Problem 1)

Abstract: The basic idea of this problem was to find the heave and pitch motions experienced by a ship transiting in head seas. Given the geometry and speed of the ship and the frequency of the waves, the first step was to calculate the added mass, damping and restoring coefficients of the system. Next, the magnitudes of the Froude-Krylov (F-K) excitation forces were determined. The following steps were based on the idea that the overall system is comprised of two linear systems in series. The first linear system acts upon the wave elevation input, and yields an output of the F-K forces. These forces are the input of the second linear system, which yields the ship's response motions as the output. The transfer functions for both of these systems were determined for both heave and pitch, uncoupled. The entire process, up to this point, was then repeated for a second input wave frequency. The transfer functions and encounter frequency were used to transform a given ambient wave spectrum into the spectra of heave and pitch response. The follow-on problem in HW9 will use these spectra to determine whether the ship met certain design criteria. Since two of the design criteria involved vertical velocity and acceleration, the heave and pitch spectra were transformed again, through the appropriate transfer functions, into velocity and acceleration spectra.

Given: The following parameters were given in the problem statement or are known physical constants.

$U := 10$ Units are m/s

$\rho := 1000$ Units are kb/m^3

$g := 9.81$ Units are m/s^2

$\omega\_o := 0.5$ Units are rad/s

Encounter frequency and wave number were both calculated based on the wave frequency. The encounter frequency formula is in the form applying to head seas in particular.

$$\omega\_e := \omega\_o + U \cdot \frac{\omega\_o^2}{g}$$ Units are rad/s

$\omega\_e = 0.755$ Units are rad/s

$k := \frac{\omega\_o^2}{g}$ $k = 0.025$ Units are m^-1

Question 1, Part a) I calculated a33 and b33 in an Excel spreadsheet and imported the numbers.

$$a33 := \begin{pmatrix} 0 \\ 11290 \\ 32201 \\ 59199 \\ 95819 \\ 95819 \\ 95819 \\ 95819 \\ 95819 \\ 59199 \\ 32201 \\ 11290 \\ 0 \end{pmatrix} \qquad b33 := \begin{pmatrix} 0 \\ 11561 \\ 35571 \\ 58692 \\ 82999 \\ 82999 \\ 82999 \\ 82999 \\ 82999 \\ 58692 \\ 35571 \\ 11561 \\ 0 \end{pmatrix}$$

I also calculated the values of the Added Mass, Damping and Restoring coefficients in Excel; they are imported here to continue the project in Mathcad.

A33 =	3422372
B33 =	3133220
A35 =	54989379
B35 =	-34223725
A53 =	-54989379
B53 =	34223725
A55 =	1052981521
B55 =	1000238628

$A33 := 3422372 \qquad A35 := 54989379$

$B33 := 3133220 \qquad B35 := -34223725$

$A55 := 1052981521 \qquad A53 := -A35$

$B55 := 1000238628 \qquad B53 := -B35$

Part b)

The restoring coefficient, C55, formula requires the vertical centers of buoyancy and gravity. The vertical center of gravity was given, but I had to calculate the vertical center of bouyancy. I did this in my Excel file by doing a vertical integration of the ship using waterplane areas at a step size of z=1 m. I drew several diagrams of the geometry and discovered that the area of the waterplane, Awp(z), was equal to the breadth at midships at that depth times what I called the effective length, 40+2z, where z is measured positive up from the waterline in meters (i.e. z will be a negative number). The half breadth at each depth was found using pythagorean's theorem, $HalfB := \sqrt{\left(10^2 - z^2\right)}$. So the overall formula used was Awp(z)=2*HalfB*(40+2z). These waterplane areas were numerically integrated vertically to determine the underwater volume and the underwater center of bouyancy, and, in turn, C55.

C33 =	7848000
C35 =C53 =	0
C55 =	1456709517

$C33 := 7848000$

$C35 := 0$

$C53 := 0$

$C55 := 1456709517$

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Project scenario:
A naval vessel is transiting in head seas at speed, U = 10 m/s
Deep water.

Density, rho = 1000 kg/m^3
Gravity, g = 9.81 m/s^2

Note: A33 (2D) is listed as a33 in this report, likewise with B.

1a) Incident wave frequency:
omega_o = 0.5 rad/s

Encounter frequency:
omega_e = 0.754842 rad/s

Station m Observed	Radius or HalfBreadth m Observed	Breadth m	x*Breadth m^2	x^2*Brdth m^3	Sectional Area, S m^2	omega_e^2*R over g	a33 over rho*S Observed	a33	x*a33	x^2*a33	b33 over rho*omega_e*S Observed	b33	x*b33	x^2*b33
30	0	0	0	0	0	0.000		0	0	0		0	0	0
25	2.5	5	125	3125	9.8	0.145	1.15	11290	282252	7056312	1.56	11561	289015	7225378
20	5	10	200	4000	39.3	0.290	0.82	32201	644026	12880530	1.2	35571	711422	14228436
15	7.5	15	225	3375	88.4	0.436	0.67	59199	887991	13319862	0.88	58692	880385	13205768
10	10	20	200	2000	157.1	0.581	0.61	95819	958186	9581858	0.7	82999	829992	8299921
5	10	20	100	500	157.1	0.581	0.61	95819	479093	2395464	0.7	82999	414996	2074980
0	10	20	0	0	157.1	0.581	0.61	95819	0	0	0.7	82999	0	0
-5	10	20	-100	500	157.1	0.581	0.61	95819	-479093	2395464	0.7	82999	-414996	2074980
-10	10	20	-200	2000	157.1	0.581	0.61	95819	-958186	9581858	0.7	82999	-829992	8299921
-15	7.5	15	-225	3375	88.4	0.436	0.67	59199	-887991	13319862	0.88	58692	-880385	13205768
-20	5	10	-200	4000	39.3	0.290	0.82	32201	-644026	12880530	1.2	35571	-711422	14228436
-25	2.5	5	-125	3125	9.8	0.145	1.15	11290	-282252	7056312	1.56	11561	-289015	7225378
-30	0	0	0	0	0	0.000		0	0	0		0	0	0
Sums:	80	160	0	26000				684474	0	90468051		626644	0	90068967

Here , I find the added mass and damping coefficients (3D) for the ship, Aij and Bij for i,j=3,5
The equations to calculate these coefficients were taken from Faltinsen page 56.
Since I have discrete values, I will use the trapezoidal rule as a substitute for integration.

h = 5 m

Trapezoidal rule = .5 * h * (a33(-30) + 2a33(-25) +2a33(-20) . . . + 2a33(25) + a33(30))
Since a33(-30)=a33(30), I can simplify the formula to h*(sum(a33 values))

A33 = 3422372
B33 = 3133220
A35 = 54989379
B35 = -34223725
A53 = -54989379
B53 = 34223725
A55 = 1052981521
B55 = 1000238628

