulse, i.e., a signal which is 1 for $k = 0$ and zero elsewhere. Be sure to label both axes of your sketch and to show relative amplitudes. Use whatever approach you find easiest to develop your sketch, but be sure to explain your method.

Problem 7 A given linear, time-invariant (LTI) system H has a *unit-step* response $s(k)$ which is



shown below

Note that the only non-zero values of $s(k)$ are at $k = 2, 3, 4$.

- a) Use any convenient method to determine the difference equation for this system in terms of an input $e(k)$ and an output $y(k)$. Please explain your approach.
- b) Now suppose that the system input is a unit sample, i.e., $e(k)=\delta(k)$. Using whatever approach you find most efficient, write an expression for the resulting output $y(k)$ which is valid for all k .
- c) Find the z-transform $S(z)$ of $s(k)$ as given in the figure above. How is this transform related to the the system transfer function $H(z)$?