Problem 1)

See following pages for mathcad solution.

The transfer functions for vertical velocity and vertical acceleration were determined as follows.

VERTICAL ACCELERATION AT POINT B, BRIDGE:

In time Domain: $\ddot{x}_{3B} = \ddot{x}_3 - x_B \ddot{x}_5$ i.e. vertical acceleration = heave acceleration − (moment arm)(pitch acceleration)

$$\left\{\begin{array}{l}\text{Sign convention is based on: } \oplus \text{ is } \uparrow \text{ for heave and vertical } \left\{\begin{array}{l}\text{motion}\\ \text{accel.}\\ \text{velocity}\end{array}\right\} \\ \oplus \text{ is down by the bow for pitch } \left\{\begin{array}{l}\text{motion}\\ \text{accel.}\\ \text{velocity}\end{array}\right\} \\ \oplus \text{ is forward of midships for locations on the ship}\end{array}\right\}$$

Looking at this in the frequency domain you can say:

$$|\ddot{X}_{3B}|\cancel{e^{i\omega t}} = -\omega^2 (|X_3| - x_B|X_5|)\cancel{e^{i\omega t}}$$

where you know $|X_3| = H_{32}(\omega)|F_3|$ and $|F_3| = H_{31}(\omega)A1$

$|X_5| = H_{52}(\omega)|F_5|$ $\quad |F_5| = H_{51}(\omega)A1$

$\therefore \quad |X_3| = H_{32}(\omega)H_{31}(\omega)A1$ and $|X_5| = H_{52}(\omega)H_{51}(\omega)A1$

$$\therefore \quad |\ddot{X}_{3B}| = -\omega^2\left[H_{32}H_{31} - x_B H_{52}H_{51}\right]A1$$

LOOK AT IT AS A SINGLE LTI SYSTEM:

$$A1e^{i\omega t} \longrightarrow \boxed{\begin{array}{c}H_B(\omega)\\ \text{LTI}\end{array}} \longrightarrow |\ddot{X}_{3B}|e^{i\omega t}$$

$$H_B(\omega) = \frac{|\ddot{X}_{3B}|}{A1} = -\omega^2\left[H_{32}H_{31} - x_B H_{52}H_{51}\right]$$

Problem 1 cont.)

SIMILARLY FOR VERTICAL VELOCITY AT POINT A, FLIGHT DECK:
{USING THE SAME SIGN CONVENTION}

$$\dot{x}_{3A}(t) = \dot{x}_3(t) - x_A \dot{x}_5(t)$$

$$|\dot{X}_{3A}| e^{i\omega t} = i\omega |X_3| e^{i\omega t} - x_A i\omega |X_5| e^{i\omega t}$$

$$|\dot{X}_{3A}| = i\omega \left[|X_3| - x_A |X_5| \right]$$

$$= i\omega \left[H_{32} \cdot H_{31} - x_A H_{52} H_{51} \right] A1$$

LOOK AT VELOCITY AS A SINGLE LTI SYSTEM:

$$A1 e^{i\omega t} \longrightarrow \boxed{\begin{array}{c}\text{LTI} \\ H_A(\omega)\end{array}} \longrightarrow |\dot{X}_{3A}| e^{i\omega t}$$

$$H_A(\omega) = \frac{|\dot{X}_{3A}|}{A1} = i\omega \left[H_{32} \cdot H_{31} - x_A H_{52} H_{51} \right]$$

Problem 1) This problem, which builds on Problem 1 of HW8, asks you to compare the responses of the evaluated ship with three different design criteria. The design criteria include one motion, one velocity and one acceleration.

Part a) First check the design criteria that the RMS pitch angle is less than 2.5 degrees. You can calculate this using only the spectrum of pitch motion. By definition, RMS is the square root of the integral of a one-sided spectrum evaluated from 0 to infinity. By definition, an integral is the area under a curve, and for a discrete function like the pitch motion spectrum, this is equal to the sum of all of the discrete values. Therefore, the RMS pitch angle, in radians, is the squareroot of the sum of the two S5 values.

$RMS\_5 := \sqrt{S5(0.755) + S5(1.323)}$ $\quad RMS\_5 = 0.069$ Units here are radians.

