13.42 HW #7 SOL'Ns

1a. $\omega_n = \sqrt{\dfrac{k}{M + m_a}} = \sqrt{\dfrac{k}{M + \rho \frac{\pi d^2}{4}}}$

b. • FIRST FIND Re

$$Re = \frac{Ud}{\nu} = \frac{(10)(0.1)}{10^{-6}} = 10^6$$

• FROM "STROUHAL NUMBER vs. REYNOLDS NUMBER" IN READING 10, THE CORRESPONDING STROUHAL # FOR A SMOOTH CYLINDER IS:

$$St \cong 0.43$$

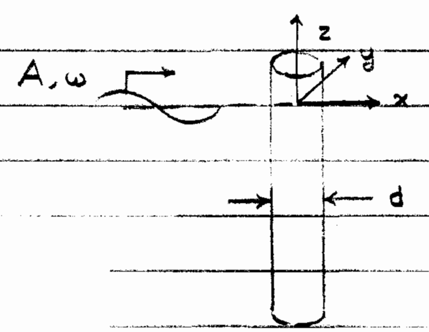
• $St = \dfrac{f_v d}{U} \Longrightarrow f_v = \dfrac{St \cdot U}{d} = \dfrac{0.43(10)}{0.1} = 43\ s^{-1}$.

c. • $f_v \sim f_n = \dfrac{\omega_n}{2\pi} = \dfrac{1}{2\pi}\sqrt{\dfrac{k}{M + \rho \frac{\pi d^2}{4}}}$

• $f_c \sim f_n$ "

• $f_{DRAG} \sim 2 f_n$

2.



• $\zeta(x,t) = A\,\mathbb{R}e\left\{ e^{-ikx + i\omega t} \right\}$

• $\phi(x,z,t) = \mathbb{R}e\left\{ \dfrac{igA}{\omega} e^{kz - ikx + i\omega t} \right\}$

a. GIVEN $\lambda \gg d$ & $A \sim 2d$, or $\frac{h}{d} \sim 4 \Longrightarrow$ USE MORISON EQ.:

$$dF_x(x=0,z,t) = \left(\rho \frac{\pi d^2}{4} + m_a\right)\frac{\partial u}{\partial t}\, dz + \frac{1}{2}\rho C_D\, d\, u|u|\, dz$$

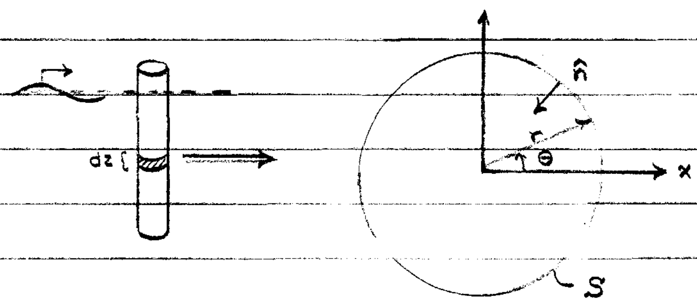
WHERE $m_a = \rho \frac{\pi d^2}{4}$ AND $u = \frac{\partial \phi}{\partial x}$.

b. THE FROUDE-KRYLOV TERM FROM a IS

$$\rho \frac{\pi d^2}{4} \frac{\partial u}{\partial t} dz \Big|_{x=0} \qquad (1)$$

... ZONTAL FORCE DUE TO UNSTEADY PRESSURE INDUCED BY THE UNDISTURBED AMBIENT WAVES IS AS FOLLOWS:

- $p = -\rho \frac{\partial \phi}{\partial t} = \mathrm{Re}\left\{ \rho g A e^{kz - ikx + i\omega t} \right\}$



- $dF_x = \iint_S p \hat{n} \cdot \hat{\imath} \, ds$

$$= \int_0^{2\pi} \mathrm{Re}\left\{ \rho g A e^{kz} e^{-ikr\cos\theta + i\omega t} \right\} \overbrace{(-\cos\theta)}^{\hat{n}\cdot\hat{\imath}} dz \, r \, d\theta \Big|_{r = d/2}$$

$$= -\rho g A e^{kz} \frac{d}{2} dz \int_0^{2\pi} \cos\left(\omega t - k\frac{d}{2}\cos\theta\right) \cos\theta \, d\theta$$

$$= -\rho g A e^{kz} \frac{d}{2} dz \int_0^{2\pi} \Big[\cos\omega t \cdot \cos\left(\frac{kd}{2}\cos\theta\right) +$$

$$+ \sin\omega t \sin\left(\frac{kd}{2}\cos\theta\right) \Big] \cos\theta \, d\theta$$