1b) Find the restoring coefficients, Cij for i,j=3,5

Area of waterplane = Awp = 800 m^2

z_g =	-7.5
C33 =	7848000
C35 =C53 =	0
C55 =	1456709517

Formulas for restoring coefficients were taken from Faltinsen page 58.
To find C55, I need to find GM_L or KB.
To find KB, I will integrate the underwater volume vertically using waterplanes at 1 m intervals.
Based on geometry, half breadth at each waterline, z, is SQRT(10^2 - z^2)
The area of each waterplane is then 2*HalfBreadth(which is a fn of z)*'Effective Length'
The 'Effective Length' is the length of the waterplane if you cut off the aft triangular portion and flip it over and put it with the forward triangular portion to make an equivalent rectangular Awp.

z m	Midships HalfBr (m)	"Effective length" (m)	Awp(z) m^2	Trapezoidal Multiplier	F(M)	Lever Arm	F(V)
0	10.00	40	800.00	0.5	400.00	0	0.00
-1	9.95	38	756.19	1	756.19	1	756.19
-2	9.80	36	705.45	1	705.45	2	1410.91
-3	9.54	34	648.68	1	648.68	3	1946.04
-4	9.17	32	586.57	1	586.57	4	2346.28
-5	8.66	30	519.62	1	519.62	5	2598.08
-6	8.00	28	448.00	1	448.00	6	2688.00
-7	7.14	26	371.35	1	371.35	7	2599.48
-8	6.00	24	288.00	1	288.00	8	2304.00
-9	4.36	22	191.79	1	191.79	9	1726.12
-10	0.00	20	0.00	0.5	0.00	10	0.00
				Sum =	4915.65		18375.09

Here, h =	1	m
Volume (m^3) = h*Sum F(M) =	4916	m^3
M_volume (m^4) = h^2*Sum F(V) =	18375	m^4
KB = M_vol/Vol =	3.74	m
z_B =	-3.74	m

Part c) Now I have to find the F-K heave exciting forces and pitch exciting moments. Because $\lambda >> B(x)$, the F-K heave exciting force per unit length, at a given x, can be approximated as $\rho g B(x)\eta(x,t)$. Therefore, I will integrate this over the length of the ship to get the F-K heave exciting force. Similarly, I will integrate $x\rho g B(x)\eta(x,t)$ over the length of the ship to get the F-K surge exciting pitch.

$i := \sqrt{-1}$ $\qquad A1 := 1$ $\qquad$ Here A1 is the amplitude of the wave, I've defined it as 1 m.

$$\eta(x,t) := A1 \cdot \mathrm{Re}\left[e^{(i\omega\_et+ik\cdot x)}\right]$$

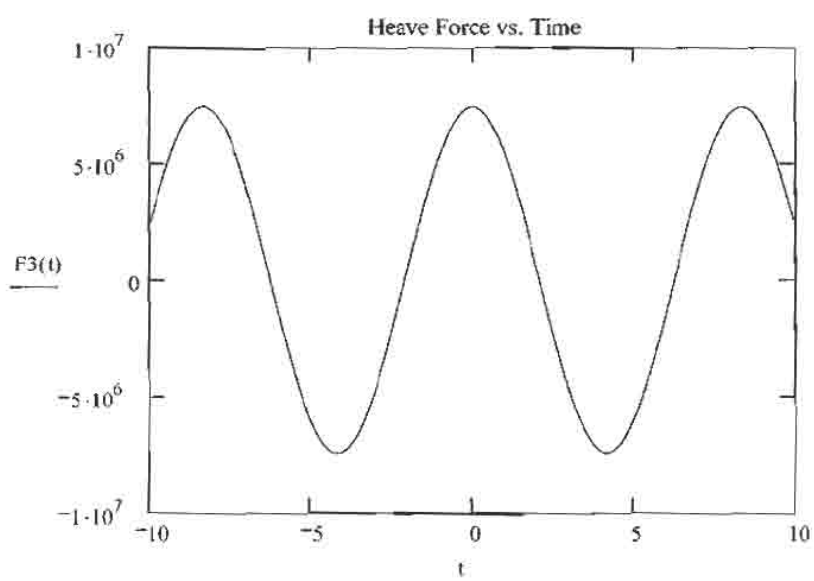
$$2 \cdot (\eta, x, t) := A1 \cdot \left(\frac{1}{2} \cdot \exp(i \cdot \omega\_e \cdot t + i \cdot k \cdot x) + \frac{1}{2} \cdot \exp\left(\overline{(i \cdot \omega\_e \cdot t + i \cdot k \cdot x)}\right)\right) \cdot \pi \cdot \Delta(\omega)$$

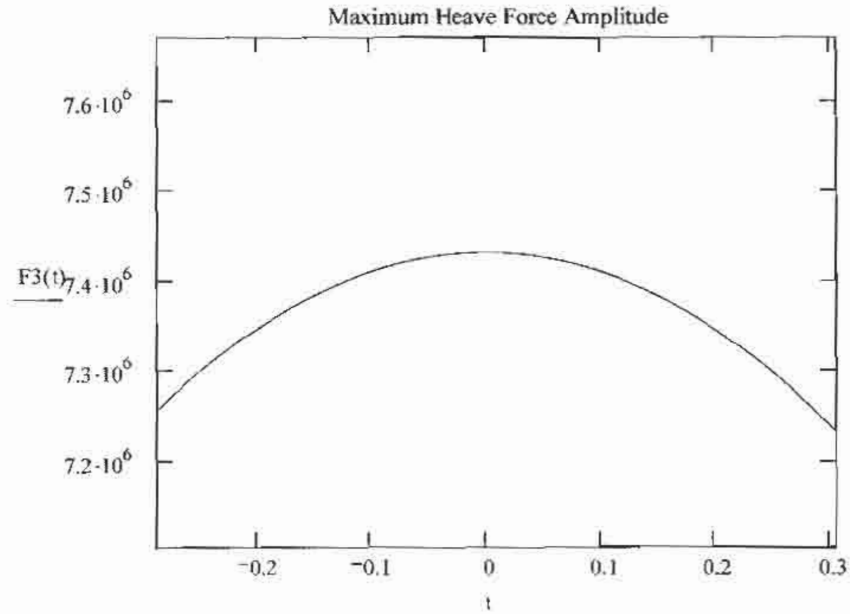
This is the Fourier Transform of $\eta(x,t)$. I will need it later to find the transfer function.

Based on the given geometry, I defined the ship's breadth as a piecewise function along the length of the ship. Note that this is the entire breadth, not the halfbreadth.

