13.42 HW #6 Waves Mini-Project
Out: 3/18/04
Due: 4/1/04

Part I: The goal of this exercise is to analyze the Bretschneider spectrum, a continuous spectrum, using Matlab or another computational tool. Using the Bretschneider spectrum given in class:

$$S^{+}(\omega) = \frac{1.25}{4}\frac{\omega_m^4}{\omega^5}\zeta^2 e^{\left\{-1.25\left(\frac{\omega_m}{\omega}\right)^4\right\}}$$

Where the significant wave height is ζ and ω_m is the modal frequency of the spectrum.

1. Plot the Bretschneider spectrum for four significant wave heights between 1 and 3 meters for *fully developed seas*.

2. Consider one significant wave height chosen in part a.
   a. Compare the spectrum in part 1 with the JONSWAP spectrum for the same significant wave height.

   b. Compare the B-S spectrum for developing, fully-developed, decaying sea states. Note that the spectrum shape can be altered by changing the modal frequency.

3. Using one case from part a:

   a. Calculate the moments of the spectrum using numerical integration (like trapezoidal integration or similar)

   b. Determine the spectrum bandwidth, ε.

   c. Calculate the 1/Nth highest wave height where N = 10, 50, and 100.

Part II: A ship is being designed to operate in the conditions you have analyzed above. The transfer function between the incoming waves and heave motion (heave transfer function) $H(\omega)$ is given by

$$H(\omega) = \frac{\omega_n^2}{-\omega^2 + 2i\beta\omega_n\,\omega + \omega_n^2}$$

where ω_n is the structure's natural frequency (rad/s) and β is the structural damping ratio.

a) Plot $\left|H(\omega)\right|^2 = H(\omega) \cdot H^*(\omega)$

b) Determine the spectrum of the heave response. Note that $\left|H(\omega)\right|^2 = H(\omega) \cdot H^*(\omega)$. Plot this spectrum. Use a structural damping ratio $\beta = 0.3$ and a $\omega_n = 1.0$ rad/s.

c) How will the response change if the structural frequency is different – illustrate your explanation with plots of the response at a higher or lower natural frequency.