$RMS\_pitch\_angle := RMS\_5 \cdot \frac{180}{\pi}$ $\quad RMS\_pitch\_angle = 3.955$ degrees

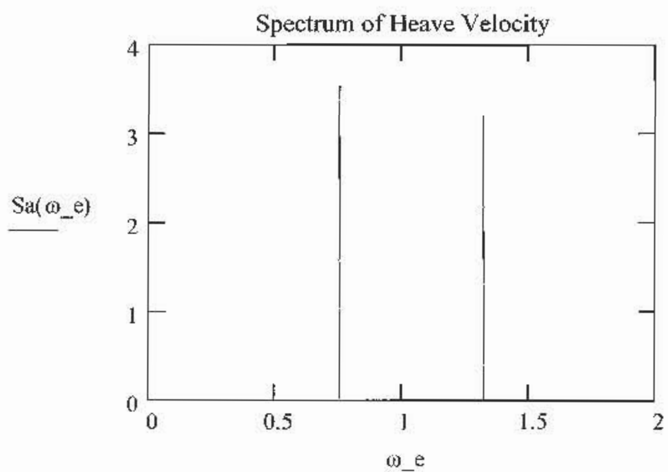
The ship fails the RMS pitch angle.

Next check the design criteria that the RMS vertical acceleration at the bridge is equal to or less than 0.2*g. There will be two components involved, the vertical acceleration due to heave acceleration and the vertical acceleration due to pitch acceleration evaluated 12 meters forward of midships.

$x\_B := 12$ Units are meters, where forward of midships is positive and aft is negative.

$$Ha(\omega\_e) := \begin{cases} -\omega\_e^2 \cdot (H32(\omega\_e) \cdot H31 - x\_B \cdot H52(\omega\_e) \cdot H51) & \text{if } \omega\_e = 0.755 \\ -\omega\_e^2 \cdot (H32(\omega\_e) \cdot H31b - x\_B \cdot H52(\omega\_e) \cdot H51b) & \text{if } \omega\_e = 1.323 \\ 0 & \text{otherwise} \end{cases}$$

$$Sa(\omega\_e) := S(\omega\_e) \cdot (|Ha(\omega\_e)|)^2$$



$RMS\_a := \sqrt{Sa(0.755) + Sa(1.323)}$

$RMS\_a = 2.593$ m/s

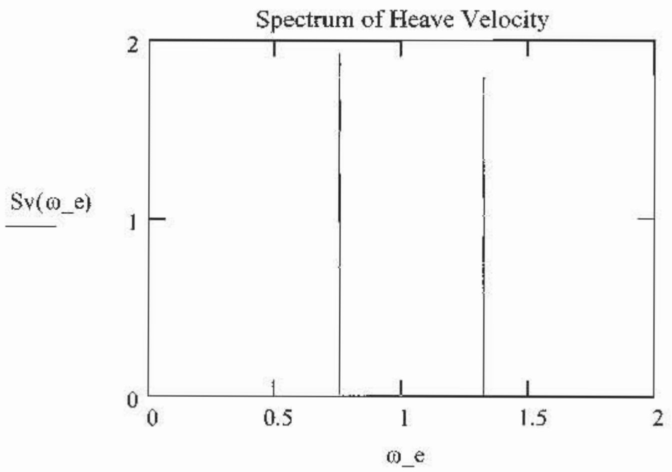
$0.2 \cdot g = 1.962$ m^2/s $\quad$ The ship fails the RMS vertical acceleration criterion.

Part b) Next check the design criteria that the RMS vertical velocity at the flight deck is equal to or less than 1.0 m/s. There will be two components involved, the vertical velocity due to heave velocity and the vertical velocity due to pitch velocity evaluated 22 meters aft of midships.

$x\_A := -22$ Units are meters, where forward of midships is positive and aft is negative.

$$Hv(\omega\_e) := \begin{cases} i \cdot \omega\_e \cdot \left(H32(\omega\_e) \cdot H31 - x\_A \cdot H52(\omega\_e) \cdot H51\right) & \text{if } \omega\_e = 0.755 \\ i \cdot \omega\_e \cdot \left(H32(\omega\_e) \cdot H31b - x\_A \cdot H52(\omega\_e) \cdot H51b\right) & \text{if } \omega\_e = 1.323 \\ 0 & \text{otherwise} \end{cases}$$

$$Sv(\omega\_e) := S(\omega\_e) \cdot \left(\left|Hv(\omega\_e)\right|\right)^2$$



$RMS\_v := \sqrt{Sv(0.755) + Sv(1.323)}$

$RMS\_v = 1.927$ m/s

The ship fails the RMS vertical velocity criterion.