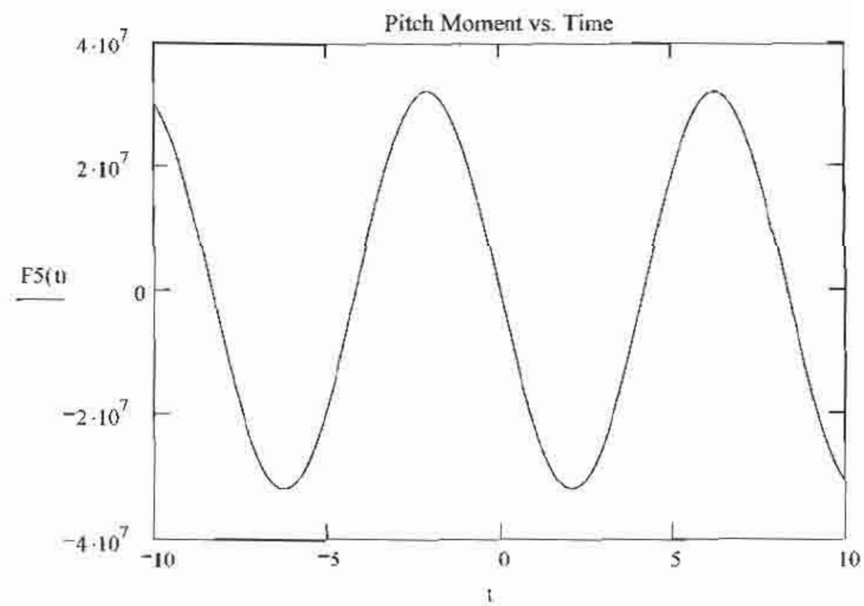
$$B(x) := \begin{cases} (x + 30) & \text{if } -30 \le x \le -10 \\ (20) & \text{if } -10 < x \le 10 \\ (-x + 30) & \text{if } 10 < x \le 30 \end{cases}$$

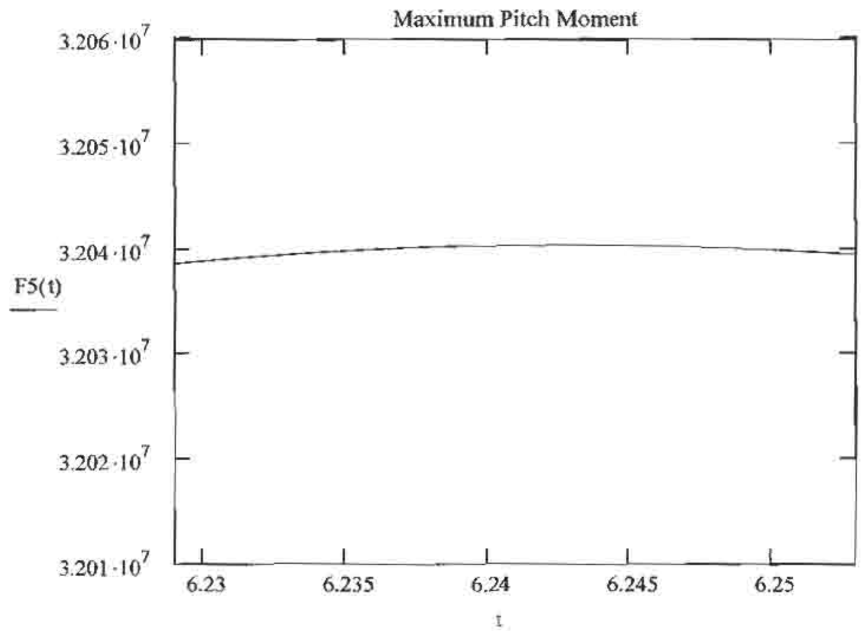
$$F3(t) := \rho \cdot g \cdot \left(\int_{-30}^{30} B(x)\,\eta(x,t)\,dx\right)$$





$$F5(t) := \rho \cdot g \cdot \int_{-30}^{30} x \cdot B(x) \cdot \eta(x,t)\, dx$$





F3(t) and F5(t) are both sinusoidal type waves, so I can redefine them in terms of amplitude, which I will call Fmax.

$$F3(0) = 7.432 \times 10^{6}$$

$F3max := F3(0)$ Visually Fmax occurs at t=0, so I will define it as such.

$$F3(t) := F3max \cdot Re\left[e^{(i\omega\_et)}\right]$$

$$2 \cdot (F3,t) := F3max \cdot \left(\frac{1}{2} \cdot \exp(i \cdot \omega\_e \cdot t) + \frac{1}{2} \cdot \exp\left(\overline{i \cdot \omega\_e \cdot t}\right)\right) \cdot \pi \cdot \Delta(\omega)$$

$$F5(6.24) = 3.204 \times 10^{7}$$

Visually Fmax occurs at t=-6.24, so I will define it as such.

$F5max := F5(6.24)$

$$F5(t) := F5max \cdot Re\left[e^{\left[i\left(\omega\_et-\frac{\pi}{2}\right)\right]}\right]$$

$$2 \cdot (F5,t) := F5max \cdot \left[\frac{1}{2} \cdot \exp\left[i \cdot \left(\omega\_e \cdot t - \frac{\pi}{2}\right)\right] + \frac{1}{2} \cdot \exp\left(i \cdot \overline{\left(\omega\_e \cdot t - \frac{\pi}{2}\right)}\right)\right] \cdot \pi \cdot \Delta(\omega)$$

Part d) Now I needed to find the transfer function for my first linear system, between $\eta(t)$ and the F-K forces, F3(t) and F5(t). I evaluated η at x=0 to find these transfer functions.

Copied from above, here are the Fourier Transforms of $\eta(x,t)$, F3(t) and F5(t).

$$2 \cdot (\eta, x, t) := A1 \cdot \left(\frac{1}{2} \cdot \exp(i \cdot \omega\_e \cdot t + i \cdot k \cdot x) + \frac{1}{2} \cdot \exp\left(\overline{(i \cdot \omega\_e \cdot t + i \cdot k \cdot x)}\right)\right) \cdot \pi \cdot \Delta(\omega)$$

Evaluated at x=0, the FT of $\eta(x,t)$ is:

$$2 \cdot (\eta, t) := A1 \cdot \left(\frac{1}{2} \cdot \exp(i \cdot \omega\_e \cdot t) + \frac{1}{2} \cdot \exp\left(\overline{(i \cdot \omega\_e \cdot t)}\right)\right) \cdot \pi \cdot \Delta(\omega)$$

$$2 \cdot (F3, t) := F3max \cdot \left(\frac{1}{2} \cdot \exp(i \cdot \omega\_e \cdot t) + \frac{1}{2} \cdot \exp\left(\overline{i \cdot \omega\_e \cdot t}\right)\right) \cdot \pi \cdot \Delta(\omega)$$

$$2 \cdot (F5, t) := F5max \cdot \left[\frac{1}{2} \cdot \exp\left[i \cdot \left(\omega\_e \cdot t - \frac{\pi}{2}\right)\right] + \frac{1}{2} \cdot \exp\left(\overline{i \cdot \left(\omega\_e \cdot t - \frac{\pi}{2}\right)}\right)\right] \cdot \pi \cdot \Delta(\omega)$$

The transfer function, $H(\omega)$, is found by dividing the output function in the frequency domain, $F5(\omega)$, by the input function in the frequency domain, $\eta(\omega)$. It is easily observed that the transfer function for each heave and pitch are constant values because all other terms drop out.

$$H31 := \frac{F3max}{A1} \qquad H51 := \frac{F5max}{A1} \cdot e^{-i\frac{\pi}{2}}$$

where A1 is the amplitude of the input wave elevation, η. For a force per unit length, I will let A1=1 m.

$$H31 = 7.432 \times 10^{6} \qquad H51 = -3.204i \times 10^{7}$$

Part e) In this project, the overall system includes two linear systems in series. The first system has wave elevation as input and incident force as output. The transfer function for this system was determined in part d. The second linear system has the force as input and the response motion of the ship is the output. The transfer function for this system will be found in this part for the uncoupled heave and pitch equations of motion.

