13.42 Design Principles for Ocean Vehicles

Homework #1: Linear Waves
Out: Thursday, February 5, 2004
Due: Thursday, February 12, 2004

(Assume density of water $\rho = 1000\ kg/m^3$ and gravity $g = 10\ m/s^2$.)

Problem 1:

a. State the necessary conditions for IDEAL flow.

b. Give two physical examples each, of a streamline, pathline and streakline.

Problem 2:

Dispersion Relationship

a. Write the dispersion relationship for linear free surface gravity waves.

b. In the limit where kH >> 1, how does this relationship simplify?

c. Discuss the significance of the dispersion relationship and give a physical example of why it is important.

Problem 3:

For designing ocean systems we need to understand when extreme environmental conditions occur. For linear free-surface gravity waves, the velocities and dynamic pressure varies harmonically in space and time.

a. Given a free surface elevation, $\eta(x,t) = a\sin(kx - \omega t)$, with wavelength, $\lambda = 24\ m$, amplitude $a = 1m$, ***non-dimensionalize and plot*** the following quantities for a fixed point in space (i.e. fixed x position @ $x = x_o$ and fixed depth $z = z_n$) but varying time (the x-axis should be non-dimensional time and y-axis(or axes) should be the non-dimensional quantities being plotted):

$$\eta(x_o,t),\ u(x_o,t),\ w(x_o,t),\ \text{and}\ p_d(x_o,t)$$

Choose the depth such that $z_n = n\ \lambda/36$ (meters), where n is the integer corresponding to the month you were born in.

Line up the plots such that you can deduce the relative phase of the variables. You can plot them on one plot or multiple subplots – just make sure to label the different curves appropriately.

b. Plot the same quantities in part a, but with x varying and at one instant in time (i.e. @ $t=t_o$). The x-axis should be non-dimensional position in space and y-axis(or axes) should be the non-dimensional quantities being plotted.

c. At the depth determined by your birth month, what is the *total* pressure under the wave crest, the wave trough, the wave nodal point (the nodal point is the point at which the wave elevation corresponds to z = 0)?

d. The added mass force on a body is proportional to the fluid acceleration. Determine when the horizontal added mass force is maximum, minimum, and zero relative to the wave elevation (i.e. at the crest, the trough, or a nodal point).

e. While we are neglecting viscous forces for the most part in this course they are still important in certain applications. In general the non-linear, viscous wave forces are proportional to the square of the fluid velocity. When is the *horizontal* viscous wave force maximum – under the wave crest, the wave trough or the wave nodal point? When is the *vertical* viscous wave force maximum?

Problem 4:

A series of monochromatic (single-frequency) linear free surface gravity waves, with amplitude, a, and frequency, ω, is propagating over a spherical instrument dome, with diameter 0.6 meters, that is fixed to the seafloor by a rigid mount (see figure 1).

a) The instrument dome is fixed halfway from the surface to the bottom. It is fitted with a pressure transducer to measure the dynamic pressure. The signal being sent to an observation station has the form $P(t) = p_o \cos(wt)$ where p_o = 19000 N/m^2 and ω = 1 rad/s. Determine the wavelength and amplitude of these waves.

b) Use the information found in part (a) to determine the average kinetic, potential and total energy per wavelength of these waves.

c) Determine the added mass force on the buoy in the vertical direction as a function of time.

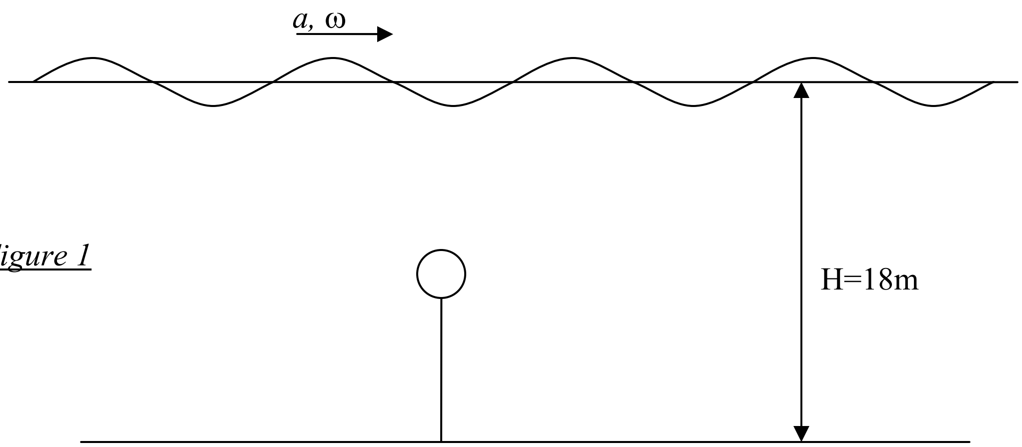

Figure 1

Problem 5:

a. Use the Euler equation to derive Bernoulli's equation for Ideal flow. State your assumptions. Explain the physical significance of each term in your final equation.

b. Linearize Bernoulli's equation for linear free surface waves. Justify your linearization by comparing the relative magnitude of the various terms.

Problem 6:

A wave maker generates a packet of monochromatic, linear free surface waves with amplitude, $a = 0.25$m, and wavelength, $\lambda = 4$ m, in a still tank. The waves propagate towards a group of 3 students dangling their feet in the water. The students, student A, student B and student C, are spaced along the tank at 5meters, 10meters and 15 meters away from the wave maker, respectively.

a. Assuming deep water, how long does it take, from the time the initial wave was generated, until each student, A, B, & C, feels the presence of the waves?

b. Can the students use the information about the time it takes the waves to reach their feet to discern anything about the velocity of the wave packet? What about the velocity of the wave crests?

c. If not, could they reposition themselves along the tank in order to determine the velocity of the wave packet and the wave crests? Given that student A is not willing to move, where would students B & C have to in order to make these determinations?