- IF $\lambda \gg d$, THEN $kd \ll 1$ AND $\cos\left(\frac{kd}{2}\cos\theta\right) \longrightarrow 1$

$$\sin\left(\frac{kd}{2}\cos\theta\right) \longrightarrow \frac{kd}{2}\cos\theta$$

• THEN

$$dF_x \simeq -\rho g A e^{kz}\, \tfrac{d}{2}\, dz \int_0^{2\pi} \left(\cos\omega t \cos\theta - \sin\omega t\, \frac{kd}{2} \cos^2\theta \right) d\theta$$

$$= -\rho g A e^{kz}\, \tfrac{d}{2}\, dz \cdot \left(\frac{kd}{2} \pi \sin\omega t \right)$$

• NOTE THAT

DISP. REL.

$$\left.\frac{\partial u}{\partial t}\right|_{x=0} = \frac{\partial^2 \phi}{\partial x \partial t} = -A\omega^2 e^{kz} \sin\omega t = -Agke^{kz} \sin\omega t$$

$$\therefore\ dF_x \simeq \rho \frac{\pi d^2}{4} \left.\frac{\partial u}{\partial t}\right|_{x=0} dz \quad ✓$$

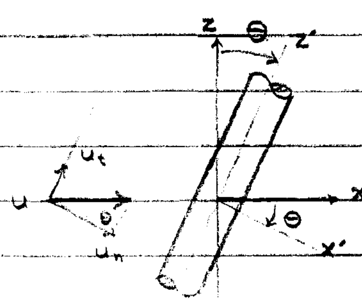
c. THE ADDED MASS TERM REPRESENTS THE DIFFRACTION FORCE.

d. THE PRESENCE OF A STRUCTURE WITH DIAMETER $\sim O(\lambda)$ WILL SIGNIFICANTLY DISTURB THE AMBIENT WAVE FIELD.

e. DEEP WATER: $\phi = \mathrm{Re}\left\{ \frac{igA}{\omega} e^{kz - ikx + i\omega t} \right\}$

$$u = \frac{\partial \phi}{\partial x} = \mathrm{Re}\left\{ \omega A e^{kz - ikx + i\omega t} \right\}$$

SEPARATION PRINCIPLE (NEGLECTING VERTICAL VELOCITIES)



· $\theta = 15°$

$u_n = u \cos\theta$

$u_t = u \sin\theta$

- $dF_n = (\rho \nabla + m_a) \frac{\partial u_n}{\partial t} dz' + \frac{1}{2} \rho C_D d |u_n| u_n dz'$

- $dF_t = \frac{1}{2} \rho C_F (\pi d) u_t |u_t| dz'$

$$dz' = dz \cos \Theta$$

LIFT: $dF_L = -dF_n \sin \Theta + dF_t \cos \Theta$

DRAG: $dF_D = dF_n \cos \Theta + dF_t \sin \Theta$

b. FOR A VERTICAL CYLINDER, THE DRAG FORCE WILL BE ENTIRELY DUE TO THE NORMAL COMPONENT.

— i.e. $dF_D = dF_n = (\rho \nabla + m_a) \frac{\partial u}{\partial t} dz + \frac{1}{2} \rho C_D d |u| u \, dz$

4a. $|F_x| \sim |F_I| = 5.6 \times 10^4$ N.

b. $F_x(t) = \underset{\text{INERTIA}}{F_I(t)} + \underset{\text{VISCOUS DRAG TERM}}{F_V(t)}$

- $|F_I| \gg |F_V|$

- SINCE $F_I \sim \frac{\partial u}{\partial t}$, WHICH IS 90° OUT-OF-PHASE WITH $\eta(t)$, THE MAXIMA OCCUR WHEN $\eta = 0$.

d. $M_{SEAFLOOR} = \int_{-H}^{0} (H + z) \, dF_x$ WHERE

$$dF_x = (\rho \nabla + m_a) \frac{\partial u}{\partial t} dz + \frac{1}{2} \rho C_D d |u| u \, dz$$

e. $$z_{cp} = \frac{\int_{-H}^{0} z \, dF_x}{\int_{-H}^{0} dF_x}$$

RELATIVE TO THE SEAFLOOR, $H + z_{cp}$.