The transfer function between the external force and the ship's response motion is of the now familiar form:

$$\frac{1}{\left[-(m + A) \cdot \omega^2 + i \cdot \omega \cdot B + C\right]}$$

I need to define the mass values, M33 and M55. M33 is the ship's mass, which is equal to that of the displaced water, so it is displaced volume times density. I calculated the displaced volume in my Excel file as 4916 m^3. M55 is given in the problem statement.

$V := 4916$ The units are m^3.

$M33 := \rho \cdot V$ $M33 = 4.916 \times 10^6$

$M55 := 1.5 \cdot 10^9$ The units are kg*m^2.

$$H32(\omega) := \frac{1}{\left[-(M33 + A33) \cdot \omega^2 + i \cdot \omega \cdot B33 + C33\right]} \qquad H32(0.755) = 2.04 \times 10^{-7} - 1.559i \times 10^{-7}$$

$$H32(\omega) \rightarrow \frac{1}{-8338372 \cdot \omega^2 + 3133220 \cdot i \cdot \omega + 7848000} \qquad |H32(0.755)| = 2.567 \times 10^{-7}$$

$$H52(\omega) := \frac{1}{\left[-(M55 + A55) \cdot \omega^2 + i \cdot \omega \cdot B55 + C55\right]} \qquad H52(0.755) = 2.536 \times 10^{-12} - 1.324i \times 10^{-9}$$

$$H52(\omega) \rightarrow \frac{1}{-2552981521.0 \cdot \omega^2 + 1000238628 \cdot i \cdot \omega + 1456709517} \qquad |H52(0.755)| = 1.324 \times 10^{-9}$$

Part f) I redid parts a through e in another Mathcad file and imported the relevant transfer functions under the names H31b, H51b, H32b and H52b.

$H31b := 5.901 \times 10^{6}$ $\quad$ $H51b := -i \cdot 6.118 \times 10^{7}$

$$H32b(\omega) := \frac{1}{-8492507 \cdot \omega^{2} + 1353808 \cdot i \cdot \omega + 7848000}$$

$H32b(1.323) = -1.338 \times 10^{-7} - 3.415i \times 10^{-8}$ $\quad$ $|H32b(1.323)| = 1.381 \times 10^{-7}$

$$H52b(\omega) := \frac{1}{-2102285755.0 \cdot \omega^{2} + 337862269 \cdot i \cdot \omega + 1456709517}$$

$H52b(1.323) = -4.324 \times 10^{-10} - 8.694i \times 10^{-11}$ $\quad$ $|H52b(1.323)| = 4.41 \times 10^{-10}$

Part f) Here I am repeating parts a through e using ω_o = 0.75 rad/s.

$U := 10$ Units are m/s

$\rho := 1000$ Units are kb/m^3

$g := 9.81$ Units are m/s^2

$\omega\_o := 0.75$ Units are rad/s

$\omega\_e := \omega\_o + U \cdot \frac{\omega\_o^2}{g}$ Units are rad/s

$\omega\_e = 1.323$ Units are rad/s

$k := \frac{\omega\_o^2}{g}$ $k = 0.057$ Units are m^-1

Note, I calculated a33 and b33 in an Excel spreadsheet and imported the numbers.

$$a33 := \begin{pmatrix} 0 \\ 6578 \\ 23562 \\ 56549 \\ 108385 \\ 108385 \\ 108385 \\ 108385 \\ 108385 \\ 56549 \\ 23562 \\ 6578 \\ 0 \end{pmatrix} \qquad b33 := \begin{pmatrix} 0 \\ 11044 \\ 22347 \\ 29233 \\ 29103 \\ 29103 \\ 29103 \\ 29103 \\ 29103 \\ 29233 \\ 22347 \\ 11044 \\ 0 \end{pmatrix}$$

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Project scenario:
A naval vessel is transiting in head seas at speed, U = 10 m/s
Deep water.

Density, rho = 1000 kg/m^3
Gravity, g = 9.81 m/s^2

Note: A33 (2D) is listed as a33 in this report, likewise with B.

1a) Incident wave frequency:
omega_o = 0.75 rad/s

Encounter frequency:
omega_e = 1.3233945 rad/s

Station m	Radius or HalfBreadth m	Breadth m	x*Breadth m^2	x^2*Brdth m^3	Sectional Area, S m^2	omega_e^2*R over g	a33 over rho*S	a33	x*a33	x^2*a33	b33 over rho*omega_e*S	b33	x*b33	x^2*b33
Observed	Observed						Observed				Observed			
30	0	0	0	0	0	0.000		0	0	0		0	0	0
25	2.5	5	125	3125	9.8	0.446	0.67	6578	164443	4111069	0.85	11044	276088	6902210
20	5	10	200	4000	39.3	0.893	0.6	23562	471239	9424778	0.43	22347	446938	8938768
15	7.5	15	225	3375	88.4	1.339	0.64	56549	848230	12723450	0.25	29233	438493	6577400
10	10	20	200	2000	157.1	1.785	0.69	108385	1083849	10838495	0.14	29103	291030	2910296
5	10	20	100	500	157.1	1.785	0.69	108385	541925	2709624	0.14	29103	145515	727574
0	10	20	0	0	157.1	1.785	0.69	108385	0	0	0.14	29103	0	0
-5	10	20	-100	500	157.1	1.785	0.69	108385	-541925	2709624	0.14	29103	-145515	727574
-10	10	20	-200	2000	157.1	1.785	0.69	108385	-1083849	10838495	0.14	29103	-291030	2910296
-15	7.5	15	-225	3375	88.4	1.339	0.64	56549	-848230	12723450	0.25	29233	-438493	6577400
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-25	2.5	5	-125	3125	9.8	0.446	0.67	6578	-164443	4111069	0.85	11044	-276088	6902210
-30	0	0	0	0	0	0.000		0	0	0		0	0	0
Sums:	80	160	0	26000				715301	0	79614830		270762	0	52112497

Here , I find the added mass and damping coefficients (3D) for the ship, Aij and Bij for i,j=3,5
The equations to calculate these coefficients were taken from Faltinsen page 56.
Since I have discrete values, I will use the trapezoidal rule as a substitute for integration.

h = 5 m

Trapezoidal rule = .5 * h * (a33(-30) + 2a33(-25) +2a33(-20) . . . + 2a33(25) + a33(30))
Since a33(-30)=a33(30), I can simplify the formula to h*(sum(a33 values))

A33 = 3576507
B33 = 1353808
A35 = 7729979
B35 = -35765069
A53 = -7729979
B53 = 35765069
A55 = 602285755
B55 = 337862269

Note: The restoring coefficients are not frequency dependent, so these are the same as for omega_o=0.5 rad/s.