Problem 2)

a) $\omega_N = \sqrt{\dfrac{k}{M + m_a}} = \sqrt{\dfrac{k}{M + \rho\frac{\pi d^2}{4}}}$

b) To FIND the Strouhal # off of the graph, first find the Reynold's #.

$$Re = \frac{Ud}{\nu} = \frac{(5)(0.1)}{10^{-6}} = 5 \times 10^5$$

From the graph, $St \cong 0.37$

$$St = \frac{f_v \cdot d}{U} \quad \therefore \quad f_v = \frac{St \cdot U}{d} = \frac{(0.37)(5\ m/s)}{0.1\ m} = 18.5\ s^{-1}$$

$$f_v = 18.5\ s^{-1}$$

c) AT LOCK-IN, The frequency of vortex shedding is equal to the natural frequency, f_n.

$$f_v = f_n = \frac{\omega_n}{2\pi} = \frac{1}{2\pi}\sqrt{\frac{k}{M + \rho\frac{\pi d^2}{4}}}$$

The frequency of vortex shedding is the same as the frequency of the cylinder response. $f_v = f_{cyl}$ so $f_{cyl} = f_n$

The frequency of the unsteady drag force is twice that of the frequency of vortex shedding

so $f_{drag} = 2 f_n$

Problem 3) Deep Water:

$\rho := 1000 \frac{kg}{m^3}$ Cm := 2.0 Cd := 1.0 $g = 9.807 \frac{m}{s^2}$ $i := \sqrt{-1}$ $\omega := 2.0 \frac{rad}{s}$

$d := 1.8 \cdot m$ $a := 1.1 \cdot m$ $h := 2 \cdot a$ $H := 50 \cdot m$ $k := \frac{\omega^2}{g}$ $k = 0.408 \frac{1}{m}$

$\eta(t) := a \cdot \cos(\omega \cdot t)$

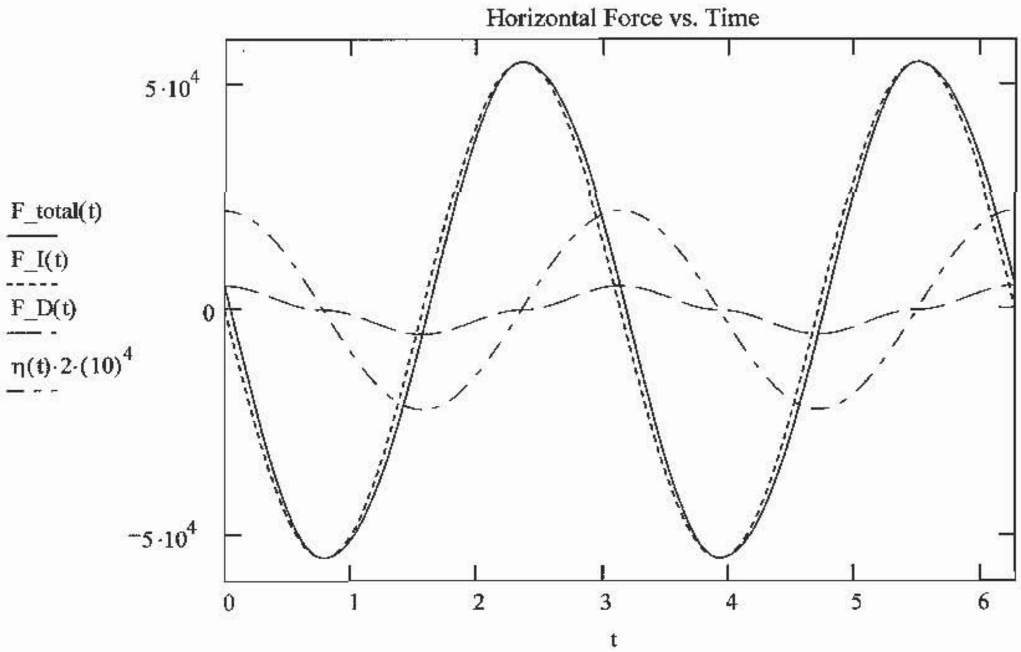
Maximum horizontal force will occur at the water's surface evaluated at x=0:

