

13.42 Exam #2

Spring 2004

Open Book

- 1) (30%) A buoy used in the local sailing regattas on Boston harbor is left out over night. At high tide, it is fully submerged due to the short mooring cable length attached to it.

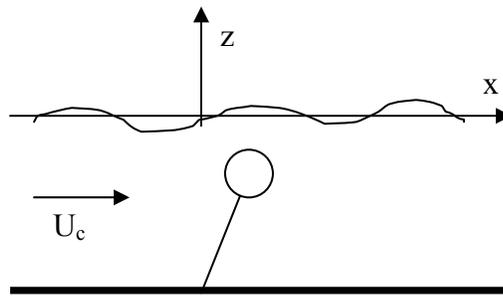
The buoy can be modeled in two dimensions, as a positively buoyant circle of radius r , and density, ρ_b , connected to the sea bottom by an inextensible mooring cable of length l . The cable length, at high tides, is such that the circle is fully submerged with its center a distance d below the average free surface elevation.

In this region a constant current is prevalent, with speed, U , oriented in the positive x -direction, in addition to regular surface waves (propagating in the x -direction). You can assume for this problem that the surge added mass and damping coefficients and the excitation force are the same functions of frequency as in the absence of the current.

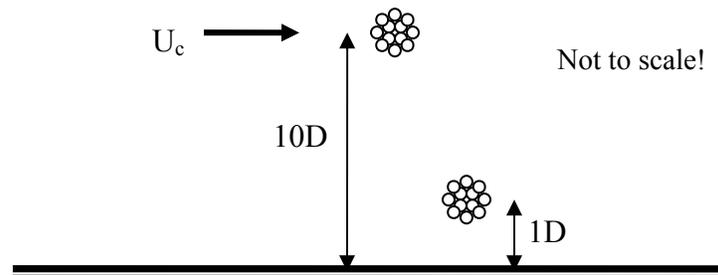
- Give the **equation of motion of the circle in surge** in the presence of the current and linear gravity waves incident on the buoy in the positive x -direction. Neglect viscous effects.
- Determine the **natural frequency of the buoy in surge**. Is your answer different for the case when $U = 0$?
- Given waves incident on the buoy in the positive x -direction, in addition to the current, **determine the wavelength(s) that will excite “resonance” in the surge motion**. How does your answer change for waves propagating in the negative x -direction (against the current)?

Extra Credit (5 points)

- If the mean drift (steady drag) force acting on the circle, due to incident waves in the absence of a current, is some function $D(\omega_0)$, what is the drift force acting on the buoy when the current is non-zero in the positive x -direction? (neglect viscous effects)



- 2) (20%) A 10cm diameter bundle of underwater fiber optic cable is installed across the channel where a strong current, with magnitude U , is prevalent. This cable is anchored 10 diameters above the sea floor, where there is little effect on the cable behavior due to the presence of the sea floor.
- Modeling the cable as a *rigid* cylinder free to move in the inline and transverse directions, determine the frequency of the inline and transverse cable vibrations, assuming that the cable bundle can be modeled as a cylinder with a rough surface, for a velocity of 0.5 m/s incident perpendicular to the long axis of the cable.
 - How do you anticipate your answer would change, over a range of inflow velocities, if the cable was modeled as a flexible cylinder pinned at both ends.
 - If the cable was mounted 1 diameter from the sea floor how do you expect your answers in parts *a* and *b* would change? Rationalize your answers physically or mathematically.



- 3) (40%) SHIP MODEL TESTING: You have been asked to evaluate the seakeeping characteristics of a model ship in the MIT towing tank. The ship model mass is **2500 kg** and the hydrostatic restoring coefficient is $c_{33} = 1.4 \times 10^4 \text{ N/m}$.

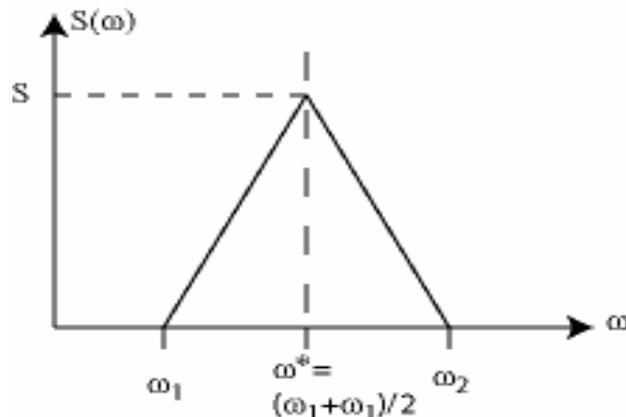
You are given results from two sets of basic tests with a stationary ship model located a distance of **2.4 meters** downstream from a wave probe used to measure the incident wave elevation. The tank is 5 meters deep. The first set of experiments, diffraction tests, were performed with waves of constant amplitude ($a = 5 \text{ cm}$) and varying frequency with a rigidly fixed model. The second set of experiments, radiation tests, involved the vessel being forced to move in heave at constant amplitude ($|x_3| = 5 \text{ cm}$) over the same range of frequencies. For the specific case of $\omega = 3.7 \text{ rad/s}$ you are given the following results for the two types of tests:

- Diffraction test force in heave: $|F_{3d}| = 1200 \text{ N}$ with a phase lag of 215° from the **wave probe** measurements.
- Radiation test force in heave: $|F_{3r}| = 1000 \text{ N}$ with a phase lag of 160° from the **imposed motion**.

Determine:

- The added mass and damping of the model in heave.
- The natural frequency of the model in heave.
- The heave response of the model (amplitude and phase) to waves of amplitude $a = 10 \text{ cm}$ and frequency $\omega = 3.7 \text{ rad/sec}$.

Next you are asked to determine the seakeeping characteristics of the vessel advancing at a forward speed U in a given sea state. The ship has length L and the ratio of full scale ship to model ship lengths is **50:1**. The full scale spectrum for unidirectional incident waves is given by the function plotted below:



Continued on next page...

Where ω_1 and ω_2 , and S are dimensional and measured relative to a fixed reference frame in space.

- d. **Determine the shape of the wave spectrum you want to generate for tank tests with your model.** Consider carefully the non-dimensional parameters you need to scale your tests. You will need to scale the testing speed and the physical characteristics of the waves.
- e. A sensor placed on your vessel indicates that the standard deviation of the heave velocity is some value α . **Determine the corresponding value for the standard deviation of heave of the full scale vessel.**

Non-dimensional parameters that *might* come in handy:

Reynolds number	$R = UL/\nu$
Froude Number	$F = U/\sqrt{gL}$
Strouhal Number	$S = fL/U$

- 4) (10%) A ship is cruising at forward speed, U , with quartering seas incident to the stern of the ship at 30° . The waves have an absolute frequency, ω_o .
 - a. Determine the combination of U and ω_o such that the ship is stationary relative to the ambient wave crests.

Extra Credit: (10 points)

- b. If the roll resonance period is T_n , determine the range over which U can vary to ensure that only one ambient wave frequency, ω_o , excites resonance in roll.