1b) Find the restoring coefficients, Cij for i,j=3,5 — Area of waterplane = Awp = 800 m^2

Formulas for restoring coefficients were taken from Faltinsen page 58.

z_g =	-7.5
C33 =	7848000
C35 =C53 =	0
C55 =	1456709517

To find C55, I need to find GM_L or KB.
To find KB, I will integrate the underwater volume vertically using waterplanes at 1 m intervals.
Based on geometry, half breadth at each waterline, z, is SQRT(10^2 - z^2)
The area of each waterplane is then 2*HalfBreadth(which is a fn of z)*'Effective Length'
The 'Effective Length' is the length of the waterplane if you cut off the aft triangular portion and flip it over and put it with the forward triangular portion to make an equivalent rectangular Awp.

z m	Midships HalfBr (m)	"Effective length" (m)	Awp(z) m^2	Trapezoidal Multiplier	F(M)	Lever Arm	F(V)
0	10.00	40	800.00	0.5	400.00	0	0.00
-1	9.95	38	756.19	1	756.19	1	756.19
-2	9.80	36	705.45	1	705.45	2	1410.91
-3	9.54	34	648.68	1	648.68	3	1946.04
-4	9.17	32	586.57	1	586.57	4	2346.28
-5	8.66	30	519.62	1	519.62	5	2598.08
-6	8.00	28	448.00	1	448.00	6	2688.00
-7	7.14	26	371.35	1	371.35	7	2599.48
-8	6.00	24	288.00	1	288.00	8	2304.00
-9	4.36	22	191.79	1	191.79	9	1726.12
-10	0.00	20	0.00	0.5	0.00	10	0.00
				Sum =	4915.65		18375.09

Here, h = 1 m
Volume (m^3) = h*Sum F(M) = 4916 m^3
M_volume (m^4) = h^2*Sum F(V) = 18375 m^4

KB = M_vol/Vol = 3.74 m
z_B = -3.74 m

I also calculated the values of the Added Mass, Damping and Restoring coefficients in Excel; they are imported here to continue the project in Mathcad.

A33 =	3576507
B33 =	1353808
A35 =	7729979
B35 =	-35765069
A53 =	-7729979
B53 =	35765069
A55 =	602285755
B55 =	337862269

$A33 := 3576507$

$B33 := 1353808$

$A35 := 7729979$

$B35 := -35765069$

$A53 := -A35$

$B53 := -B35$

$A55 := 602285755$

$B55 := 337862269$

C33 =	7848000
C35 =C53 =	0
C55 =	1456709517

$C33 := 7848000$

$C35 := 0$

$C53 := 0$

$C55 := 1456709517$

Part c) Now I have to find the F-K heave exciting forces and pitch exciting moments. Because $\lambda >> B(x)$, the F-K heave exciting force per unit length, at a given x, can be approximated as $\rho g B(x)\eta(x,t)$. Therefore, I will integrate this over the length of the ship to get the F-K heave exciting force. Similarly, I will integrate $x\rho g B(x)\eta(x,t)$ over the length of the ship to get the F-K surge exciting pitch.

$i := \sqrt{-1}$ $\qquad A1 := 1$ $\qquad$ Here A1 is the amplitude of the wave, I've defined it as 1 m.

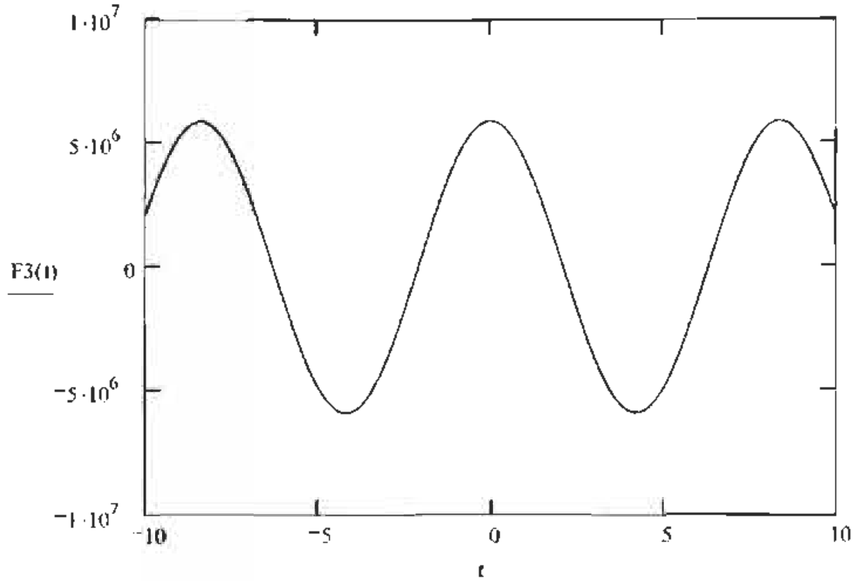
$\eta(x,t) := A1 \cdot \mathrm{Re}\left[e^{(i\omega\_ot + ik \cdot x)}\right]$ $\qquad \eta(0,t) \rightarrow \frac{1}{2} \cdot \exp(.75 \cdot i \cdot t) + \frac{1}{2} \cdot \exp\left(.75 \cdot i \cdot \bar{t}\right)$

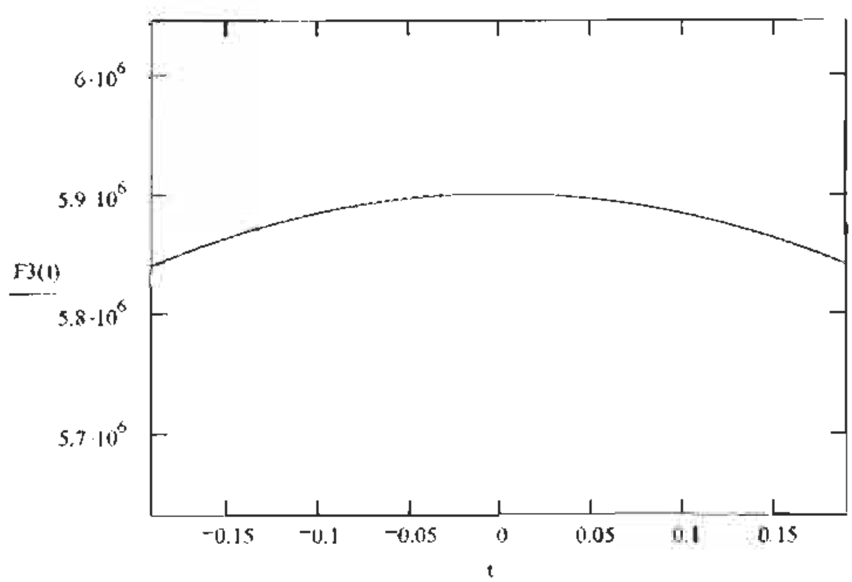
$$2 \cdot (\eta, x, t) := A1 \cdot \left(\frac{1}{2} \cdot \exp(i \cdot \omega\_o \cdot t + i \cdot k \cdot x) + \frac{1}{2} \cdot \exp\left(\left(\overline{i \cdot \omega\_o \cdot t + i \cdot k \cdot x}\right)\right)\right) \cdot \pi \cdot \Delta(\omega)$$