At x = 0 and z = 0: $u(t) := a \cdot \omega \cdot \cos(\omega \cdot t)$ $u\_dot(t) := -a \cdot \omega^2 \cdot \sin(\omega \cdot t)$

$F\_I(t) := -\rho \cdot Cm \cdot \frac{\pi}{4} \cdot d^2 \cdot \frac{h}{2 \cdot k} \cdot \omega^2 \cdot \sin(\omega \cdot t)$ Formula (3.8) in the Morrison's notes.

$$F\_D(t) := \frac{1}{2} \cdot \rho \cdot Cd \cdot d \cdot \frac{h^2 \cdot \omega^2}{4} \cdot \frac{\frac{\sinh(2 \cdot k \cdot H)}{4} + \frac{k \cdot H}{2}}{k \cdot (\sinh(k \cdot H))^2} \cdot \cos(\omega \cdot t) \cdot |\cos(\omega \cdot t)|$$ Formula (3.10).

$F\_total(t) := F\_I(t) + F\_D(t)$



From this plot, you can see that the inertial force component dominates the drag force component in the total force. You can see that the maximum total forces occur when the drag force is zero and inertial force is at the maximum. Wave elevation is included magnified by a factor of 2*10^4 so that it shows up on the same graph.

a) $F\_total(0.75\cdot\pi\cdot s) = 5.49 \times 10^{4}\,N$ This is maximum total force.

b) $T := \frac{2\cdot\pi}{\omega}$ $T = 3.142\,s$ $L := 0.75\cdot\pi\cdot s$ $F\_total(L) = 5.49 \times 10^{4}\,N$

The horizontal force is maximum at 3π/4 + π*n and it is minimum at 3π/4 + π*(n-1/2).

c) When you look at total, inertial or drag force over one period, they are all about zero because they are sinusoidal functions.

$$F\_I\_period := \int_0^T F\_I(t)\,dt \qquad F\_I\_period = -5.84 \times 10^{-12}\,\frac{kg\,m}{s}$$

$$F\_D\_period := \int_0^T F\_D(t)\,dt \qquad F\_D\_period = -8.795 \times 10^{-13}\,\frac{kg\,m}{s}$$

$$F\_total\_period := \int_0^T F\_total(t)\,dt \qquad F\_total\_period = 0\,\frac{kg\,m}{s}$$

When you look at the absolute value of total, inertial or drag force over one period, you can see that the inertial force is an order of magnitude larger than drag, so it dominates overall.

$$F\_I\_period\_abs := \int_0^T (|F\_I(t)|)\,dt \qquad F\_I\_period\_abs = 1.098 \times 10^{5}\,\frac{kg\,m}{s}$$

$$F\_D\_period\_abs := \int_0^T (|F\_D(t)|)\,dt \qquad F\_D\_period\_abs = 8.388 \times 10^{3}\,\frac{kg\,m}{s}$$

$$F\_total\_period\_abs := \int_0^T |F\_total(t)|\,dt \qquad F\_total\_period\_abs = 1.103 \times 10^{5}\,\frac{kg\,m}{s}$$

d) Find the moment of forces about the bottom of the cylinder.

$$dF\_over\_dz(z,t) := \left(-\rho\cdot Cm\cdot\frac{\pi\cdot d^2}{4}\cdot e^{k\cdot z}\cdot a\cdot\omega^2\cdot\sin(\omega\cdot t) + Cd\cdot\rho\cdot\frac{1}{2}\cdot d\,a^2\cdot\omega^2\cdot e^{2\cdot k\cdot z}\cdot\cos(\omega\cdot t)\cdot|\cos(\omega\cdot t)|\right)$$

$$M\_seafloor(t) := \int_{-50\cdot m}^{0\cdot m} (H + z)\cdot dF\_over\_dz(z,t)\,dz$$

e) $Z\_cf(t) := \frac{M\_seafloor(t)}{F\_total(t)}$ $Z\_cf(0.75\cdot\pi\cdot s) = 47.548\,m$ Above the seafloor.