This is the Fourier Transform of $\eta(x,t)$

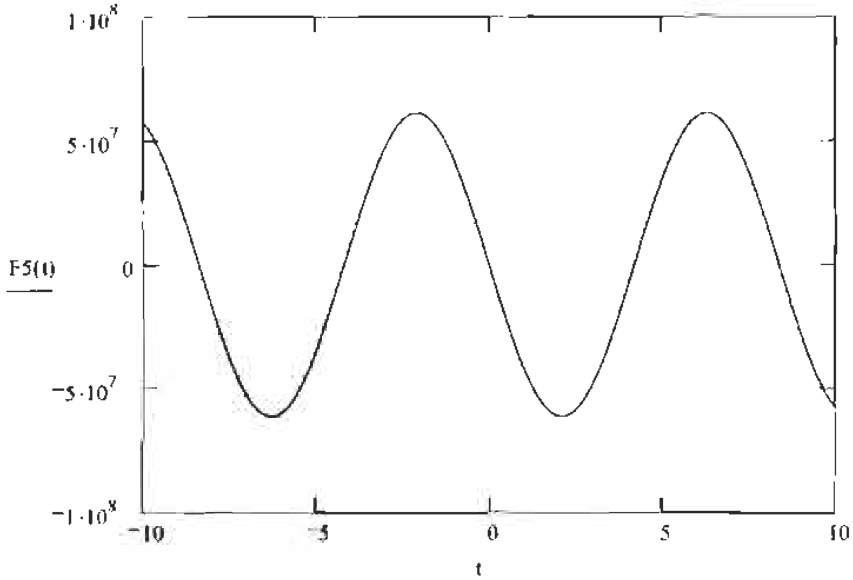
$$B(x) := \begin{cases} (x + 30) & \text{if } -30 \le x \le -10 \\ (20) & \text{if } -10 < x \le 10 \\ (-x + 30) & \text{if } 10 < x \le 30 \end{cases}$$

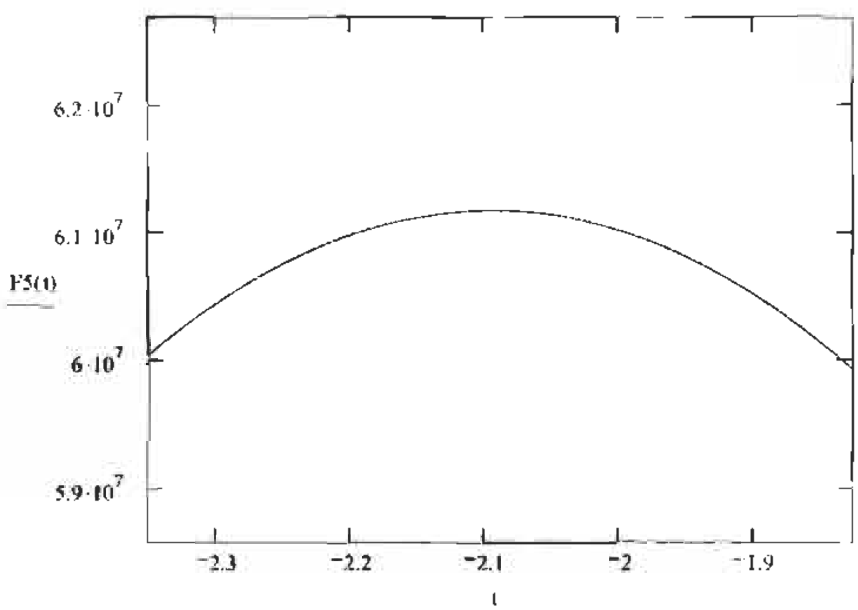
$$F3(t) := \rho \cdot g \cdot \left(\int_{30}^{30} B(x)\, \eta(x,t)\, dx \right)$$





$$F5(t) := \rho \cdot g \cdot \int_{-30}^{30} x \cdot B(x) \cdot \eta(x,t)\,dx$$





Based on visual observation, I will redefine F3(t) and F5(t) in terms of Fmax.

$$F3(0) = 5.901 \times 10^{6}$$

$$F3max := F3(0)$$

$$F3(t) := F3max \cdot Re\left[e^{(i\omega\_ot)}\right]$$

$$2 \cdot (F3,t) := F3max \cdot \left(\frac{1}{2} \cdot \exp(i \cdot \omega\_o \cdot t) + \frac{1}{2} \cdot \exp\left(\overline{i \cdot \omega\_o \cdot t}\right)\right) \cdot \pi \cdot \Delta(\omega)$$

$$F5(-3.14) = 4.331 \times 10^{7}$$

$$F5max := F5(-2.1)$$

$$F5(t) := F5max \cdot Re\left[e^{\left[i\left(\omega\_ot - \frac{\pi}{2}\right)\right]}\right]$$

$$2 \cdot (F5,t) := F5max \cdot \left[\frac{1}{2} \cdot \exp\left[i \cdot \left(\omega\_o \cdot t - \frac{\pi}{2}\right)\right] + \frac{1}{2} \cdot \exp\left(\overline{i \cdot \left(\omega\_o \cdot t - \frac{\pi}{2}\right)}\right)\right] \cdot \pi \cdot \Delta(\omega)$$

Part d) Now I need to find the transfer function between $\eta(t)$ (I'm letting x=0) and F3(t) and F5(t) respectively.

Copied from above, here are the Fourier Transforms of $\eta(x,t)$, F3(t) and F5(t).

$$2 \cdot (\eta,x,t) := A1 \cdot \left(\frac{1}{2} \cdot \exp(i \cdot \omega\_o \cdot t + i \cdot k \cdot x) + \frac{1}{2} \cdot \exp\left(\left(\overline{i \cdot \omega\_o \cdot t + i \cdot k \cdot x}\right)\right)\right) \cdot \pi \cdot \Delta(\omega)$$

Evaluated at x=0, the FT of $\eta(x,t)$ is:

$$2 \cdot (\eta,t) := A1 \cdot \left(\frac{1}{2} \cdot \exp(i \cdot \omega\_o \cdot t) + \frac{1}{2} \cdot \exp\left(\left(\overline{i \cdot \omega\_o \cdot t}\right)\right)\right) \cdot \pi \cdot \Delta(\omega)$$

$$2 \cdot (F3,t) := F3max \cdot \left(\frac{1}{2} \cdot \exp(i \cdot \omega\_o \cdot t) + \frac{1}{2} \cdot \exp\left(\overline{i \cdot \omega\_o \cdot t}\right)\right) \cdot \pi \cdot \Delta(\omega)$$

$$2 \cdot (F5,t) := F5max \cdot \left[\frac{1}{2} \cdot \exp\left[i \cdot \left(\omega\_o \cdot t - \frac{\pi}{2}\right)\right] + \frac{1}{2} \cdot \exp\left(\overline{i \cdot \left(\omega\_o \cdot t - \frac{\pi}{2}\right)}\right)\right] \cdot \pi \cdot \Delta(\omega)$$

The transfer function, $H(\omega)$, is found by dividing the output function in the frequency domain, $F5(\omega)$, by the input function in the frequency domain, $\eta(\omega)$. It is easily observed that the transfer function for each heave and pitch are constant values.

$$H31 := \frac{F3max}{A1} \qquad \text{and} \qquad H51 := \frac{F5max}{A1} \cdot e^{-i \cdot \frac{\pi}{2}}$$

where A1 is the amplitude of the input wave elevation, η. For a force per unit length, I will let A1=1 m.

$$H31 = 5.901 \times 10^{6} \qquad H51 = -6.118i \times 10^{7}$$

Part e) In this problem the overall system includes two linear systems in series. The first system has wave elevation as input and incident force as output. The transfer function for this system was determined in part d. The second linear system has the force as input and the response motion of the ship is the output. The transfer function for this system will be found in this part for the uncoupled heave and pitch equations of motion.

The transfer function between the external force and the ship's response motion is of the now familiar form:

$$\frac{1}{\left[-(m + A) \cdot \omega^2 + i \cdot \omega \cdot B + C\right]}$$

I need to define the mass values, M33 and M55. M33 is the ship's mass, which is equal to that of the displaced water. I calculated the displaced volume in my Excel file as 4916 m^3. M55 is given in the problem statement.

$V := 4916$ The units are m^3.

$M33 := \rho \cdot V \qquad M55 := 1.5 \cdot 10^{9}$ The units are kg*m^2.

$$H32(\omega) := \frac{1}{\left[-(M33 + A33) \cdot \omega^2 + i \cdot \omega \cdot B33 + C33\right]}$$

$$H32(\omega) \rightarrow \frac{1}{-8492507 \cdot \omega^2 + 1353808 \cdot i \cdot \omega + 7848000}$$

$$H52(\omega) := \frac{1}{\left[-(M55 + A55) \cdot \omega^2 + i \cdot \omega \cdot B55 + C55\right]}$$

$$H52(\omega) \rightarrow \frac{1}{-2102285755.0 \cdot \omega^2 + 337862269 \cdot i \cdot \omega + 1456709517}$$

Part g) Now I will transform the given wave spectrum, $S(\omega\_o)$, into a wave spectrum with respect to encounter frequency, $S(\omega\_e)$, using the equation given in the encounter frequency lecture. Then I will use the Weiner-Kinchine relations to determine and plot the spectra of heave and pitch responses.

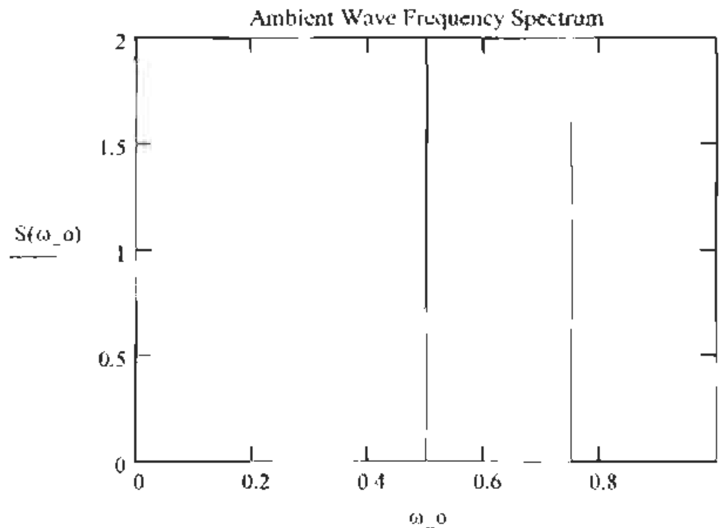
$\omega\_o := 0, 0.0001 .. 1$

Because the spectrum is comprised of unit impulses, I defined ω_o in very small steps. Otherwise it distorts the shape of the spectrum.

$$S(\omega\_o) := \begin{cases} 2 & \text{if } \omega\_o = 0.5 \\ 1.6 & \text{if } \omega\_o = 0.75 \\ 0 & \text{otherwise} \end{cases}$$

$S(\omega\_o)$ is the ambient wave spectrum given in the problem.

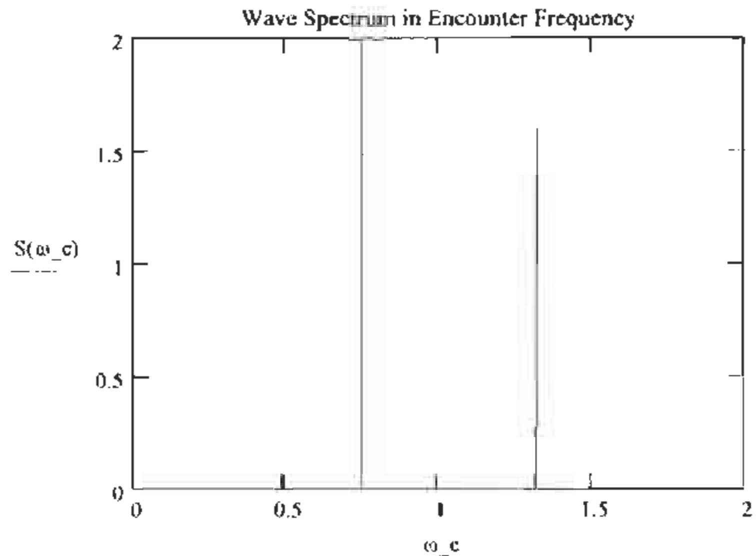
$S(0.5) = 2$ $S(0.75) = 1.6$



$\omega\_e := 0, 0.0001 .. 2$

Once again I defined ω_e in small intervals so as not to distort the shape of the spectrum.

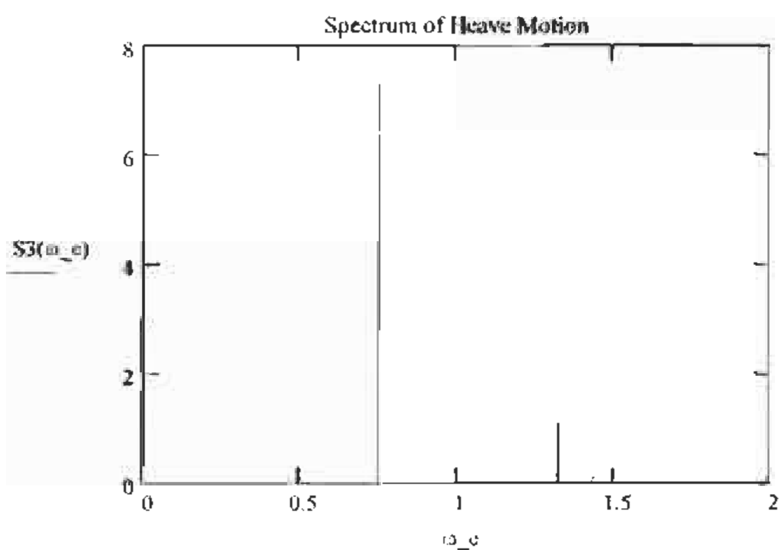
$$S(\omega\_e) := \begin{cases} 2 & \text{if } \omega\_e = 0.755 \\ 1.6 & \text{if } \omega\_e = 1.323 \\ 0 & \text{otherwise} \end{cases}$$



Apply the Wiener-Kinchine relations for heave:

$$H3(\omega\_e) := \begin{cases} (|H31|)^2 \cdot (|H32(0.755)|)^2 & \text{if } \omega\_e = 0.755 \\ (|H31b|)^2 \cdot (|H32b(1.323)|)^2 & \text{if } \omega\_e = 1.323 \\ 0 & \text{otherwise} \end{cases}$$

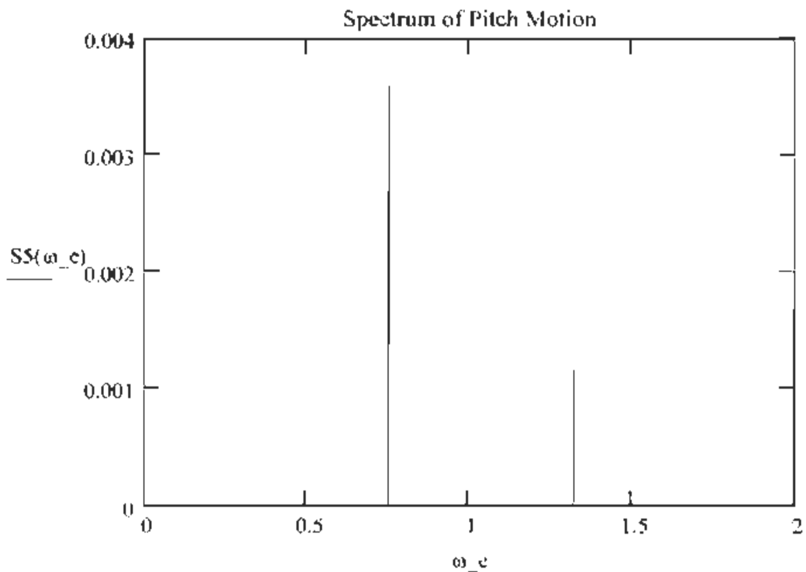
$$S3(\omega\_e) := S(\omega\_e) \cdot H3(\omega\_e)$$



Now apply the Weiner-Kinchine relations for pitch:

$$H5(\omega\_e) := \begin{cases} (|H51|)^2 \cdot (|H52(0.755)|)^2 & \text{if } \omega\_e = 0.755 \\ (|H51b|)^2 \cdot (|H52b(1.323)|)^2 & \text{if } \omega\_e = 1.323 \\ 0 & \text{otherwise} \end{cases}$$

$$S5(\omega\_e) := S(\omega\_e) \cdot H5(\omega\_e)$$



Problem 2)

a) SURGE EXCITATION FORCE:

$$dF_1 = \left(\rho \frac{\pi d^2}{4} + A_{11}\right) \frac{\partial u}{\partial t}\Big|_{x=0} dz$$

$$\frac{\partial u}{\partial t}(x=0, z, t) = \omega^2 A e^{kz} \sin \omega t$$

$$F_1(t) = \int_{-T}^{0} \left(\rho \frac{\pi d^2}{4} + A_{11}\right) \omega^2 A e^{kz} \sin \omega t \, dz$$

$$= \left(\rho \frac{\pi d^2}{4} + A_{11}\right) \omega^2 A \sin \omega t \left[\frac{1}{k}\left(1 - e^{-kT}\right)\right]$$

PITCH EXCITATION MOMENT:

$$F_5(t) = \int_{-T}^{0} (-z) dF_1 = \left(\rho \frac{\pi d^2}{4} + A_{11}\right) \omega^2 A \sin \omega t \int_{-T}^{0} (-z) e^{kz} dz$$

$$= -\left(\rho \frac{\pi d^2}{4} + A_{11}\right) \omega^2 A \sin \omega t \left[\frac{1}{k} z e^{kz} - \frac{1}{k^2} e^{kz}\right]_{-T}^{0}$$

$$= -\left(\rho \frac{\pi d^2}{4} + A_{11}\right) \omega^2 A \sin \omega t \left[-\frac{1}{k^2} + \frac{T}{k} e^{-kT} + \frac{1}{k^2} e^{-kT}\right]$$

b) $A_{11} = \int_{-T}^{0} a_{11} \, dz = \rho \frac{\pi d^2}{4} T$

$A_{33} = \frac{2}{3} \rho \frac{\pi d^3}{8}$ $\qquad A_{55} = \int_{-T}^{0} a_{11} z^2 dz = \rho \frac{\pi d^2}{4} \frac{T^3}{3}$

$A_{15} = A_{51} = \int_{-T}^{0} a_{11} z \, dz = \rho \frac{\pi d^2}{4} \frac{T^2}{2}$

$A_{31} = A_{13} = A_{35} = A_{53} = C_{11} = C_{13} = C_{31} = C_{15} = C_{51} = C_{35} = C_{53} = 0$

$C_{33} = \rho g A_{wp} = \rho g \frac{\pi d^2}{4}$

$C_{55} = \rho g \forall (z_B - z_G) + \rho g \iint_{A_{wp}} x^2 ds = \rho g \forall \left[-\frac{T}{2} - \left(-\frac{3T}{4}\right)\right] + \rho g \frac{\pi d^4}{64}$

Problem 2 cont.)

c) $$\left[\omega_N^{(3)}\right]^2 = \frac{C_{33}}{m + A_{33}} = \frac{\rho g \, \pi d^2/4}{m + 2/3\, \rho \pi d^3/8}$$

d) $$\sum_{j=1,3,5} \left[(M_{ij} + A_{ij}) \ddot{x}_j + B_{ij} \dot{x}_j + C_{ij} x_i \right] = F_i(t) \quad \text{for } i = 1, 3, 5$$

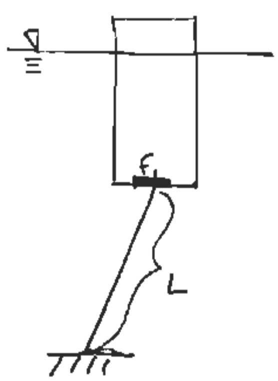
e) A_{ij} are the same as calculated in part b.

$$C_{13} = C_{31} = C_{35} = C_{53} = 0$$

$$C_{11} = k_{11} \qquad C_{15} = k_{15} \qquad C_{51} = k_{51}$$

$$C_{33} = k_{33} + \rho g \frac{\pi d^2}{4}$$

$$C_{55} = k_{55} + \rho g \forall z_B - m g z_G + \rho g \iint_{A_{wp}} x^2 \, ds$$

f) 

The restoring force due to the cable is $f_1 = k_{11} L\theta = P \sin\theta$

For small x_j's, $\sin\theta \simeq \theta$

so $k_{11} L \theta \simeq P\theta$

$$k_{11} \simeq \frac{P}{L}$$

$$\omega_{n_1}^2 = \frac{k_{11}}{m + A_{11}} = \frac{P}{L\left(m + \rho \frac{\pi d^2}{4} T\right)}$$